Mass, Stiffness, and Damping Matrix Estimates from Structural Measurements

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ABSTRACT

Modal testing has traditionally been used to confirm the validity of finite element models of structures. This is normally done by identifying the modal properties of a structure from test data, and then comparing them with the modal properties of the finite element model.

In this paper an alternative approach of directly comparing the mass, stiffness, and damping matrices of the dynamic model is explored. A new algorithm which estimates the mass, stiffness, and damping matrices of a structure from Frequency Response Function (FRF) measurements is also presented. These matrix estimates are compared to the matrices of an analytical model of the same structure, and the differences are noted.

Clearly, if the dynamics of the structure are represented by FRF measurements, and some significant part of the measurement data is left out of the estimation process, the matrix estimates will be deficient. The effect upon the accuracy of the matrix estimates is illustrated by examples, for cases when an inadequate frequency range of FRF data is used, and when a reduced number of degrees-of-freedom (DOFs) is used.

NOMENCLATURE

n = number of DOFs of the model
m = number of modes
r = number of reference points of the FRF measurements
t = time variable
s = Laplace variable

$[M]$ = mass matrix $(n \times n)$
$[C]$ = damping matrix $(n \times n)$
$[K]$ = stiffness matrix $(n \times n)$

$\{x(t)\}$ = vector of displacements $(n \times 1)$
$\{x'(t)\}$ = vector of velocities $(n \times 1)$
$\{x''(t)\}$ = vector of accelerations $(n \times 1)$
$\{f(t)\}$ = vector of externally applied forces $(n \times 1)$
$\{X(s)\}$ = vector of Laplace transforms of displacements $(n \times 1)$
$\{ICs\}$ = vector of initial condition terms $(n \times 1)$

$[B(s)]$ = system matrix $(n \times n)$
$[H(s)]$ = transfer matrix = $[B(s)]^{-1}$ $(n \times n)$

$[H(t)]$ = r columns of the matrix of impulse responses $(n \times r)$
$[L]$ = matrix of modal participation factor $(n \times r)$
$[U]$ = matrix of mode shapes $(n \times m)$

p_k = pole location for the k^{th} mode = $-\sigma_k + j\omega_k$
σ_k = damping of the k^{th} mode
ω_k = damped natural frequency of the k^{th} mode, $k = 1,\ldots,m$

INTRODUCTION

During the past ten years, the majority of the research activity on the testing of structures has centered around the development of new methods for identifying the modal properties of structures. While the modes of vibration are a good means for comparing experimental and analytical (finite element model) results, many times it would be more useful to obtain the mass, damping, and stiffness properties of the structure directly from measured data.

In cases such as automatic control system design, accurate estimates of the mass, stiffness, and damping properties for only a few DOFs may be all that is required in order to build an effective controller.

The method presented here is the outgrowth of our recent experiences with a new multiple reference algorithm for identifying modal parameters. This algorithm was first described in [2]. From examination of the curve fitting equations, (shown later), it is clear that the curve fitting operation itself actually yields two matrices which are then used to form an eigenvalue problem, which is then solved to find the modal properties of the structure. In this paper, however, it is shown how these intermediate matrix solutions can be used to recover the mass, damping, and stiffness properties instead of the modal properties.

In a previous paper [4], it was shown how the mass, stiffness, and damping matrices can be recovered from the modal properties of a structure. This approach turned out to be very sensitive to errors in the modal properties themselves, and also gave poor results if an inadequate number of modes was used. The approach taken here avoids the use of modes altogether, and hence avoids these causes of error.

In his recent thesis [1] on this subject, J. Leuridan named this approach to parameter identification the "Direct Parameter Identification" method. He has categorized a variety of time-domain and frequency-domain approaches for solving this problem. His primary concern, however, was with the identification of the modal properties of a structure.

