Modal Testing using the Slinky™ Method

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Topics of this Presentation

- TRN Chain testing *makes ODS & Modal testing easier*
- TRN Chain testing *only requires a pair of response sensors*
- TRN Chain testing *does not require a (long) wire* from one of the sensors to the acquisition system
- A Slinky™ test *minimizes* the number of *sensor moves*
- A TRN Chain can be *“seeded”* with an *Auto spectrum, Cross spectrum, Fourier spectrum, or FRF* resulting in a *single-reference set* of those measurements
- One *Uni-axial* & one *Tri-axial* accelerometer and a *4-channel* acquisition system will yield *3D ODS’s & Mode Shapes*

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ODS's & Mode Shapes from a Set of Single-Reference Measurements

- A frequency-based ODS is obtained at any frequency in a set of single-reference ODS FRFs, Cross Spectra, Fourier spectra, or FRFs
- Experimental mode shapes are obtained by curve fitting a set of these single-reference measurements
- A set of these single-reference measurements can be calculated in two ways,

  1. Roving Force Test: A response sensor is fixed and an excitation force is applied at different DOFs (points & directions) between data acquisitions
  2. Roving Response Test: The excitation is fixed and one or more response sensors are moved between acquisitions

- A set of single-reference measurements requires that either the response of the excitation sensor remain fixed during the test
- Limitation of Single-Reference Testing: Since one sensor must remain fixed, a long wire may be required to connect it to the acquisition system

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Difference Between an FRF & a Transmissibility

FRF: The Fourier spectrum of a structural response *divided by* the Fourier spectrum of the force that caused the response

FRF: The magnitude of the response *divided by* the magnitude of the force, together with the phase angle between the response & force

$$FRF(2{:}1) = {}^{FFT(2)}\!/_{FFT(1)} = {}^{Accel}\!/_{Force}$$

Transmissibility: The Fourier spectrum of a response *divided by* the Fourier spectrum of a response at a different DOF (point & direction)

Transmissibility: The magnitude of a response *divided by* the magnitude of another response, together with the phase angle between the two responses

$$TRN(2{:}1) = {}^{FFT(2)}\!/_{FFT(1)} = {}^{Accel}\!/_{Accel}$$

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Difference Between an FRF & a Transmissibility

FRF: Requires *simultaneous acquisition* of all excitation forces and a response caused by the excitation forces

FRF: Requires a 2-channel acquisition system

$$FRF(2:1) = {FFT(2)}/{FFT(1)} = {Accel}/{Force}$$

Transmissibility: Requires *simultaneous acquisition* of two responses

Transmissibility: Requires a 2-channel acquisition system

$$TRN(2:1) = {FFT(2)}/{FFT(1)} = {Accel}/{Accel}$$

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Unique Transmissibility Properties

The Product of two Transmissibilitys is another Transmissibility

$$TRN(3:1) = TRN(3:2)\ x\ TRN(2:1)$$

The Inverse of $TRN(2:1)$ is $TRN(1:2)$

$$TRN(1:2) = \left({}^{1}/_{TRN(2:1)}\right)$$

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TRN (Transmissibility) Chain

Each Transmissibility must contain *one DOF*
that is also contained in another Transmissibility

***TRN* (2:1)**
***TRN* (3:2)**
***TRN* (4:3)**
***TRN* (5:4)**
***TRN* (6:4)**

Single-Reference TRN Chain

A single-reference TRN Chain is created by ***recursive multiplication*** **of Transmissibility's**

$$TRN\ (3{:}1) = TRN\ (3{:}2)\ x\ TRN\ (2{:}1)$$

$$TRN\ (4{:}1) = TRN\ (4{:}3)\ x\ TRN\ (3{:}1)$$

$$TRN\ (5{:}1) = TRN\ (5{:}4)\ x\ TRN\ (4{:}1)$$

$$TRN\ (6{:}1) = TRN\ (6{:}5)\ x\ TRN\ (5{:}1)$$

$$TRN\ (7{:}1) = TRN\ (7{:}2)\ x\ TRN\ (2{:}1)$$

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Benefits of a TRN Chain

A TRN Chain is measured using ***two response sensors*** and a ***multi-channel acquisition*** system

- Two ***uni-axial*** sensors and a ***2-channel*** system
- One ***uni-axial*** and one ***tri-axia***l sensor and a ***4-channel*** system
- Two ***tri-axial*** sensors and a ***6-channel*** system

***TRN* (2:1)**
***TRN* (3:2)**
***TRN* (4:3)**
***TRN* (5:4)**
***TRN* (6:4)**

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Benefits of a TRN Chain

- Excitation Forces are ***not acquired***
- Excitation Force levels ***can change*** between acquisitions
- A TRN chain can be ***"seeded"*** with an Auto spectrum, Cross spectrum, Fourier spectrum or FRF to yield a set of ***single-reference measurements***

***TRN* (2:1)**
***TRN* (3:1)**
***TRN* (4:1)**
***TRN* (5:1)**
***TRN* (6:1)**

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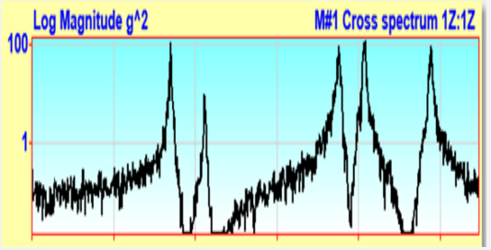
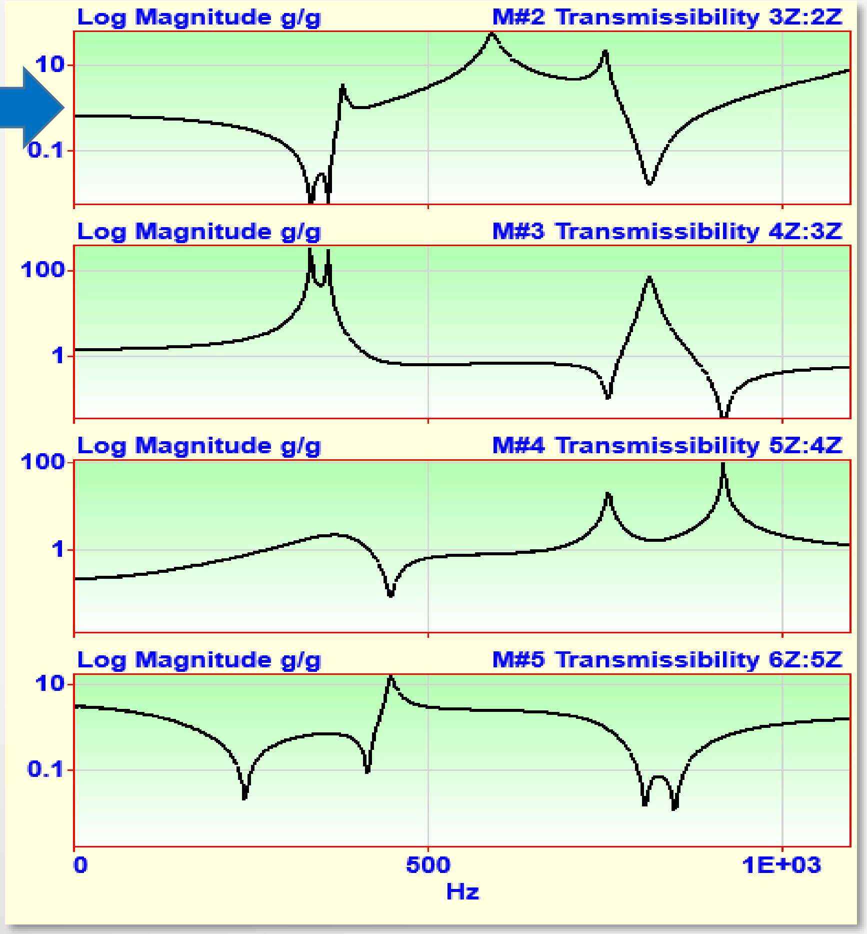
Seeding a TRN Chain with a Cross Spectrum

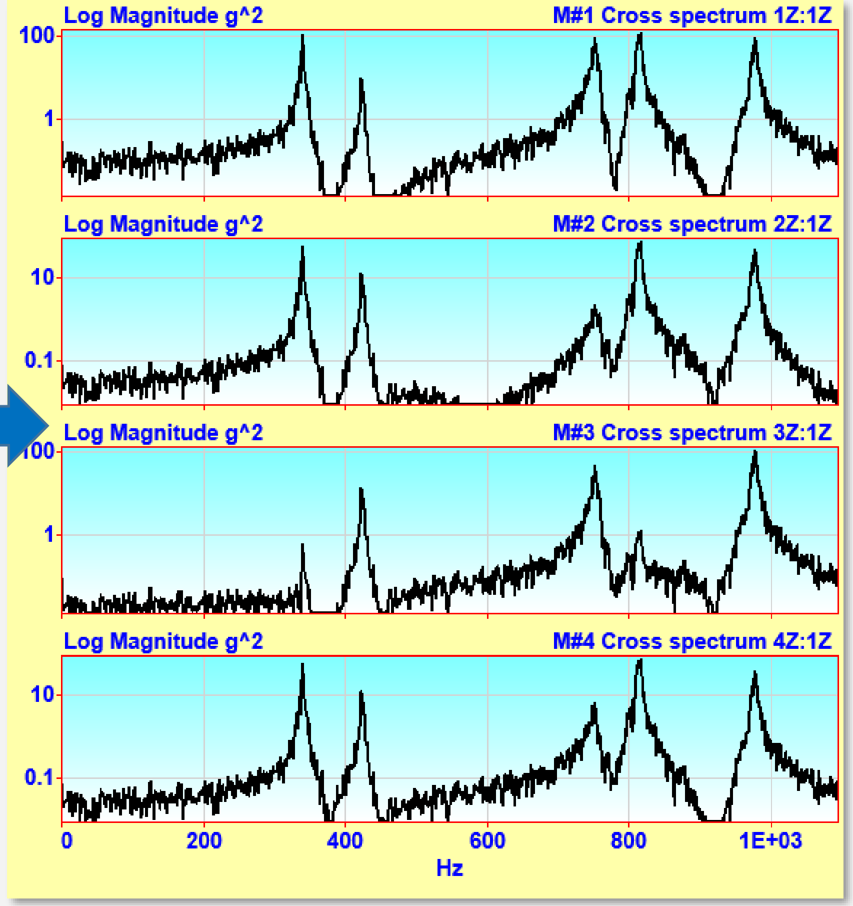
$$XPS(4:20) \rightarrow \begin{matrix} TRN(2:1) \\ TRN(3:2) \\ TRN(4:3) \\ TRN(5:4) \\ TRN(6:5) \end{matrix}$$

$$XPS(3:20) = XPS(4:20)\ x\ TRN(3:4)$$
$$XPS(2:20) = XPS(3:20)\ x\ TRN(2:3)$$
$$XPS(1:20) = XPS(2:20)\ x\ TRN(1:2)$$
$$XPS(5:20) = XPS(4:20)\ x\ TRN(5:4)$$
$$XPS(6:20) = XPS(5:20)\ x\ TRN(6:5)$$

