Structural Modifications Using Higher Order Elements

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ABSTRACT

The Structural Dynamics Modification (SDM) method is very useful for solving the so-called forward variational problem for structures. That is, given changes in a structure's mass, stiffness, or damping properties, SDM efficiently calculates the corresponding changes in its modal properties.

Although this method is very useful for exploring potential modifications to real structures using experimentally derived modal data, its practical use has been limited to date, because only simple linear spring, damper, and point mass modification elements have been available in commercial software.

In this paper, we show how all of the most commonly used elements of finite element analysis (FEA) can also be used to model structural modifications. These include ***rods***, ***bars***, ***triangular*** *and* ***quadrilateral*** *plate and shell elements, and* ***tetrahedron***, ***prism***, *and* ***brick*** *solid elements.*

An example flat plate structure with a rib stiffener attached to its centerline was tested and modeled using SDM, with both ***plate*** *and* ***bar*** *elements. The modal data for the unmodified structure (plate without rib) and the element properties are used as input data to the SDM method. The modes of the modified structure (plate with rib) calculated by SDM, are then compared with both test and FEA results.*

INTRODUCTION

The SDM method [6] is useful for performing "What If?" analyses on a structure. More specifically, "If a stiffener or a tuned absorber is added to a structure, how will its modes change?", or "How will it vibrate differently?" For addressing these kinds of problems, SDM has two advantages compared to using an FEA package directly. They are,

1. *Experimental* or *analytical* modal data can be used to define the dynamics of the unmodified structure.
2. SDM solves for the new modes of the modified structure fast and efficiently, so many more potential modifications can be tried.

However, the most conspicuous weakness of SDM in past implementations has been its lack of elements with which to model realistic physical changes to a structure. For the most part, all commercial structural modification packages have heretofore been restricted to the use of simple point masses, linear springs, and linear dampers to model modifications.

In this paper, we demonstrate that the same elements used in most popular finite element modeling packages can also be used with the SDM method to model more complex and realistic structural modifications. A variety of popular finite element types have been implemented in the Structural Modifications option to the ME'scope™ software available from Vibrant Technology, Inc. The use of quadrilateral plate elements (*quads*) and *bar* (or beam) elements is demonstrated in this paper.

A rectangular aluminum plate was modeled using the NASTRAN for Windows* finite element package, and then tested using the impact method. This simple structure is easy to test and model, and we obtained very good correlation between its experimental and analytical modes.

Then, we bolted a rib stiffener to the centerline of the plate and,

1. Re tested it,
2. Added the rib to the NASTRAN model, and solved for the new analytical modes,
3. Modeled the rib using both Quads and Bar elements with SDM.

The experimental and FEA results for the unmodified structure were first compared. Then, the experimental, FEA, and SDM results of the modified structure were compared. Results were compared in three different ways,

1. Modal frequencies
2. Modal Assurance Criterion (MAC) of mode shapes.
3. Synthesized versus measured FRFs.

THEORETICAL BACKGROUND

The SDM method was first made available in commercial software in 1980 [1]. Since then, many technical papers have been written about it, and the underlying method (called local eigenvalue modification) has been well documented [2], [6].

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All structural modifications can be represented in terms of changes to the mass, stiffness, and damping properties of a structure. Once the mass, stiffness, and damping of the modified structure are known, a straightforward eigensolution calculation is used to find the modes of the modified structure.

A key advantage of the SDM method is that only the modes of the unmodified structure, plus the mass, stiffness, and damping *changes* of the structural modification are required. The mass, stiffness, and damping of the unmodified structure are not required.

Thus, SDM can be used once a valid set of modes of the unmodified structure is obtained, either experimentally or analytically. The remaining difficulty, however, lies in defining the structural modification(s) correctly. If Structural Modifications is to be a useful tool, it must offer a variety of modeling elements that model real world modifications.

A variety of elements have been developed for use in finite element analysis programs over the past twenty years, that are now accepted as "standards" in the industry. Text books are now available that document the details of these elements [3, 4, 5].