BACKGROUND

Most analyses of the dynamics of structures are based upon the use of a set of linear second-order differential equations. For a structural model with *n* degrees-of-freedom, the equations can be written in the following form:

$$[M]\{x''(t)\}+[C]\{x'(t)\}+[K]\{x(t)\}=\{f(t)\} \qquad (n\times 1) \quad (1)$$

These equations are a statement of Newton's Second law involving all of the DOFs which are chosen for the model. The coefficient matrices, $([M], [C], \& [K])$, contain constants which represent the mass, damping, and stiffness properties of the structure, at least for the DOFs which are included in the model.

Since the equations of motion are linear, we can transform them into the Laplace domain without losing any information:

$$s^2[M]\{X(s)\}+s[C]\{X(s)\}+[K]\{X(s)\}=\{F(s)\}+\{ICs\} \qquad (n\times 1) \quad (2)$$

All of the physical properties of the structure are preserved on the left-hand side of the equations, while all of the applied forces and initial conditions (ICs) appear on the right-hand side. The initial conditions can be treated as a special form of the applied forces, and hence can be dropped from consideration in the following development without loss of generality.

To focus our attention on the physical properties, namely, the mass, damping, and stiffness properties, the equations can be written:

$$[B(s)]\{X(s)\}=\{F(s)\} \qquad (n\times 1) \quad (3)$$

where:

$$[B(s)]=s^2[M]+s[C]+[K] \qquad (n\times n) \quad (4)$$

$[B(s)]$ is called the **System Matrix.**

Alternatively, the transformed equations of motion can be written:

$$\{X(s)\}=[H(s)]\{F(s)\} \qquad (n\times 1) \quad (5)$$

where $[H(s)]$ is called the **Transfer Matrix,** or the matrix of transfer functions.

Clearly, the System Matrix and the Transfer Matrix are inverses of one another. That is, they satisfy the equation:

$$[B(s)][H(s)]=[I] \qquad (n\times n) \quad (6)$$

where $[I]$ is the identity matrix.

The above equation is true for all values of the *s*-variable, and in particular for its values along the frequency axis ($j\omega$ -axis) in the *s*-plane. Hence, for all values of frequency, the following is true:

$$[B(j\omega)][H(j\omega)]=[I] \qquad (n\times n) \quad (7)$$

The matrix $[H(j\omega)]$ is called the Frequency Response Function Matrix, or simply the **FRF Matrix.** This relationship is particularly useful since it involves only the mass, damping, and stiffness matrices, and FRFs, which can be measured on a structure with any modern-day multi-channel FFT analyzer.

CURVE FITTING METHOD

This curve fitting method is the outgrowth of some recent research at the University of Cincinnati, the results of which were presented in a recent IMAC paper [2]. The original implementation of the method did not explicitly include the conjugate (negative frequency) poles associated with each mode. The equations shown here, which do include the negative frequency poles, have yielded more consistent answers than those which so not include these extra terms.

The inverse Laplace transform of the Transfer Matrix is a matrix of **Impulse Response Functions,** which we will denote by $[H(t)]$. For the cases of multiple input locations, $[H(t)]$ becomes a rectangular matrix, with the number of columns equal to the number of input references. Hence, if a set of measurements is made on a structure with *n* DOFs and *r* references, then $[H(t)]$ is an $(n\times r)$ matrix.

Writing out this matrix in terms of modal parameters yields:

$$[H(t)] = [U][e^{pt}][L] + [U^*][e^{p^*t}][L^*] \quad (n \times r) \ (8)$$

where $[U]$ is an $(n \times m)$ matrix of complex mode shapes (m= the number of modes). $[e^{pt}]$ is an $(m \times m)$ diagonal matrix, with each "**p**" in the diagonal elements corresponding to a mode's complex pole location (frequency and damping values). $[L]$ is an $(m \times r)$ matrix of complex **modal participation factors.** These factors can be shown to be proportional to the mode shape values at the references.