Seeding a TRN Chain with a Cross Spectrum







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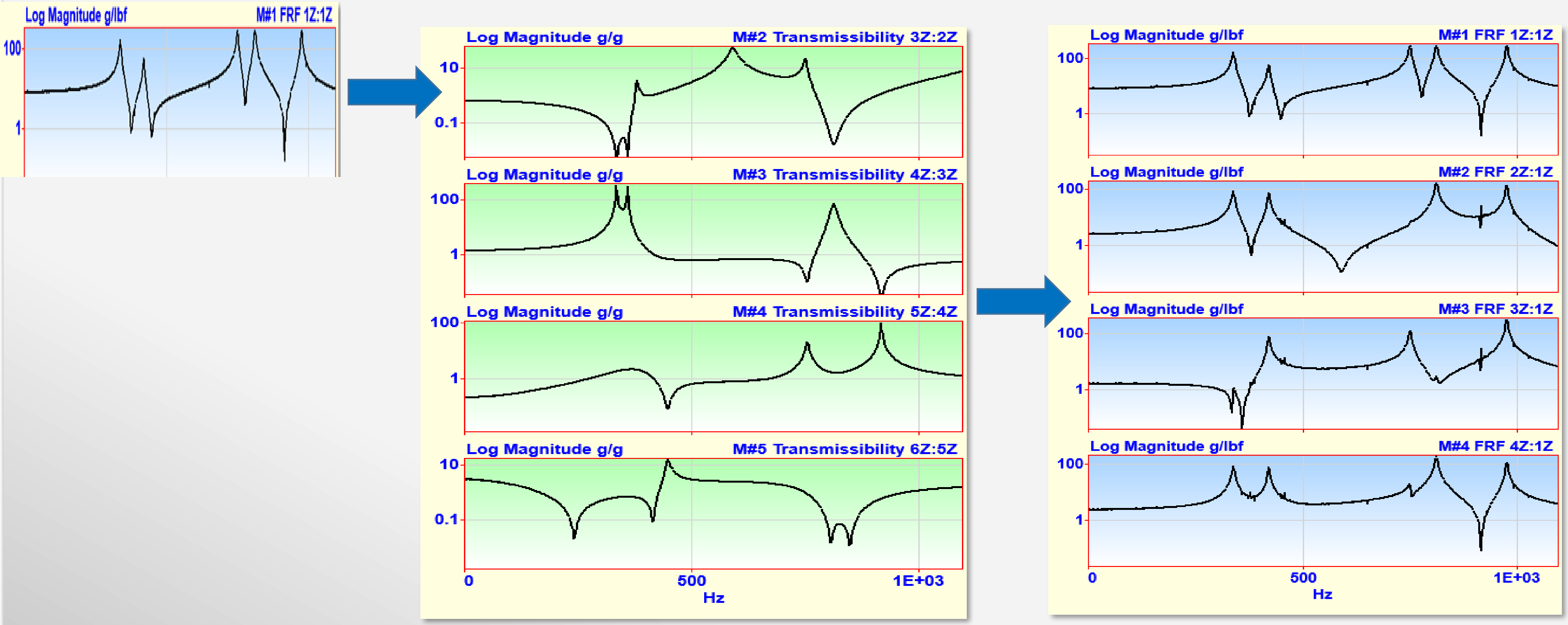
Seeding a TRN Chain with an FRF

$$\boldsymbol{FRF(4:20)} \Rightarrow \begin{array}{l} \boldsymbol{TRN(2:1)} \\ \boldsymbol{TRN(3:2)} \\ \boldsymbol{TRN(4:3)} \\ \boldsymbol{TRN(5:4)} \\ \boldsymbol{TRN(6:5)} \end{array}$$

$$\boldsymbol{FRF(3:20) = FRF(4:20)\ x\ TRN(3:4)}$$
$$\boldsymbol{FRF(2:20) = FRF(3:20)\ x\ TRN(2:3)}$$
$$\boldsymbol{FRF(1:20) = FRF(2:20)\ x\ TRN(1:2)}$$
$$\boldsymbol{FRF(5:20) = FRF(4:20)\ x\ TRN(5:4)}$$
$$\boldsymbol{FRF(6:20) = FRF(5:20)\ x\ TRN(6:5)}$$

Seeding a TRN Chain with an FRF



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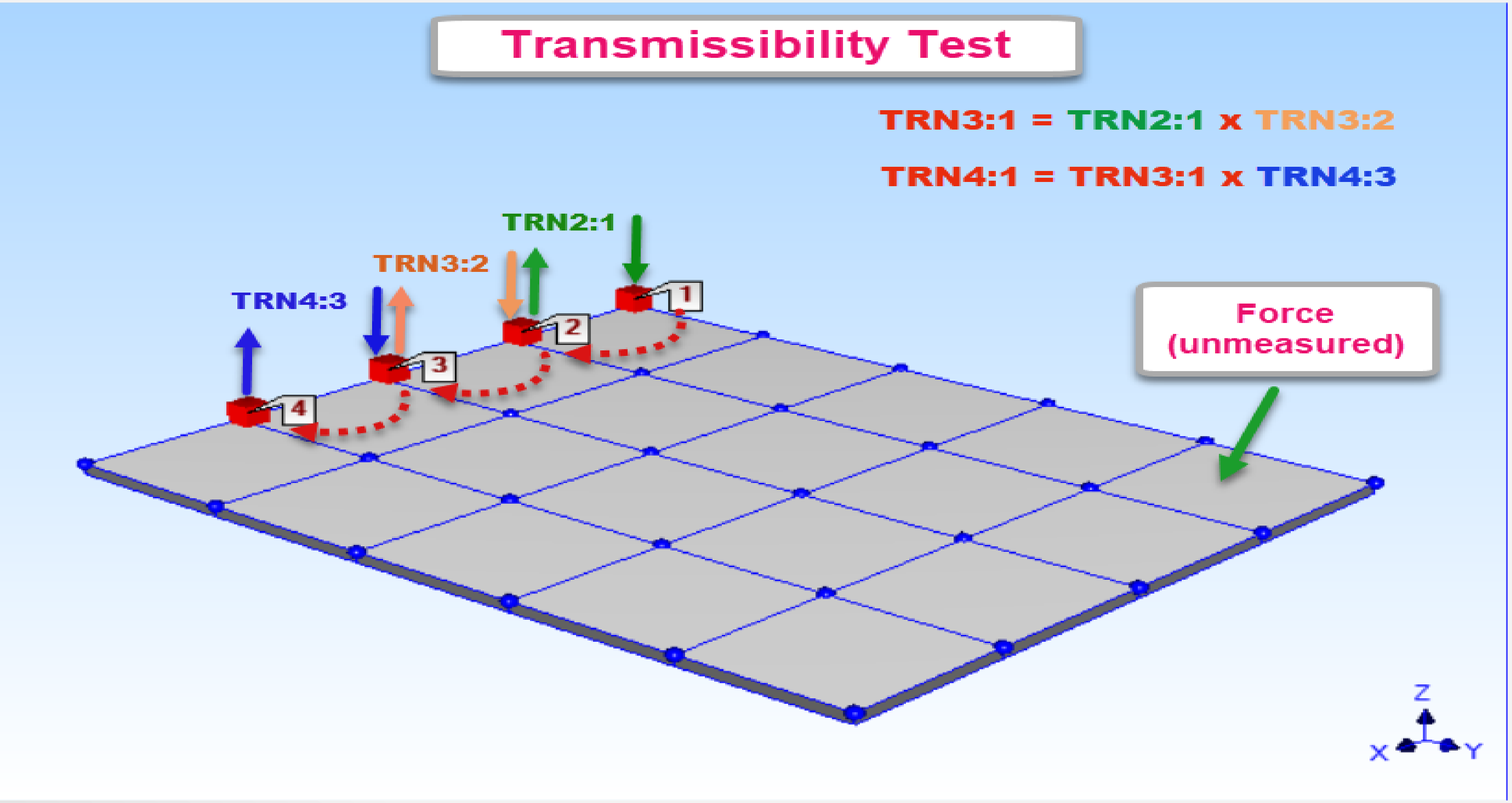
TRN Chain Measurement