In addition to simple point masses, linear springs, and dampers, the most common elements used in finite element modeling include **rods**, **bars** (or beams), **triangular** and **quadrilateral** plates and shells, and solid elements like **tetrahedrons**, **prisms**, and **brick** elements.

In this paper, we demonstrate the use of both **Bar** and **Quad** elements to model a rib stiffener modification to a flat plate structure. Both of these elements add mass and stiffness to the structure.

Bar Elements

Bar elements have 2 nodes, with 6 degrees of freedom per node. Bar elements are able to transfer axial loads and torsional loads, as well as shearing forces and moments due to bending. The stiffness matrix formulation for bars assumes constant cross-sectional properties and linear, isotropic materials. Since the cross sectional properties of the bar are constant, the mass matrix for the bar simply lumps half of the mass at either end of the bar.

Quad Elements

Quadrilateral elements have 4 co-planar nodes with up to 5 degrees of freedom per node. Supported degrees of freedom are all translations, both in the plane and normal to the plane of the quadrilateral, and rotation along the sides of the quadrilateral. The quadrilateral element does not, however, support rotations at a point normal to its plane.

There are three types of quadrilateral elements currently in use; the membrane, the plate bending element, and the shell element. The membrane element, has 2 degrees of freedom per node and only sustains loads in the plane of the element. This element typically assumes a plane stress displacement model.

The plate bending element has 3 degrees of freedom per node; displacement normal to the element face, and rotation along each edge of the element. Shell elements are simply membrane elements and plate bending elements merged together.

We used membrane elements to model the rib stiffener in this paper.

Static Condensation

In finite element models, all of the degrees of freedom (displacements) in each element, and in the model are known, or can be calculated. When using finite elements with SDM, all of the degrees of freedom for an element are rarely known, especially the rotational degrees of freedom.

Static condensation [3] is the application of Gaussian elimination to remove degrees of freedom that are not required by SDM. In finite element analysis, static condensation is employed in substructure analysis, where large structures can be created by assembling together smaller substructures. Using static condensation, the degrees of freedom internal to a substructure can be removed, condensing the structure down to only those degrees of freedom shared by the connected substructures. This same method can be applied to condense out any degrees of freedom that are not part of an SDM solution.

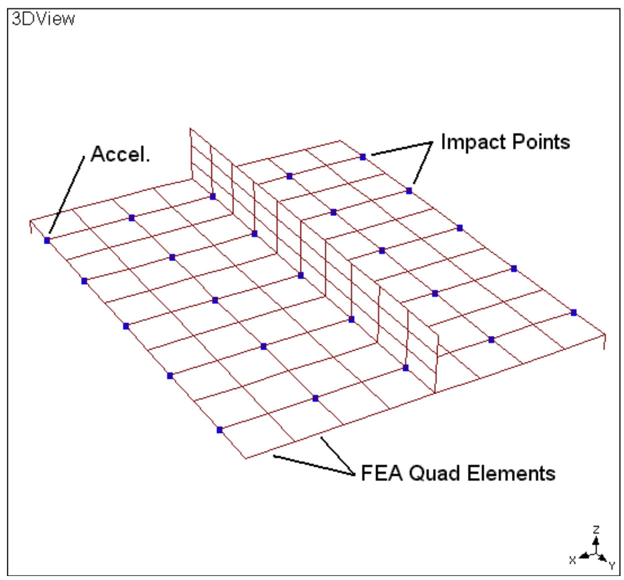

Figure 1. The Test Article.

TEST ARTICLE

A rectangular plate structure (20 inches by 25 inches by 3/8 inch thick) was constructed out of 6061 T6511 aluminum, as a test article. A rib (25 inches by 3 inches by 3/8 inch thick) was attached to the centerline of the plate with 5 cap screws. A drawing of the plate and rib is shown in Figure 1

This type of structure is very convenient for impact testing when suspended with bungee cords in a free-free condition. A single reference accelerometer was attached to point #1., and 25 FRF measurements were made by impacting the plate at each point in a (5 by 5) grid of points, spaced 5 inches apart.

EXPERIMENTAL MODES

Figure 2 shows the sum of magnitudes (imaginary parts squared) of the 25 measurements for the plate without rib, showing the modal peaks in the frequency range (0 to 1.25 kHz).