Taking the time derivation of equation (8) yields:

$$[H'(t)] = [U][pe^{pt}][L] + [U^*][p^*e^{p^*t}][L^*] \quad (n \times r) \ (9)$$

At time t=0, the initial displacement impulse response and its time derivative can then be written:

$$[H(t=0)] = [U][L] + [U^*][L^*] \quad (n \times r) \ (10)$$

$$[H(t=0)] = [U][p][L] + [U^*][p^*][L^*] \quad (n \times r) \ (11)$$

Now, returning to the Laplace domain, the Transfer Matrix can also be written in terms of modal parameters:

$$[H(s)] = [U][T(s)] + [U^*][T^*(s)] \quad (n \times r) \ (12)$$

where:

$$[T(s)] = [s - p]^{-1}[L] \quad (n \times r) \ (13)$$

The matrix $[s-p]$ is an $(m \times m)$ diagonal matrix, each term containing the pole location of a mode. Using equations (10), (11), and (12) above, we can now write the following identities:

$$s[H(s)] - [H(t=0)] = [U][p][T(s)] + [U^*][p^*][T^*(s)] \quad (n \times r) \ (14)$$

$$s^2[H(s)] - s[H(t=0)] - [H'(t=0)] = [U][p^2][T(s)] + [U^*][p^{*2}][T^*(s)] \quad (n \times r) \ (15)$$

Returning, for a moment, to the equations of motion (2), the modal properties are actually solutions to the homogeneous equations:

$$[U][p^2] + [A_1][U][p] + [A_0][U] = [0] \quad (n \times n) \ (16)$$

where:

$$[A_0] = [M]^{-1}[K] \quad (n \times n) \ (17)$$

$$[A_1] = [M]^{-1}[C] \quad (n \times n) \ (18)$$

The conjugate modal parameters ($[U^*]$, and $[p^*]$) are also solutions to the homogeneous equations. This can be written in a manner similar to equation (16).

Finally, we can pre-multiply equations (12), (14), and (15) by the matrices $[A_0]$, $[A_1]$, and $[I]$, and write the result as:

$$[A_0][H(s)] + [A_1][s[H(s)]] - [H(t=0)] + [I][s^2[H(s)]] - s[H(t=0)] - [H'(t=0)] = [0] \quad (n \times r) \ (19)$$

Or, the above equation can be rewritten as:

$$[A_0][H(s)] + [A_1][s[H(s)]] + [B_0] + s[B_1] = -s^2[H(s)] \quad (20)$$

where:

$$[B_0] = [A_1][H(t=0)] - [H'(t=0)] \qquad [B_1] = -[H(t=0)] \quad (n \times r) \ (21)$$

Equation (20) **is the curve fitting equation,** and is valid for all values of the s-variable, in particular, those along the frequency axis $(s = j\omega)$. This equation is set up using measured FRF data $([H(j\omega)])$, and solved for the unknown real-valued matrices, $[A_0]$, $[A_1]$, $[B_0]$, and $[B_1]$.

Typically, a least squared error solution to equation (20) is found, by solving a slightly different set of linear equations, which allow any amount of FRF data to be used.

RECOVERING THE MASS, STIFFNESS, AND DAMPING MATRICES

The matrices $[A_0]$ and $[A_1]$ are estimated directly from the measured FRF data. Once these are known, the modal parameters $[U]$ and $[p]$ can be found by solving equation (16), as first described in [2]. Alternatively, the matrices $[A_0]$ and $[A_1]$ can be used, together with some additional assumptions, to recover the mass, stiffness, and damping matrices.

In general, there are more unknown elements in the mass, stiffness, and damping matrices than there are knowns in the

$[A_0]$ and $[A_1]$ matrices. To solve the problem, **three additional assumptions** will be made:

1) the stiffness matrix is symmetric

2) the mass matrix can be approximated by a diagonal matrix

3) the total mass of the structure is known

Using the second assumption, each element in the i^{th} row and j^{th} column of the $[A_0]$ matrix can be written as:

$$a_{ij} = k_{ij}/m_i \qquad i, j = 1,2,...,n \tag{22}$$

where:

k_{ij} = the element in the i^{th} row and j^{th} column of the unknown stiffness matrix, and

m_i = the i^{th} element of the diagonal mass matrix

But, since $[K]$, is assumed to be symmetric,

$$a_{ij}/a_{ji} = m_j/m_i \qquad i, j = 1,2,...,n \tag{23}$$

It is clear that equation (23) can be written $(n-1)$ times, yielding relative values for all of the diagonal mass matrix elements. If the total mass of the structure is known, all of the mass matrix elements can be determined by using the one additional equation:

$$\sum m_i = M \tag{24}$$

where M is the total mass.