Does not require **acquisition of the** ***excitation forces***

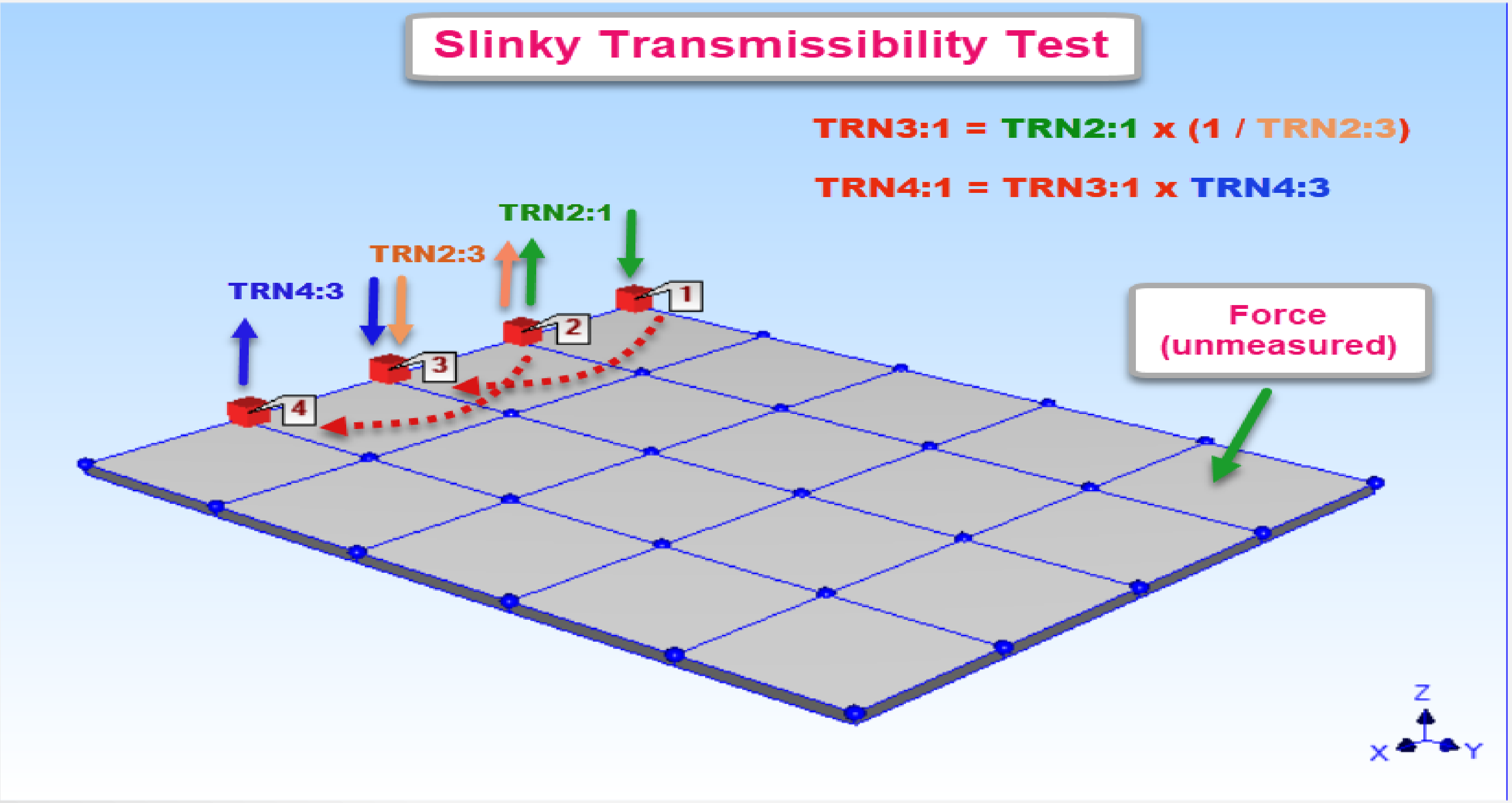
Seeding **a TRN Chain gives a set of** ***single-reference*** **ODS FRFs, Cross spectra, Fourier spectra, or FRFs**



Slinky Test

Only one sensor must move **between acquisitions**

Either or both sensors **can be moved as long as the** ***chain of DOFs*** **is** ***not broken***



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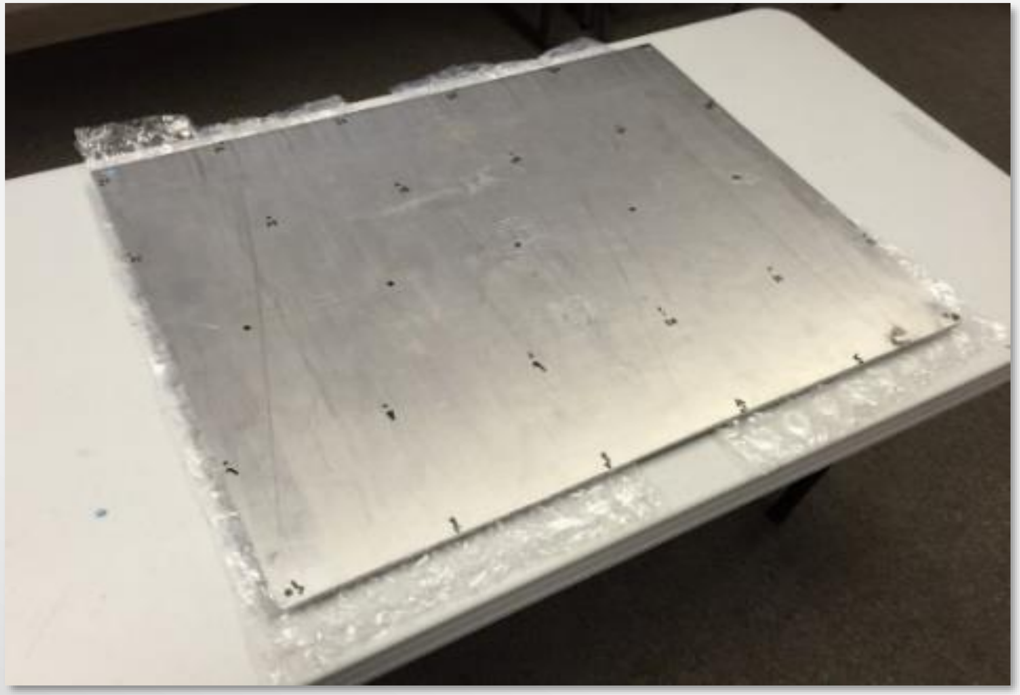
Round-Trip Slinky™ Test of an Aluminum Plate



Aluminum Plate Resting on Bubble Wrap

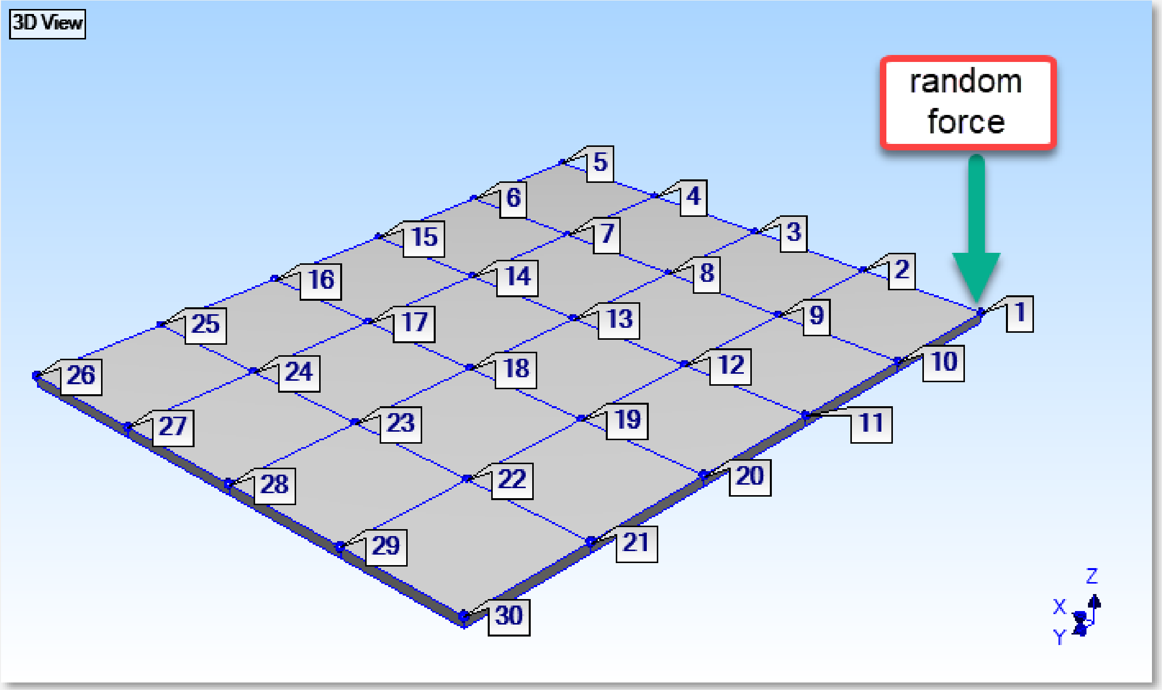
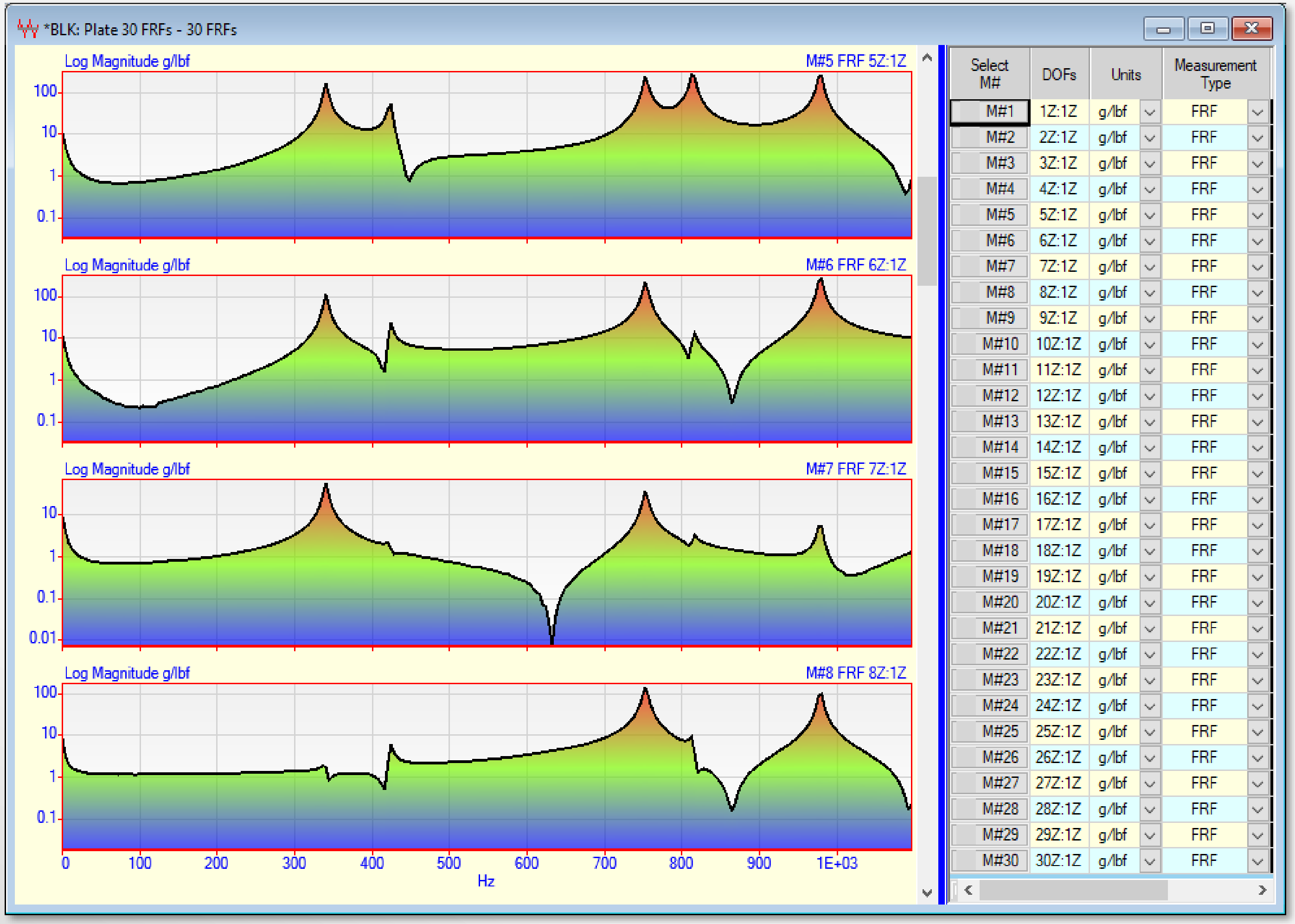