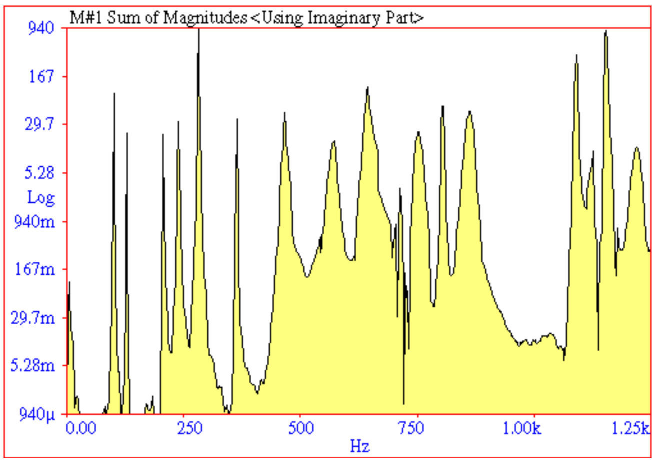

Figure 2. Sum of Magnitudes for Plate Without Rib.

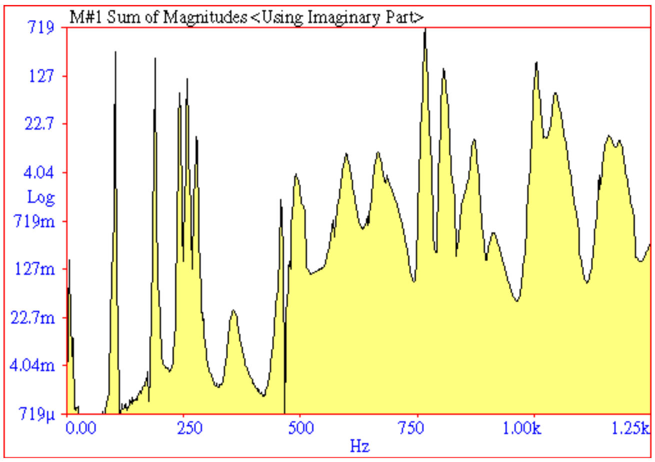

Figure 3. Sum of Magnitudes for Plate With Rib.

Figure 3 shows the sum of magnitudes of the 25 measurements for the plate with rib. Clearly, the rib has caused the modal frequencies to shift, as expected.

Both sets of measurements were curve fit to identify the modes of the unmodified and modified structures. For the unmodified structure, 17 modes were found. Figure 4 shows an FRF that was synthesized using the modal parameters, overlaid on the original measurement data. This data has been double integrated (to units of in/lb), and is displayed in log magnitude format.

Structural modifications uses the modal data in displacement units, so modifications are typically influenced more strongly by the lowest frequency modes.

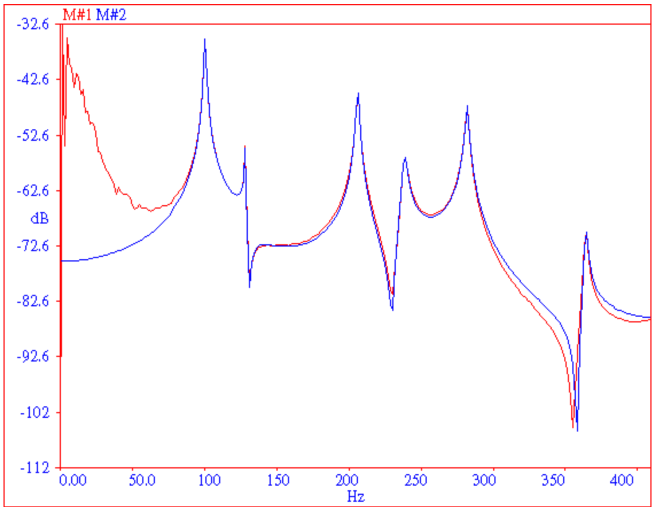

Figure 4. Synthesized Overlaid on Measured FRF (Showing First 6 Modes).