After the mass matrix elements are calculated, the stiffness matrix can be calculated using equation (17). Similarly, the damping matrix can be recovered using equation (18).

A FIVE DEGREE-OF-FREEDOM EXAMPLE

In order to demonstrate the feasibility of this method for estimating mass, stiffness, and damping parameters, we will begin with a simple five degree-of-freedom model, as shown in Figure 1. The five DOFs are five discrete masses, with mass values of 0.4, 0.8, 1.2, 1.6, and 2.0. These five point masses are connected in series by springs (with constants of 36,000), and dashpots (with constants of 12.0). The fifth mass is also connected to ground.

The mass, stiffness, and damping matrices for the five degree-of-freedom structure are shown in Table 1. The modal

Figure 1. The Five Degree-of-Freedom Model

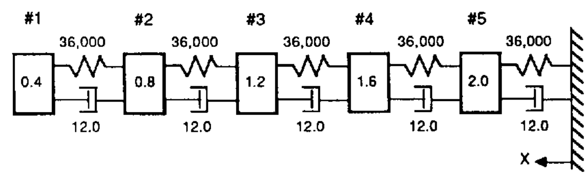


Table 1. Mass, Stiffness, and Damping Matrices of the Five DOF Model

Mass Matrix (diagonals only)	0.4	0.8	1.2	1.6	2.0
Stiffness Matrix	36000.0	- 36000.0	0.0	0.0	0.0
	− 36000.0	72000.0	- 36000.0	0.0	0.0
	0.0	- 36000.0	72000.0	- 36000.0	0.0
	0.0	0.0	- 36000.0	72000.0	- 36000.0
	0.0	0.0	0.0	- 36000.0	72000.0
Damping Matrix	12.0	− 12.0	0.0	0.0	0.0
	− 12.0	24.0	− 12.0	0.0	0.0
	0.0	− 12.0	24.0	− 12.0	0.0
	0.0	0.0	− 12.0	24.0	− 12.0
	0.0	0.0	0.0	− 12.0	24.0

frequencies, damping, and mode shapes for the structure are shown in Table 2. FRF measurements were synthesized, using the modal data, for three reference points (at masses 1, 2, and 3). An example of one of the FRFs is shown in Figure 2.

The FRF measurements were curve fit using a least squared error version of equation (20), to yield estimates of the matrices $[A_0]$ and $[A_1]$. The results are listed in Table 3. During the curve fitting process, FRF data from all five response DOFs, all three references, and in 2 Hz frequency ranges centered around each of the five resonance peaks, were used.

The mass, stiffness, and damping matrix estimates, (recovered by using equations (17), (18), and (22) through (24), are shown in Table 4. The percentage of error of these estimates, as compared with the correct values in Table 2, are all **less than .005%.** Clearly, these errors are all very small, and can be attributed mainly to the computational accuracy of the computer.

Table 2. Modal Parameters at the Five DOF Model

		1	2	3	4	5
Mode Number		1	2	3	4	5
Frequency (Hz)		9.11	23.56	35.81	46.14	63.89
Damping (%)		0.95	2.47	3.75	4.83	6.69
Mode Shapes	DOF#1	0.580	0.635	0.593	0.623	1.011
	DOF#2	0.559	0.480	0.259	0.041	-0.799
	DOF#3	0.497	0.092	-0.366	-0.617	0.252
	DOF#4	0.380	-0.364	-0.373	0.453	-0.050
	DOF#5	0.209	-0.459	0.459	-0.170	0.007

Table 3. Estimated $[A_0]$ and $[A_1]$ Matrices

$[A_0]$ Matrix

90002.15	- 90002.30	- 0.09	0.09	0.34
- 45001.59	90001.69	- 44999.75	- 0.26	– 0.21
0.29	- 30000.20	59999.70	- 29999.82	0.01
- 0.03	- 0.11	- 22499.80	44999.93	– 22499.98
0.09	- 0.11	- 0.01	- 17999.97	36000.04