Plate Model Showing Test Points and Excitation Force

FRFs from a Roving Impact Test of the Aluminum Plate



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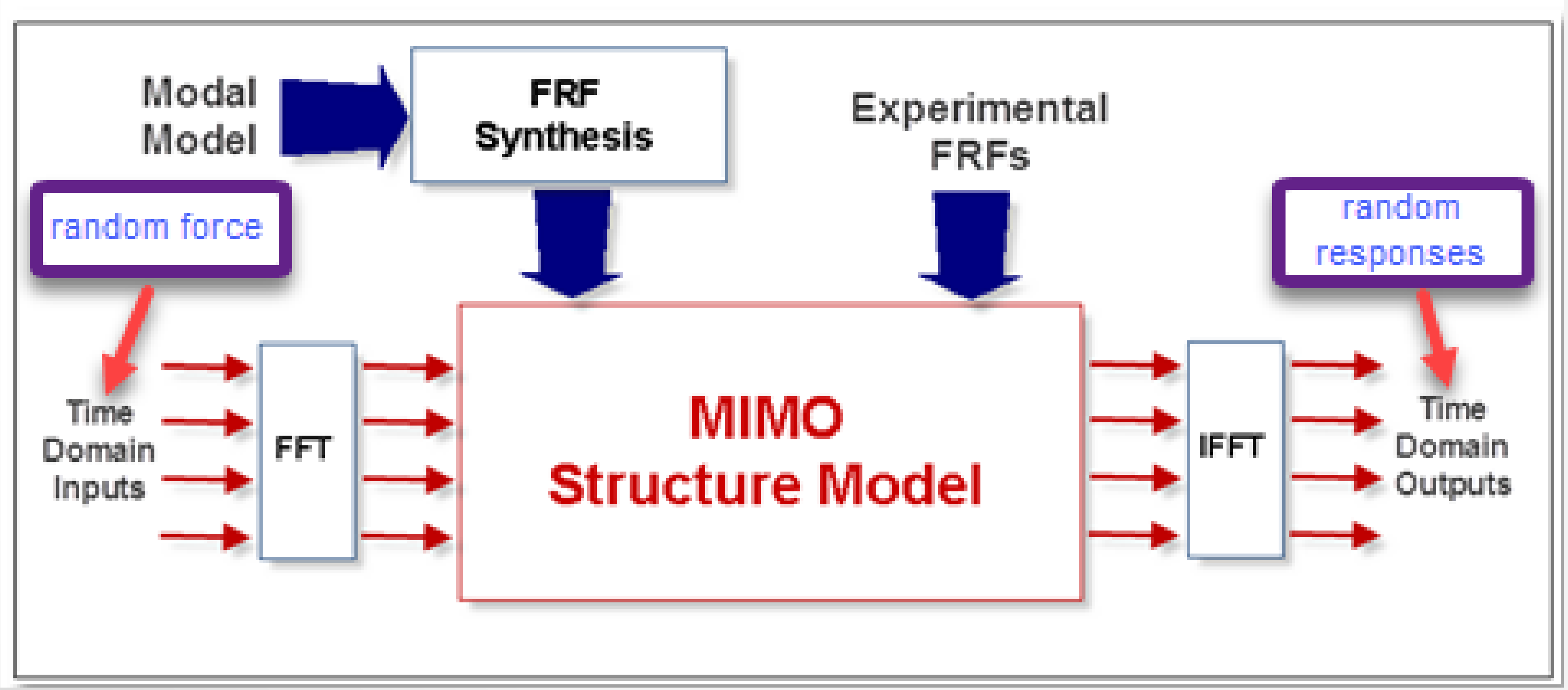
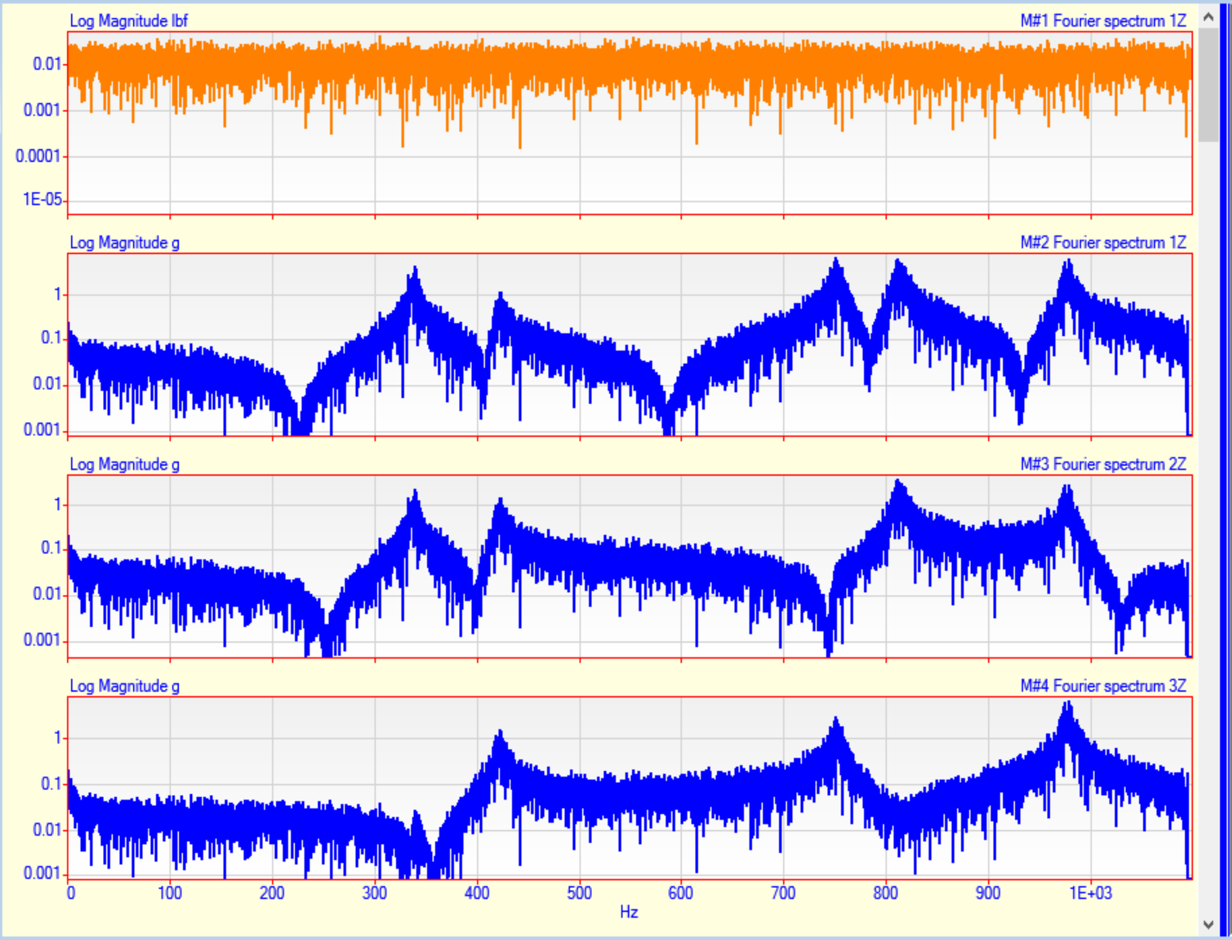
Random Response Calculation using Experimental FRFs



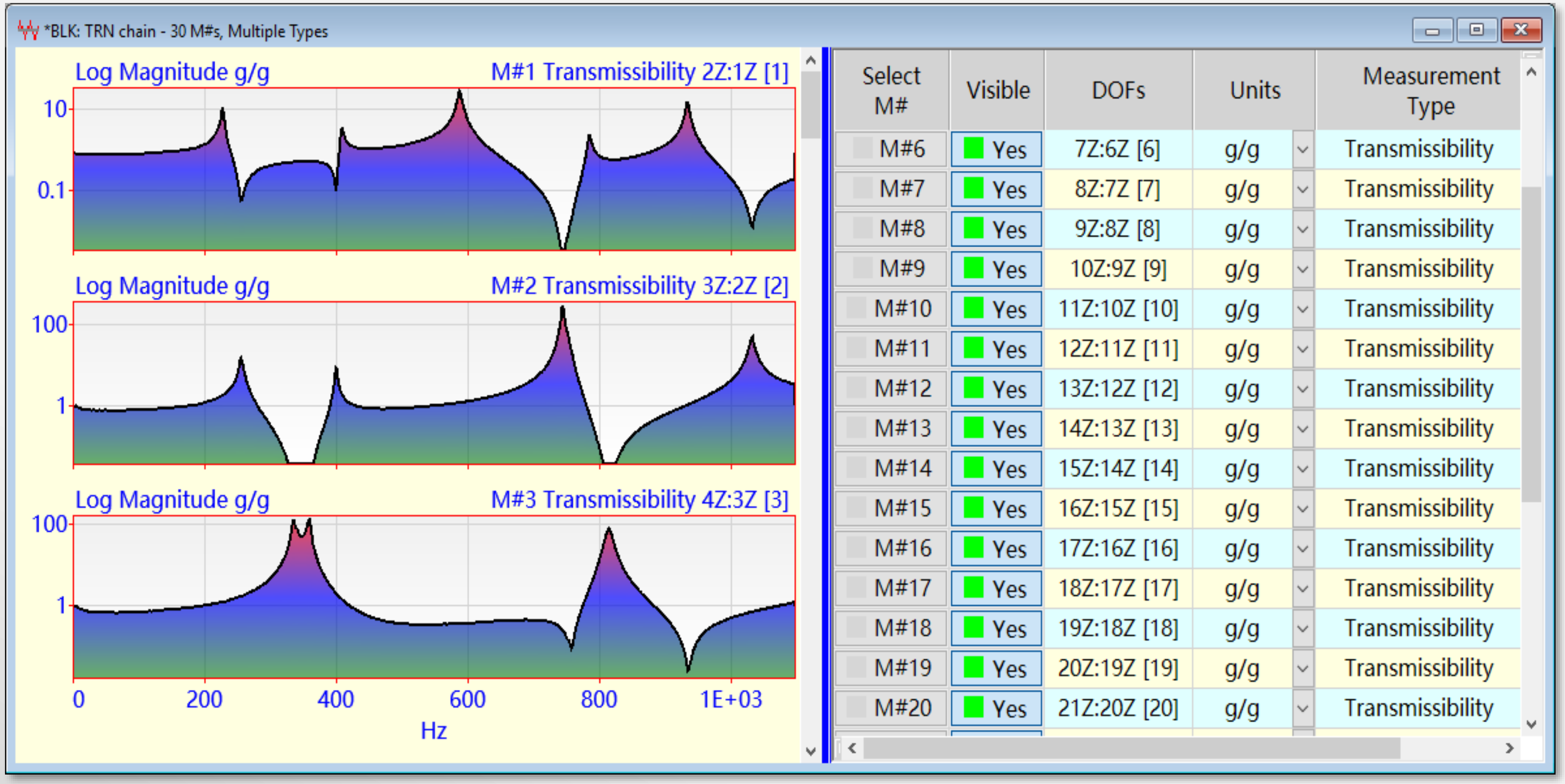
Random Force & Response Spectra of the Aluminum Plate