FEA MODEL OF THE PLATE

The unmodified aluminum plate (without the rib) was modeled using quadrilateral plate elements (Quads). Each Quad was defined between nodes 2.5 inches apart. This formed a (10 by 8) grid of Quad elements, as shown in Figure 1. The following element properties were used,

Modulus of Elasticity: 10^7 lb/in^2
Poissons Ratio: 0.33
Density: 0.101 lb/in^3

FEA MODEL OF THE RIB

The rib was modeled using 3 rows of Quad elements **(**1 inch high by 2.5 inches wide by 3/8 inch thick), for a total of 30 elements. These elements were added to the plate without rib model, yielding the complete model shown in Figure 1.

ANALYTICAL MODES

The first 25 modes of the finite element models were found using NASTRAN for Windows, for both the plate without rib and the plate with rib models.

The first three modes (which were rigid body modes), plus all rotational DOFS, and translational DOFs in the X and Y directions were deleted from the analytical mode shapes. This left only the translational DOFs in the Z (vertical) direction. This data was compared with the experimental modes.

COMPARISON OF EXPERIMENTAL & ANALYTICAL MODES

Table 1 shows a comparison of modal frequencies of the experimental and analytical modes that had matching mode shapes. That is, they had mode shapes with MAC values of 0.95 or greater. Also shown are the experimental damping values of the matching modes.

Exp Freq. (Hz)	Exp. Damp. (%)	FEA Freq. (Hz)	MAC
100.22	0.575	98.31	0.995
128.34	0.409	118.49	0.984
205.89	0.389	195.25	0.993
238.19	0.702	221.31	0.993
281.36	0.469	254.03	0.994
363.28	0.280	339.92	0.953
570.12	1.995	505.97	0.985
641.46	0.817	568.73	0.964
748.89	1.873	636.10	0.973
802.55	0.417	681.48	0.980
861.68	1.523	750.44	0.954

Table 1. Comparison of Experimental and FEA Modes.

Figure 5 shows an FRF synthesized between the same two DOFs as the one in Figure 4, but using the FEA modes and the experimental damping shown in Table 1. Clearly, the frequencies are in error, but the shape data matches well. MAC values above 0.95 indicate a *strong similarity* of mode shapes.

MODELING THE RIB WITH BAR ELEMENTS

To model the rib using Bars, Bar elements were added between all point pairs down the centerline of the plate, as shown in Figure 6.

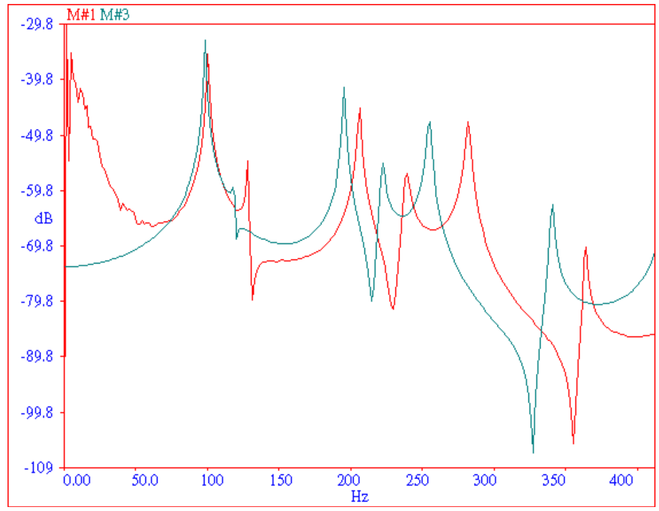

Figure 5. Synthesized Versus Measured FRF Using FEA Modes and Experimental Damping.

Bar Properties

The physical properties of the Bar are the same as those of an FEA plate elements.