$[A_1]$ Matrix

30.00008	– 30.00099	0.00035	0.00101	– 0.00073
- 15.00031	30.00075	– 15.00006	– 0.00097	0.00060
0.00020	– 10.00029	19.99981	– 9.99953	– 0.00014
- 0.00010	0.00013	– 7.49992	14.99966	– 7.49984
- 0.00002	– 0.00009	0.00009	– 5.99995	11.99987

Table 4. Estimated Mass, Stiffness, and Damping Matrices

Mass Matrix (diagonals only)

0.40001	0.80001	1.19999	1.60000	2.00000

Stiffness Matrix

36001.47	- 36001.53	- 0.03	0.03	0.13
- 36001.53	72001.86	- 36000.05	– 0.21	- 0.17
0.35	- 36000.05	71999.28	- 35999.61	0.01
- 0.04	- 0.17	- 35999.61	71999.74	- 35999.89
0.00	0.00	0.00	- 35999.89	71999.95

Damping Matrix

12.00024	- 12.00060	0.00014	0.00040	– 0.00029
- 12.00033	24.00077	- 12.00013	– 0.00077	0.00048
0.00024	- 12.00029	23.99965	- 11.99938	– 0.00016
– 0.00016	0.00021	- 11.99986	23.99941	- 11.99972
0.00004	– 0.00017	0.00018	- 11.99989	23.99969

EFFECTS OF LIMITED FREQUENCY RANGE IN THE MEASUREMENTS

Experimental FRF data which is measured with a multi-channel FFT analyzer will typically contain frequency responses from close to DC (zero frequency) up to the cutoff frequency of the anti-aliasing filters on the front end of the analyzer. Even though the cutoff frequency used during the measurement process will typically be less than the highest modal frequency of the test structure, this does not necessarily mean that we have completely lost the characteristics of the high frequency modes. In fact, the high frequency modes will still contribute to the measured FRF data in the lower frequency range, even though their contribution will not be as great as that of the lower frequency modes. The purpose of this section of the paper is to demonstrate the reduced effects of the high frequency modes on the mass, stiffness, and damping matrix estimates.

Synthesized FRF data of the five degree-of-freedom model (shown in Figure 1) was used in the following two analyses. All five measurements points, and all three references, were used in both examples.

Example #1: In this example the mass, stiffness, and damping matrices were estimated using FRF data in the range from 8.0 Hz to 47.0 Hz. This includes the 2 Hz frequency ranges centered at the resonance peaks of the **first four modes,** but excludes the fifth mode.

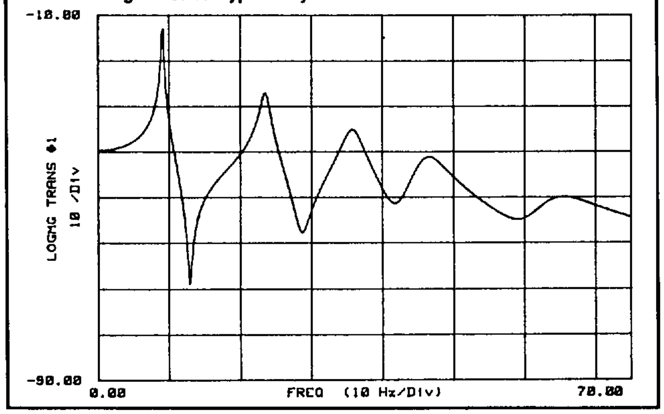
Errors in the matrix estimates are shown in Table 5. All of the errors are within 0.1% for the mass and stiffness, and within 0.6% for the damping matrix.

Example #2: In this example the mass, stiffness, and damping matrices were estimated using FRF data in the range from 8.0 Hz to 38.8 Hz. This includes the 2 Hz frequency ranges centered at the resonance peaks of only the **first three modes,** but excludes the fourth and fifth modes.