Select M#	Input Output	DOFs	Units
M#1	Input	1Z	lbf
M#2	Output	1Z	g
M#3	Output	2Z	g
M#4	Output	3Z	g
M#5	Output	4Z	g
M#6	Output	5Z	g
M#7	Output	6Z	g
M#8	Output	7Z	g
M#9	Output	8Z	g
M#10	Output	9Z	g
M#11	Output	10Z	g
M#12	Output	11Z	g
M#13	Output	12Z	g
M#14	Output	13Z	g
M#15	Output	14Z	g
M#16	Output	15Z	g
M#17	Output	16Z	g
M#18	Output	17Z	g
M#19	Output	18Z	g
M#20	Output	19Z	g
M#21	Output	20Z	g
M#22	Output	21Z	g
M#23	Output	22Z	g
M#24	Output	23Z	g
M#25	Output	24Z	g
M#26	Output	25Z	g
M#27	Output	26Z	g
M#28	Output	27Z	g
M#29	Output	28Z	g
M#30	Output	29Z	g
M#31	Output	30Z	g

TRN Chain from Random Response Pairs

- A Transmissibility is a ***different complex waveform*** than an FRF
- ***Peaks*** in a Transmissibility are ***not resonance peaks***
- Transmissibility's ***cannot be curve fit*** using FRF-based curve fitting



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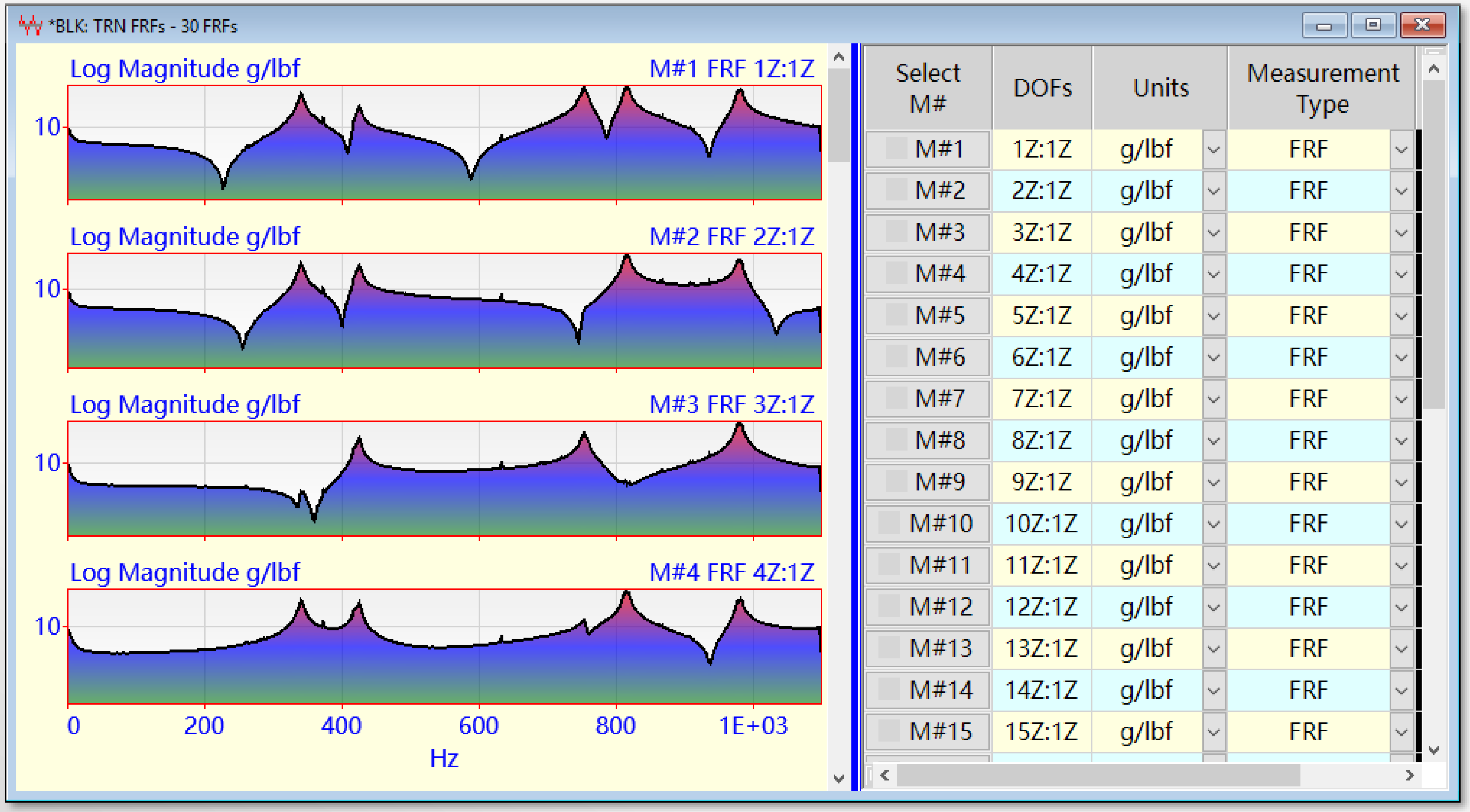
Single-Reference FRFs from *"Seeding"* the TRN Chain

- A TRN Chain can be ***seeded*** with ***any FRF***, provided that the Roving DOF of the seed matches one of the DOFs in the TRN Chain



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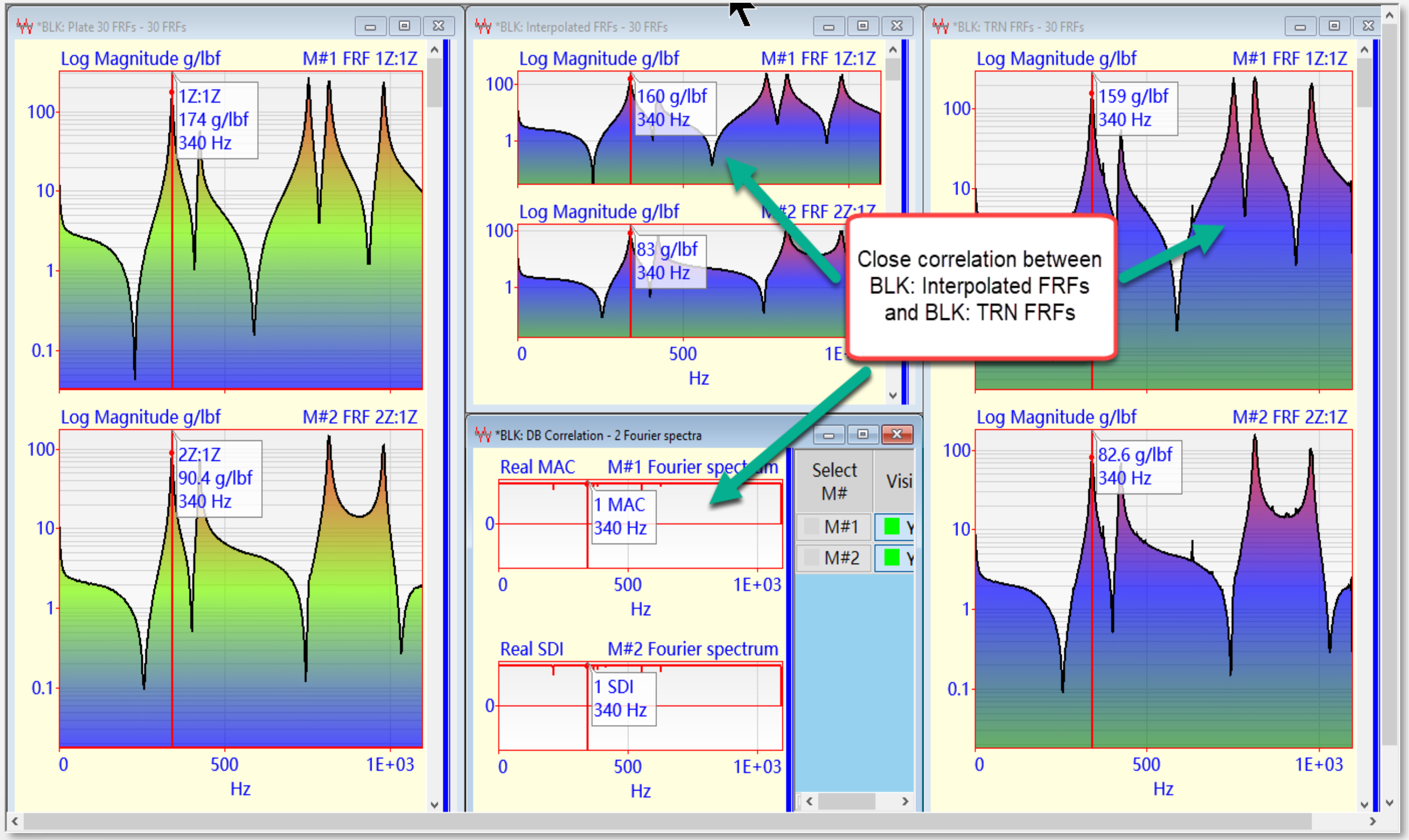
Comparing Experimental & Slinky™ FRFs of the Aluminum Plate