Modulus of Elasticity: 10^7 lb/in^2
Poissons Ratio: 0.33
Density: 0.101 lb/in^3

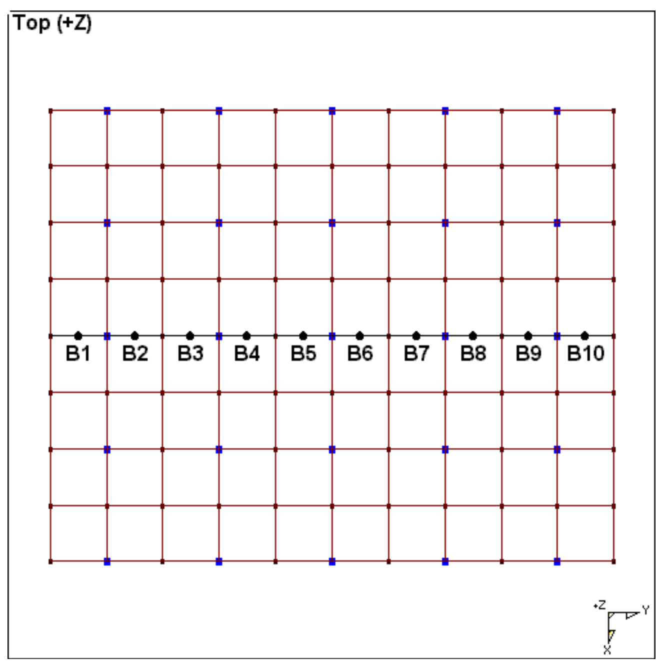

Figure 6. Top View of Plate Showing Bar Elements.

Bar Cross Section

The Bar elements have rectangular cross sections, as shown in Figure 7. The cross sectional area is simply the width times the height,

AREA = (3 in.) x (3/8 in.) = 1.125 in.^{2}

The Y-axis is the horizontal axis of the Bars. And since they are attached to the plate along their bottom edges, only the inertia Iyy shown in Figure 6 needs to be specified.

Iyy = (1/3) x (3/8 in) x (3 in)^3 = 3.375 in.^{4}

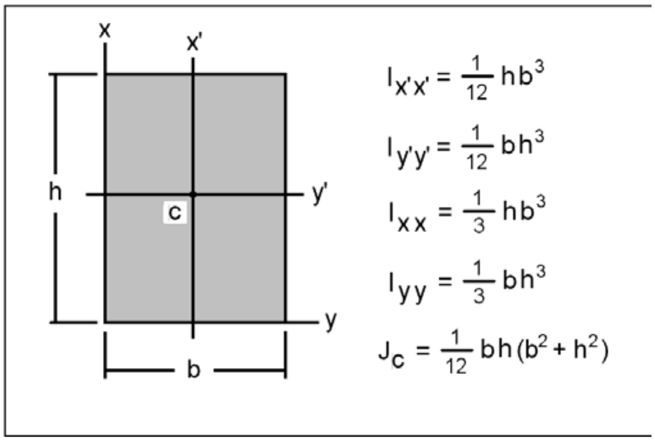

Figure 7. Rectangular Bar Cross Section.

Comparison of Bar With FEA Modes

Table 2 contains a comparison of the SDM modes obtained with Bar elements, the FEA modes of the plate with rib.

FEA Freq. (Hz)	Bar Freq. (Hz)	MAC
103.559	98.353	0.999
175.424	210.208	0.852
229.420	221.277	0.984
235.542	266.524	0.865
262.336	284.315	0.976
420.103	413.177	0.999
451.394	469.303	0.987
470.122	505.968	0.997
537.343	568.739	0.997
673.419	681.484	0.982
698.029	729.951	0.910
736.098	750.445	0.963

Table 2. Comparison of Bar and FEA Modes.

Comparison of Bar With Experimental Modes

Table 3 contains a comparison of the modes obtained with SDM and Bar elements, and the experimental modes of the plate with rib.

Exp. Freq. (Hz)	Bar Freq. (Hz)	MAC
103.341	100.267	0.996
188.005	216.755	0.991
240.111	229.756	0.987
257.069	300.843	0.984
276.693	376.169	0.873

Table 3. Comparison of Bar and Experimental Modes.

MODELING THE RIB WITH QUAD ELEMENTS

We modeled the rib a second time, using Quad elements instead of Bar elements. In order to add the quad elements, a row of points must be defined along the top edge of the rib. Then, the quad elements can be added between the points on the centerline of the plate, and the points on the top edge of the rib, as shown in Figure 8.

