Detection and Location of Structural Cracks using FRF Measurements

by
M.A. Mannan
Royal Institute of Technology, Stockholm, Sweden
And

Mark H. Richardson
Structural Measurement Systems, Milpitas, California

Abstract

This is follow-on work to an IMAC paper given last year [1] where it was shown that modal testing can be used to detect "faults" in mechanical structures. By "faults", we mean any of the following occurrences:

- failure of the structural material, e.g. cracking, breaking, or delamination.
- loosening of assembled parts, e.g. loose bolts, rivets, or glued joints.
- flaws, voids, cracks, thin spots, etc. caused during manufacturing.
- improper assembly of parts during manufacturing.

The underlying principle behind this fault detection method is that vibration is a sensitive indicator of the physical integrity of any mechanical structure. Or, more specifically, if any of the mass, stiffness, of damping properties of the structure change due to a structural fault, then its vibrational response will change, and this change can be accurately measured using standard modal testing methods.

In this paper, we carry this approach one step farther, and discuss the problem of not only *detecting,* but also *locating,* or at least *localizing,* a structural fault. We present a method for determining the mass, stiffness, and damping properties of the structure from measured Frequency Response Functions (FRFs), and show how changes in these parameters can be used to localize the fault.

Introduction

The linear dynamics of structures are commonly represented by the "force balance" shown in Figure 1. This equation balances the internal forces within a structure, which are functions of its mass, damping and stiffness properties, with any externally applied forces, which are written on the right hand side of the equation.

Structures begin to vibrate when external forces are applied to them and the resulting energy becomes trapped within the their boundaries. This energy is then "traded back and forth" between the inertial (mass) properties and the restoring (stiffness) properties.

If the external forces are removed, the structure will continue to vibrate, but eventually, the dissipative (damping) properties will dissipate the energy, and the structure will stop vibrating.

The structural faults listed above will all have an effect on the mass, damping, and stiffness properties of a structure. All of them should cause a decrease in the structure's stiffness, and some will also affect its mass and damping properties. Therefore, structural faults should *always, at a sufficient level of severity,* cause a change in a structure's vibrational behavior.

Equivalent Forms of Structural Dynamics

In addition to its differential equations of motion, a structure's linear dynamics can be represented in several other equivalent forms, as shown in Figure 2. FRFs, Impulse Responses, or modal parameters each fully represent the linear dynamic properties of a structure. Consequently, if any of the mass, damping or stiffness properties of a structure should change, we should expect its dynamic response, and also its FRFs, Impulse Responses, and modal parameters to change.