- Experimental FRFs have **275 samples**. Slinky (TRN FRFs) have **1000 samples**
- MAC measures the ***co-linearity*** of two shapes
- SDI measures the ***difference*** between two shapes



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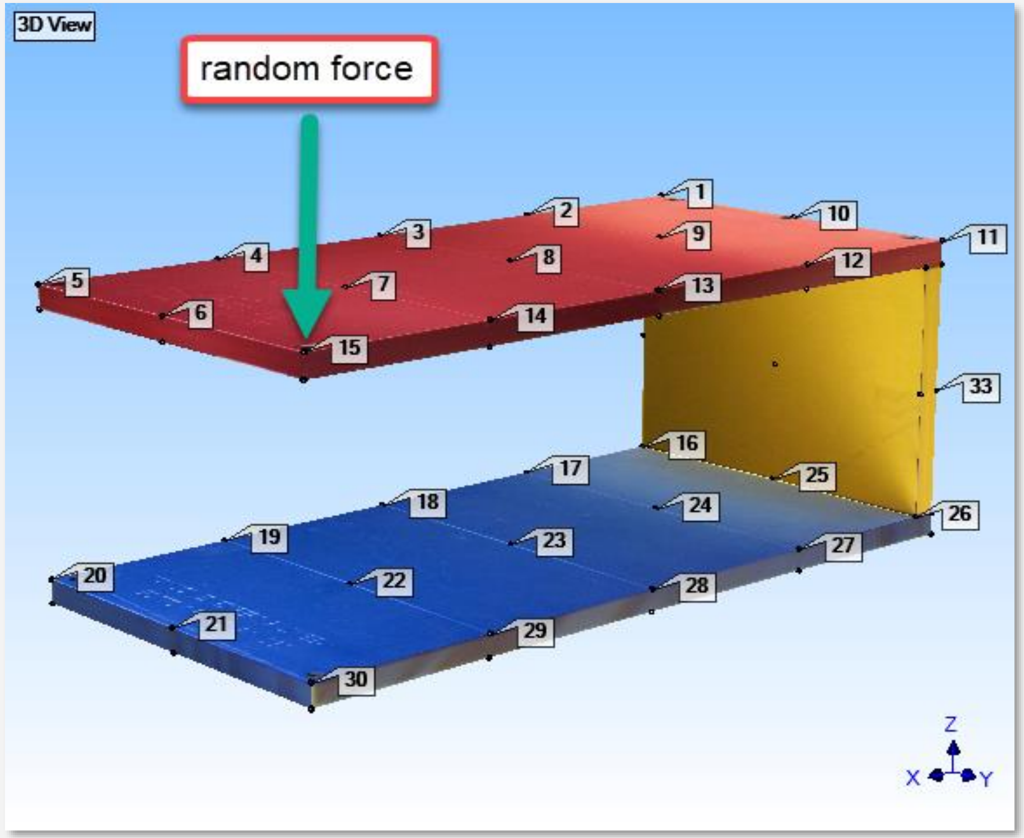
Round Trip Slinky™ Test of the Jim Beam

Impact Test with Roving Tri-Axial Accel



Jim Beam Model Showing Test Points and Excitation Force

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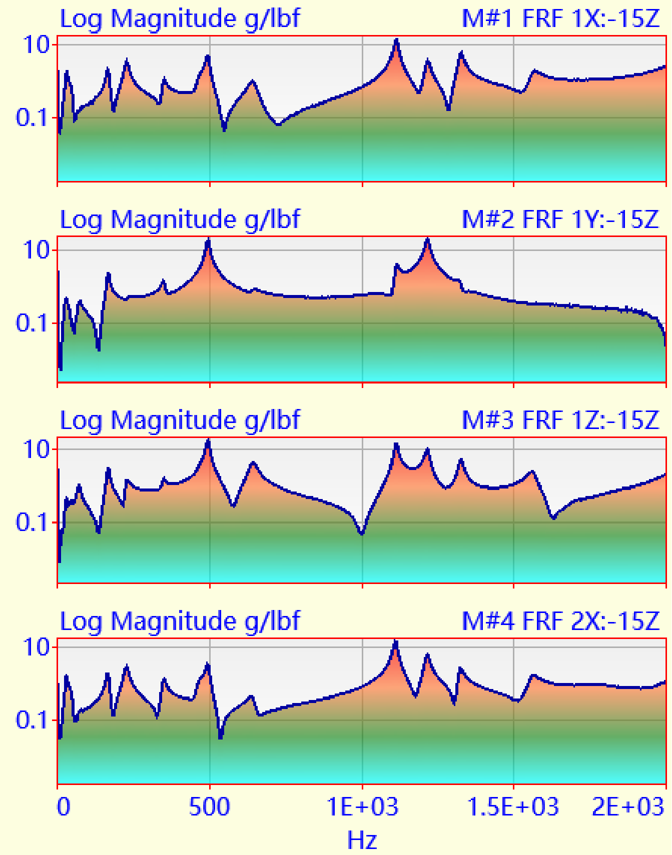
FRFs from the Impact Test of the Jim Beam

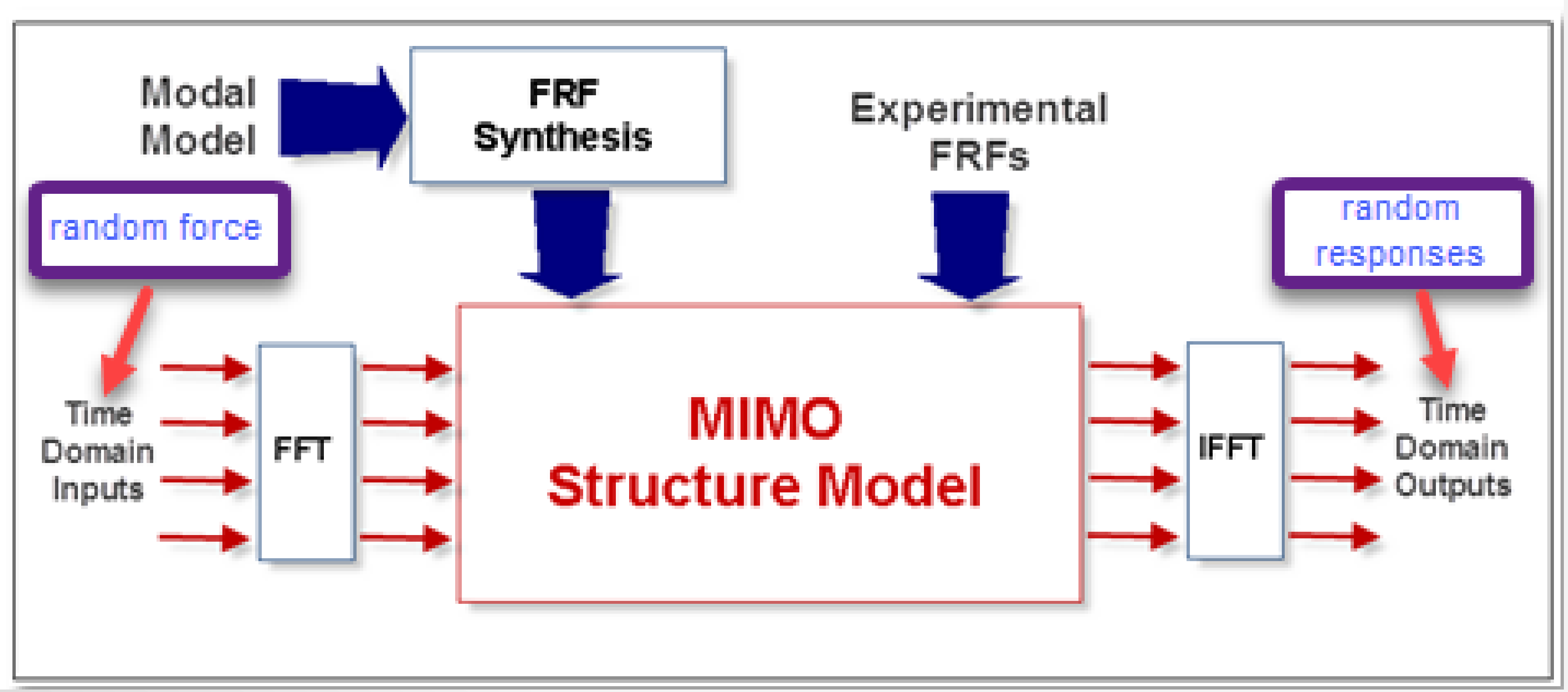
*BLK: 99 FRFs - 99 FRFs



Select M#	Visible	DOFs	Units	Measurement Type
M#1	Yes	1X:-15Z	g/lbf	FRF
M#2	Yes	1Y:-15Z	g/lbf	FRF
M#3	Yes	1Z:-15Z	g/lbf	FRF
M#4	Yes	2X:-15Z	g/lbf	FRF
M#5	Yes	2Y:-15Z	g/lbf	FRF
M#6	Yes	2Z:-15Z	g/lbf	FRF
M#7	Yes	3X:-15Z	g/lbf	FRF
M#8	Yes	3Y:-15Z	g/lbf	FRF
M#9	Yes	3Z:-15Z	g/lbf	FRF
M#10	Yes	4X:-15Z	g/lbf	FRF
M#11	Yes	4Y:-15Z	g/lbf	FRF
M#12	Yes	4Z:-15Z	g/lbf	FRF
M#13	Yes	5X:-15Z	g/lbf	FRF
M#14	Yes	5Y:-15Z	g/lbf	FRF
M#15	Yes	5Z:-15Z	g/lbf	FRF
M#16	Yes	6X:-15Z	g/lbf	FRF
M#17	Yes	6Y:-15Z	g/lbf	FRF
M#18	Yes	6Z:-15Z	g/lbf	FRF