Errors in the matrix estimates are shown in Table 6. Errors for the mass and stiffness have now grown to within 0.4%, while the maximum error for the damping matrix is 13.2%.

These results, though still very idealized when compared with an actual test situation, nevertheless suggest that accurate mass, stiffness, and damping estimates can still be obtained from FRF data that is frequency limited. It is also noteworthy that damping cannot be estimated as accurately as mass and stiffness. This certainly agrees with our previous parameter estimation experience.

Figure 2. A Typical Synthesized FRF Measurement



EFFECTS OF A LIMITED NUMBER OF DOFs

Experimental FRF data is typically taken from a relatively small number of discrete points and directions (DOFs) on a structure, and these measurements may not include all of the "physically significant" DOFs of the structure.

In applications such as automatic control, a mass, stiffness, and damping model involving only a small number of DOFs may be all that is needed to build an effective controller. However, questions related to whether the mass or stiffness distribution of the structure can be adequately modeled with such a truncated model have to be answered before it could be used. In this section, the effects of reducing the number of measurement locations, of DOFs, in the parameter estimation process are demonstrated.

Synthesized FRF data for the five degree-of-freedom structure in Figure 1 was again used in the following two examples. In both cases, FRF data in 2 Hz frequency bands centered around each modal resonance peak, was used.

Table 5. Errors Due to Truncation of One Modal Frequency Band

Mass Matrix (diagonals only)	– 0.1%	– 0.1%	0.0%	0.0%	0.0%
Stiffness Matrix	- 0.1%	0.1%	0.0%	0.0%	0.0%
	0.1%	- 0.1%	0.1%	0.0%	0.0%
	- 0.0%	0.1%	0.0%	0.0%	0.0%
	0.0%	0.0%	0.0%	0.0%	0.0%
	0.0%	0.0%	0.0%	0.0%	0.0%
Damping Matrix	- 0.1%	0.0%	0.0%	0.0%	0.0%
	- 0.2%	0.1%	- 0.1%	0.0%	0.0%
	0.2%	- 0.6%	0.1%	- 0.1%	0.0%
	0.0%	- 0.1%	0.1%	0.0%	0.0%
	- 0.1%	0.2%	- 0.1%	0.0%	0.0%

Table 6. Errors Due to Truncation of Two Modal Frequency Bands

Mass Matrix (diagonals only)	– 0.1%	– 0.2%	– 0.2%	0.1%	0.1%
Stiffness Matrix	0.2%	- 0.4%	0.1%	- 0.1%	0.0%
	- 0.4%	0.3%	- 0.3%	0.1%	0.0%
	0.1%	- 0.3%	- 0.1%	0.2%	0.0%
	0.0%	- 0.1%	0.2%	0.0%	0.0%
	0.0%	0.0%	0.0%	0.0%	0.1%
Damping Matrix	- 4.6%	7.4%	- 1.8%	0.5%	- 0.1%
	11.7%	- 9.2%	8.6%	- 1.1%	0.2%
	- 4.2%	13.2%	- 2.9%	1.3%	- 0.1%
	- 0.2%	0.5%	- 0.1%	0.3%	- 0.8%
	0.6%	- 0.8%	0.3%	0.0%	0.0%

Example #1: In this first example, FRF data was used for the **first four** of the five point masses. In other words, measurements from the grounded point mass were excluded. The total mass of the structure was also taken as 4.0, instead of 6.0, since the fifth mass was excluded.

The percentage errors for the mass, stiffness, and damping matrices of the four DOF model are shown in Table 7. Errors in all three matrices are within 0.4%, except for the elements in the fourth row of the stiffness and damping matrices. Those estimates are too erroneous to be useful. However, the estimates in the fourth column of these matrices, (which are assumed to be symmetric), are within acceptable accuracy.

Example #2: In this example, FRF data for **only the first three point masses** was used. The total mass of the structure was taken to be 2.4, since the fourth and fifth masses were excluded.

The errors of the mass, stiffness, and damping estimates are listed in Table 8. The errors of all the elements of all three matrices are within 0.9%, with the exception, again, of the last row of the stiffness and damping matrices.