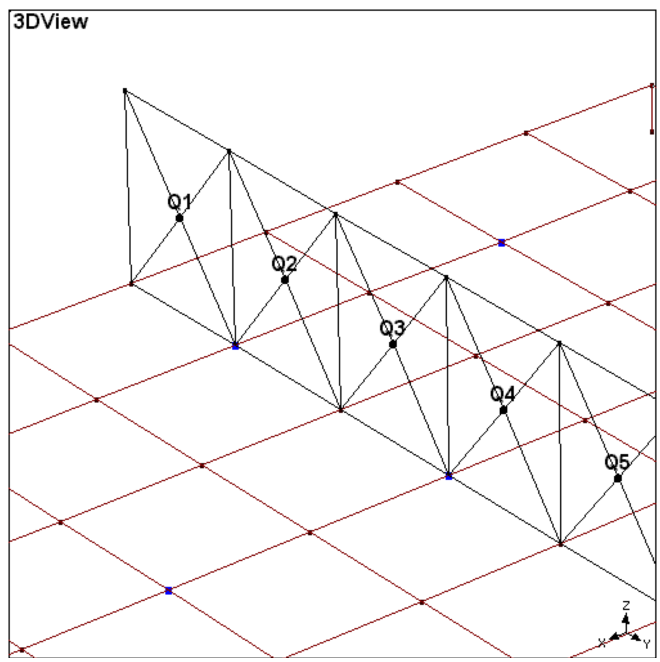

Figure 8. Zoomed View of Plate Showing Quad Elements.

Quad Properties

The following properties were used to define the Quad elements,

Thickness: 0.375 in
Elasticity: 10^7 lb/in^2
Poissons Ratio: 0.33
Density: 0.101 lb/in^3

Comparison of Quad With FEA Modes

Table 4 contains a comparison of the SDM modes obtained with Quad elements, and the FEA modes of the plate with rib.

FEA Freq. (Hz)	Quad Freq. (Hz)	MAC
103.559	98.353	0.999
175.424	194.361	0.881
229.420	221.264	0.983
235.542	256.233	0.862
262.336	262.576	0.988
420.103	413.176	0.999
451.394	463.337	0.989
470.122	505.967	0.997
537.343	568.738	0.997
673.419	681.484	0.982
698.029	704.119	0.963
736.098	750.439	0.963

Table 4. Comparison of Quad and FEA Modes.

Comparison of Quad With Experimental Modes

Table 5 contains a comparison of the SDM modes obtained with Quad elements, and the experimental modes of the plate with rib.

Exp. Freq. (Hz)	Quad Freq. (Hz)	MAC
103.341	100.264	0.996
188.005	208.138	0.993
240.111	237.608	0.997
257.069	287.354	0.986
276.693	391.631	0.913

Table 5. Comparison of Quad and Experimental Modes.

CONCLUSIONS

We modeled and tested an aluminum plate with rib, and used both FEA and experimental modes to evaluate the SDM method using higher order elements.

The FEA mode shapes for the unmodified structure agreed closely with the experimental mode shapes, but the higher modal frequencies didn't match well. Using a micrometer, we found variations in the thickness of the test specimen, which could explain some of this discrepancy.

Bar and Quad finite elements were used with SDM to model the rib stiffener modification to the plate structure. Both element types yielded acceptable results, when compared to the experimental and FEA results.

To evaluate the structural modification results objectively, we used the FEA modes of the unmodified plate, and compared the SDM results with the FEA modes of the plate with rib. Likewise we used the experimental modes of the unmodified plate, and compared the SDM results with the experimental results for the plate with rib.

Looking carefully at Figures 6 and 8, reveals another very useful property of this method. The Bar and Quad elements were connected between points along the centerline that had no modal data available for them. In this implementation of SDM, static condensation is used to automatically remove from the model all DOFs that don't have modal data.

In other words, points that had no modal data were treated as if they weren't there. The modification elements simply "bridged the gap" between points that had no modal data.

This very useful result allowed us to compare analytical results with experimental results, even though the analytical modes were defined on a grid with twice the point density as the experimental modes.

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