Vibrant TECHNOLOGY

Random Response Calculation using Experimental FRFs



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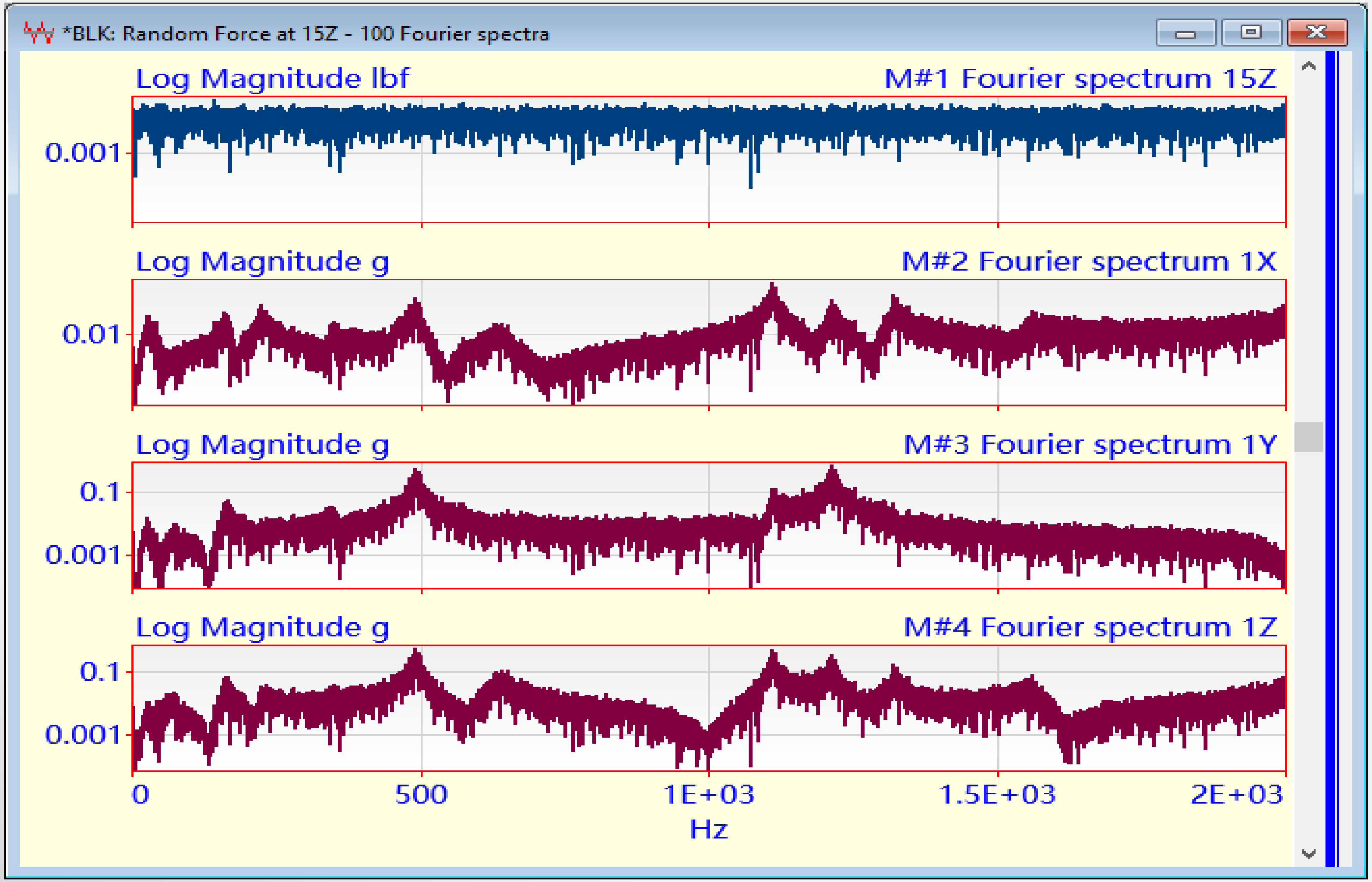
Random Force & Response Spectra of the Jim Beam



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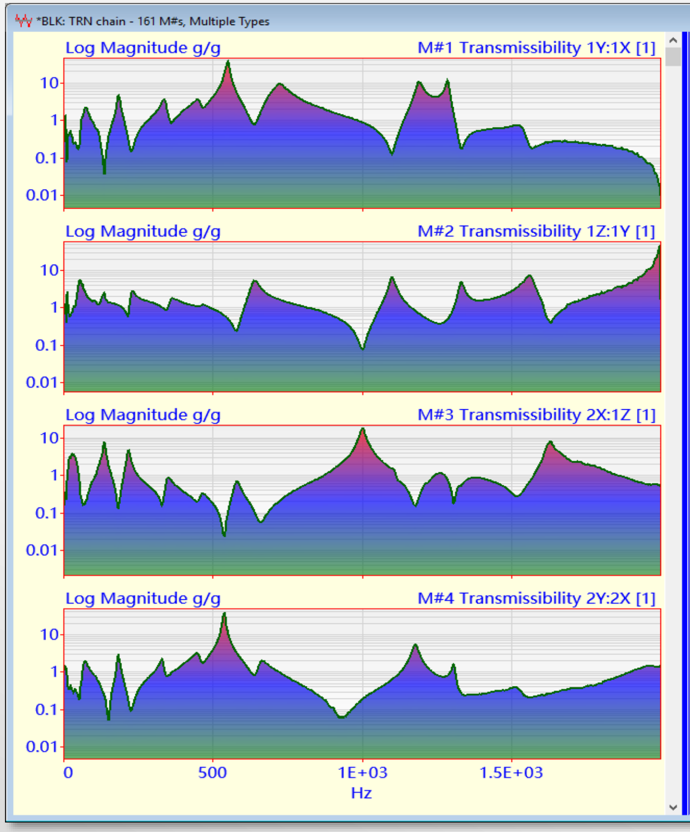
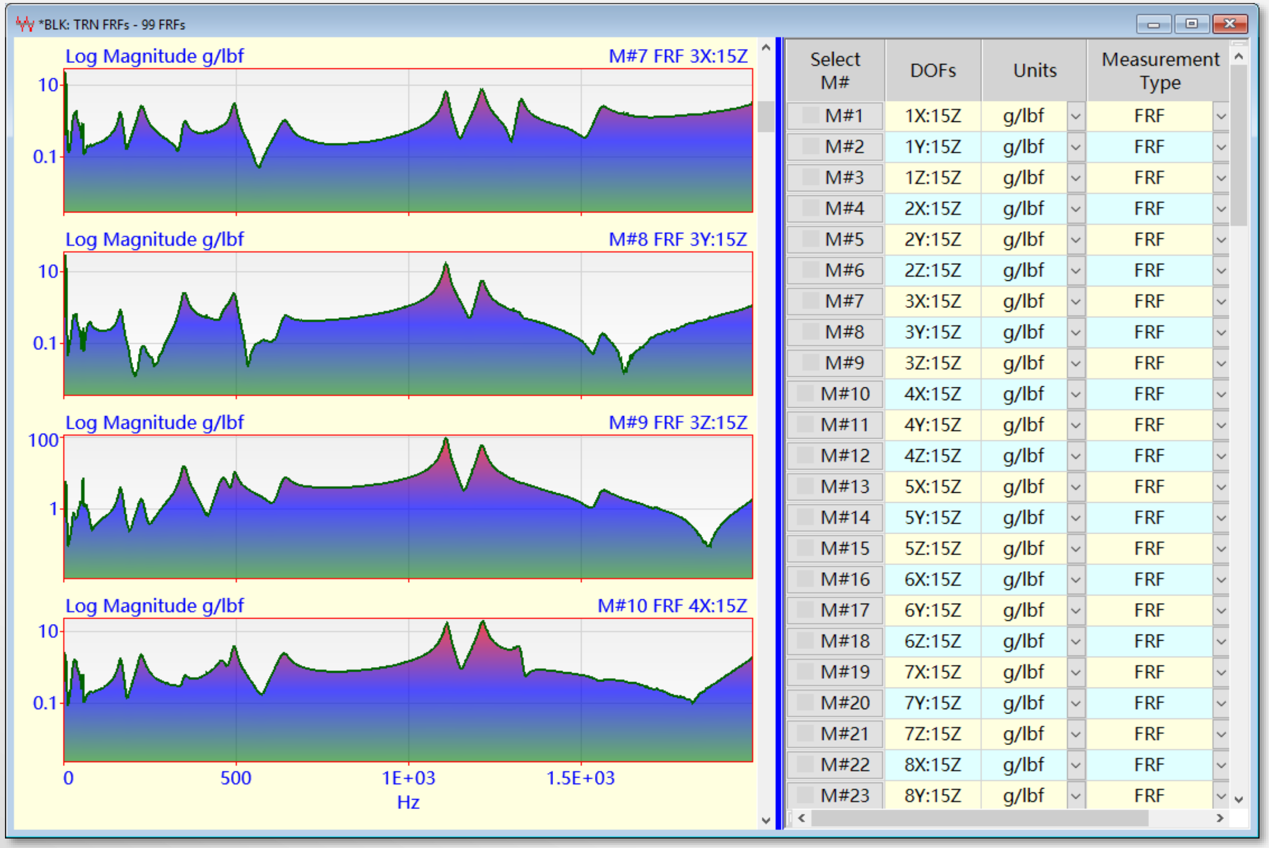


Select M#	Visible	DOFs	Units	Measurement Type
M#1	Yes	1Y:1X [1]	g/g	Transmissibility
M#2	Yes	1Z:1Y [1]	g/g	Transmissibility
M#3	Yes	2X:1Z [1]	g/g	Transmissibility
M#4	Yes	2Y:2X [1]	g/g	Transmissibility
M#5	Yes	2Z:2Y [1]	g/g	Transmissibility
M#6	Yes	2Y:2X [2]	g/g	Transmissibility
M#7	Yes	2Z:2Y [2]	g/g	Transmissibility
M#8	Yes	3X:2Z [2]	g/g	Transmissibility
M#9	Yes	3Y:3X [2]	g/g	Transmissibility
M#10	Yes	3Z:3Y [2]	g/g	Transmissibility
M#11	Yes	3Y:3X [3]	g/g	Transmissibility
M#12	Yes	3Z:3Y [3]	g/g	Transmissibility
M#13	Yes	4X:3Z [3]	g/g	Transmissibility
M#14	Yes	4Y:4X [3]	g/g	Transmissibility
M#15	Yes	4Z:4Y [3]	g/g	Transmissibility
M#16	Yes	4Y:4X [4]	g/g	Transmissibility
M#17	Yes	4Z:4Y [4]	g/g	Transmissibility
M#18	Yes	5X:4Z [4]	g/g	Transmissibility
M#19	Yes	5Y:5X [4]	g/g	Transmissibility
M#20	Yes	5Z:5Y [4]	g/g	Transmissibility
M#21	Yes	5Y:5X [5]	g/g	Transmissibility
M#22	Yes	5Z:5Y [5]	g/g	Transmissibility
M#23	Yes	6X:5Z [5]	g/g	Transmissibility
M#24	Yes	6Y:6X [5]	g/g	Transmissibility
M#25	Yes	6Z:6Y [5]	g/g	Transmissibility
M#26	Yes	6Y:6X [6]	g/g	Transmissibility
M#27	Yes	6Z:6Y [6]	g/g	Transmissibility