These results indicate that the mass, stiffness, and damping properties of a structure can still be accurately identified with this estimation method, even when measurements from significant parts of the structure are not used. In the first example, the values of four point masses, plus the three springs and dampers connecting them together, were accurately identified. In the second example, the values of three point masses, plus the two springs and dampers that connect them together, were again correctly identified.

Table 7. Errors Due to Truncation of One Measurement Location

Mass Matrix (diagonals only)				
	0.4%	0.4%	0.4%	− 0.6%

Stiffness Matrix				
	0.4%	- 0.4%	0.0%	0.0%
	- 0.4%	- 0.4%	- 0.4%	0.0%
	0.0%	- 0.4%	0.4%	0.4%
	2.2%	3.9%	- 0.4%	- 35.4%

Damping Matrix				
	0.4%	- 0.4%	0.0%	0.0%
	- 0.4%	0.4%	- 0.4%	0.0%
	0.0%	- 0.4%	0.4%	- 0.4%
	52.8%	13.2%	- 306.6%	77.2%

Table 8. Errors Due to Truncation of Two Measurement Locations

Mass Matrix (diagonals only)			
	− 0.9%	− 0.9%	0.9%

Stiffness Matrix			
	− 0.9%	0.9%	0.0%
	0.9%	− 0.9%	0.9%
	10.9%	0.9%	- 49.3%

Damping Matrix			
	− 0.9%	0.9%	0.0%
	0.9%	− 0.9%	0.9%
	- 13.1%	- 174.5%	85.2%

CONCLUSIONS

We have applied a curve fitting technique which was originally developed for finding the modal properties of a structure from experimental FRF data [2], to the estimation of the physical mass, stiffness, and damping properties of the structure instead.

We tried out the method on an admittedly very simple analytical lumped-parameter model at first, so that we could compare its answers to the correct results. In anticipation of its use on real world problems, though, we simulated two different conditions which will occur in testing situations; namely, its use with FRF measurements which were taken:

1) over **limited frequency ranges,** and

2) from a **limited number of DOFs** on the structure

In both tests, the method yielded very usable results.

Our plan is to apply this method next to a real structure, and also to build a finite element model of the structure so that we can compare results.

A key advantage of this technique is that it can be used on the type of data, namely FRF data, that is commonly measured in a structural testing laboratory using a multi-channel FFT analyzer system. Another advantage is that it does not require the use of modal parameters, which we have found through prior experience [4] to be a significant source of errors. Also, the algorithm will handle single reference as well as multiple reference FRF data [3], which gives it the advantage of yielding more accurate results in the presence of noise and other measurement errors.

One drawback of this method is that an estimate of the total mass of the structure, only for the DOFs where measurements are made, must be known beforehand. On the other hand, when a total mass value is estimated correctly, the correct mass distribution, as well as the stiffness and damping matrices result.

Another drawback of this method is that large problem sizes (many DOFs) yield very large internal matrices. This not only requires large amounts of computer memory, but can also render this method infeasible for use on the desktop computers which are commonly used in testing laboratories today.

Further development is needed in order to make this a reliable tool for use in structural testing laboratories.

REFERENCES

[1] Leuridan, J. "Some Direct Parameter Identification Methods Applicable for Multiple Input Modal Analysis", Ph.D. Thesis, University of Cincinnati, Mar, 1984.

[2] Lembregts, F., Leuridan, J., Zhang, L., and Kanda, H. "Multiple Input Modal Analysis of Frequency Response Functions Based on Direct Parameter Identification", Proceedings of the Fourth IMAC, Los Angeles, CA, Feb 3-6, 1 986.

[3] Shye, K., VanKarsen, C., and Richardson, M. "Modal Testing using Multiple References", Proceedings of the Fifth IMAC Conference, Imperial College, London, England, April 6-9, 1987.

[4] Potter, R. and Richardson, M. "Mass, Stiffness, and Damping Matrices from Measured Modal Parameters", ISA International Instrumentation - Automation Conference, New York, NY, Oct 28-31, 1974.