Single-Reference FRFs of Jim Beam from *"Seeding"* the TRN Chain

- A TRN Chain can be ***seeded*** with ***any FRF***, provided that the Roving DOF of the seed matches one of the DOFs in the TRN Chain



Select M#	DOFs	Units	Measurement Type
M#1	1X:15Z	g/lbf	FRF
M#2	1Y:15Z	g/lbf	FRF
M#3	1Z:15Z	g/lbf	FRF
M#4	2X:15Z	g/lbf	FRF
M#5	2Y:15Z	g/lbf	FRF
M#6	2Z:15Z	g/lbf	FRF
M#7	3X:15Z	g/lbf	FRF
M#8	3Y:15Z	g/lbf	FRF
M#9	3Z:15Z	g/lbf	FRF
M#10	4X:15Z	g/lbf	FRF
M#11	4Y:15Z	g/lbf	FRF
M#12	4Z:15Z	g/lbf	FRF
M#13	5X:15Z	g/lbf	FRF
M#14	5Y:15Z	g/lbf	FRF
M#15	5Z:15Z	g/lbf	FRF
M#16	6X:15Z	g/lbf	FRF
M#17	6Y:15Z	g/lbf	FRF
M#18	6Z:15Z	g/lbf	FRF
M#19	7X:15Z	g/lbf	FRF
M#20	7Y:15Z	g/lbf	FRF
M#21	7Z:15Z	g/lbf	FRF
M#22	8X:15Z	g/lbf	FRF
M#23	8Y:15Z	g/lbf	FRF

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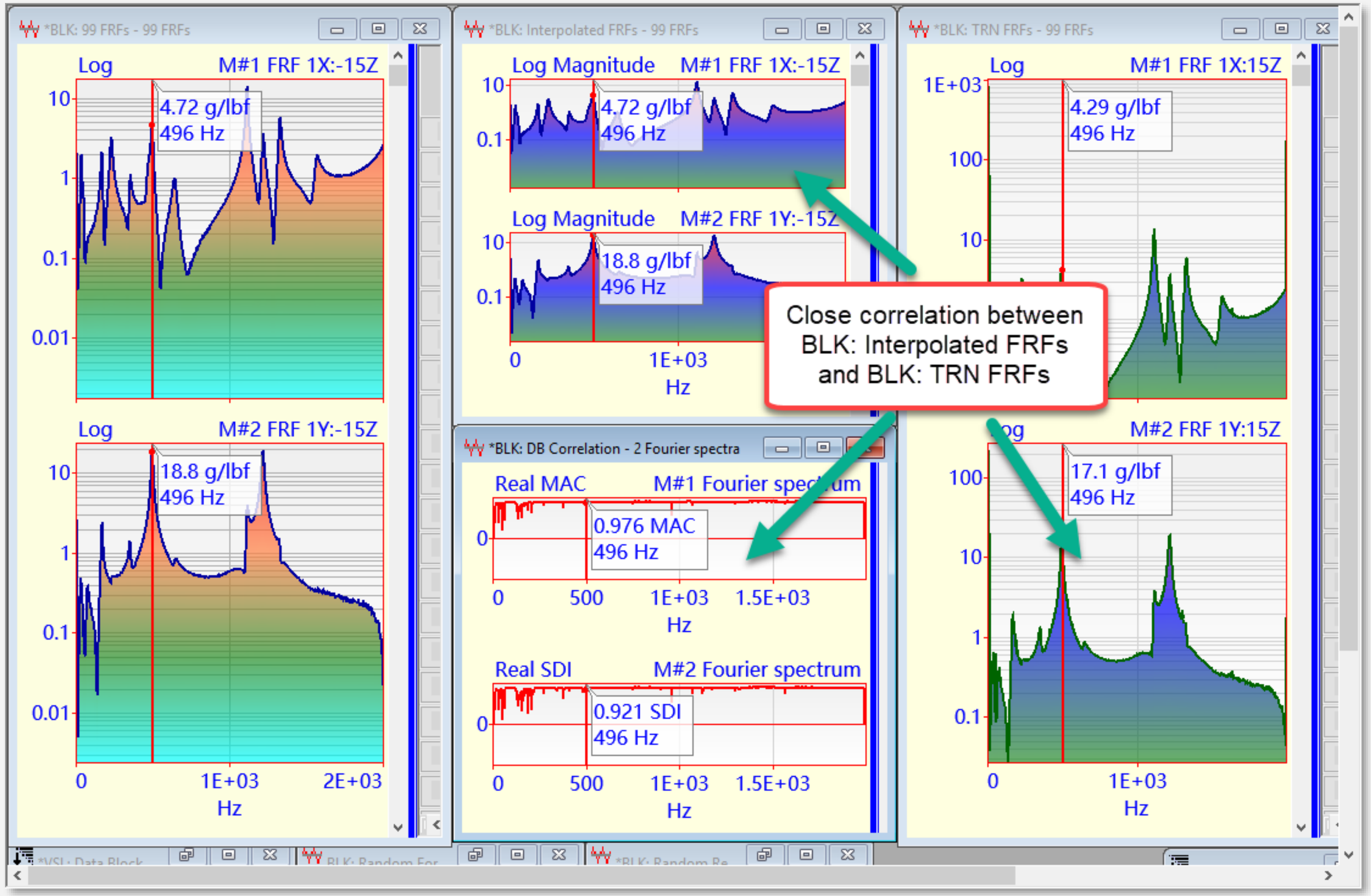

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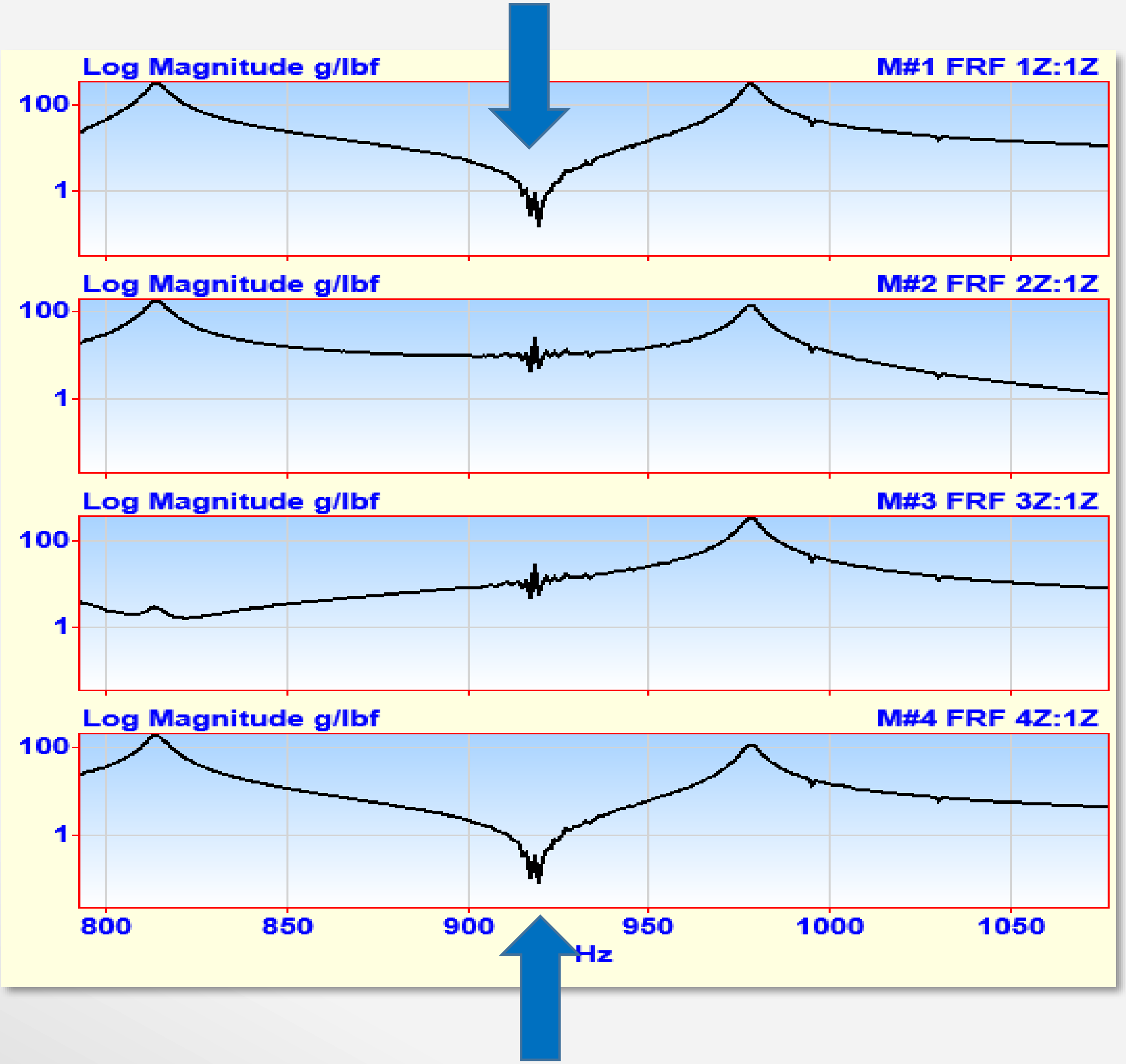
Noise Propagation from the FRF Seed



Summary

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- TRN Chain testing *only requires a pair of response sensors*
- TRN Chain testing *does not require a (long) wire* from one of the sensors to the acquisition system
- A Slinky™ test *minimizes* the number of *sensor moves*
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