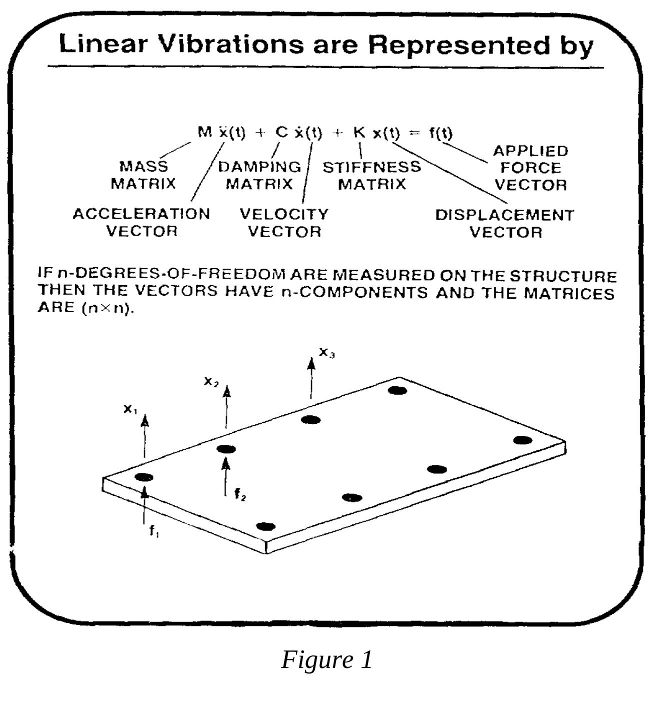

Figure 1

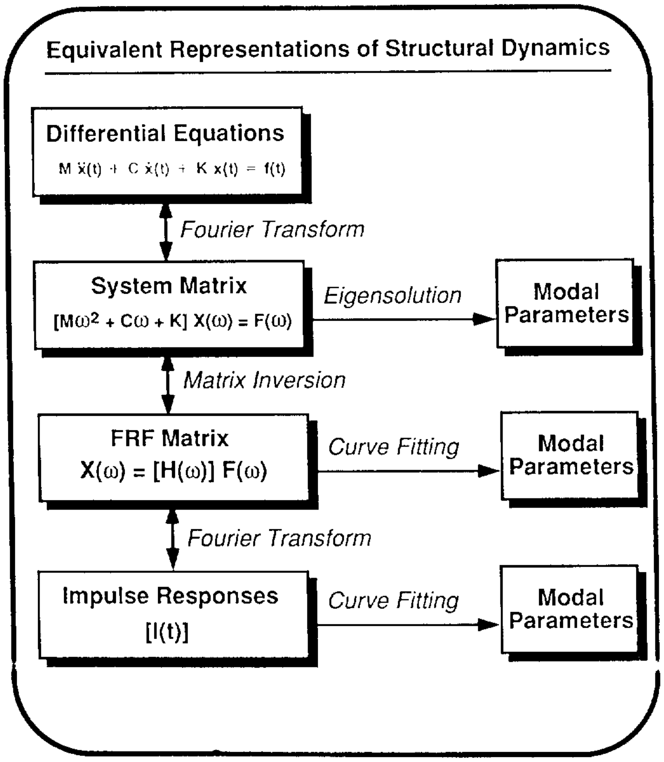

Figure 2

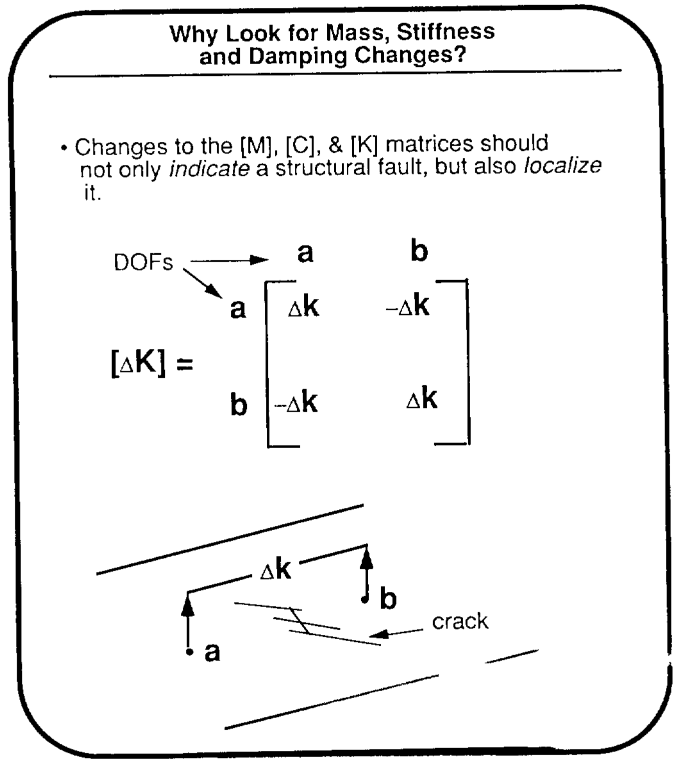

Figure 3

Conversely, if the measured FRFs, Impulse Responses, or modal parameters of a structure were to change, then we can expect that the mass, damping, and stiffness properties should have changed also.

Structural Faults and Changes in Modal Parameters

In a previous IMAC paper [1], it was shown that significant and easily detectable changes in the modal parameters of a plate-with-rib structure occurred when a bolt attaching the rib to the plate was removed. Changes of the modal frequencies were easily found by curve fitting FRF measurements taken from the structure. Furthermore, since the modes are "global" properties of the structure, their frequency shifts can be detected from FRF measurements which are taken from practically any point on the structure.

Secondly, changes in the mode shapes from before and after the fault was induced, were detected by using the Modal Assurance Criterion (MAC) on the mode shapes. The MAC calculation, which essentially measures the amount of correlation, or likeness, between two mode shapes, has proven to be a very sensitive indicator of changes in the mode shapes. Extensive testing performed recently by NASA on an aircraft structure [2], has also demonstrated the usefulness of modal frequency shifts and the MAC calculation as means of detecting structural faults.

Why Consider Mass, Stiffness, and Damping Changes?

Although changes in the modal parameters are a sensitive means of *detecting* structural faults, unless the modal parameters measured are those of "local" modes, their changes cannot be used to *locate* the fault. (Local modes usually occur at higher frequencies, and are the result of energy being trapped in local areas of a structure). Modal parameters, especially for the lower frequency modes, are "global" properties of the structure, and hence will change "all over" the structure, even though the fault may have occurred locally. This was demonstrated in [1] where the removal of a bolt, i.e. a local fault, could not be pinpointed by examining the mode shapes, even though some of them changed substantially.

From an analytical standpoint, a stiffness change between two DOFs on a structure is modeled as shown in Figure 3. This results from a simple application of Hooke's Law between the two DOFs, and is the principle used in both the Finite Element Method and the Structural Dynamics Modification (SDM) technique for modeling stiffness changes in structures.

Hence, if, for instance, a crack occurred in a local area of a structure, one would expect the stiffnesses between those DOFs in close proximity to the crack to change more than the stiffnesses between DOFs far away from the crack. Similarly, if mass were removed from a local area, due to a casting void or break-off of a part, then the mass matrix of the structure should change more for those DOFs which are close to the mass removal point than for other DOFs.

This, then, is the motivation for examining changes in the mass, damping, and stiffness matrices of a structure.

Why Start With FRF Measurements?

A variety of estimation methods can be used for estimating the $[M]$, $[C]$, and $[K]$ matrices from measured data. For this paper, $[M]$, $[C]$, and $[K]$ were computed with formulas which involve the pseudo-inverse of the mode shape matrix. (See reference [3]). The modal parameters were obtained by curve fitting a set of FRF measurements in the normal manner of a modal test.

In comparison to any "direct" form of parameter estimation, either time domain or frequency domain based, measuring FRFs first, and then processing them to obtain the desired results, offers a number of advantages:

- A variety of multi-input / multi-output FFT analyzers are commercially available for making FRF measurements.
- A variety of broad band excitation methods can be used, employing low level random, sine, or transient signals.
- Measurement noise can be removed by using frequency domain averaging methods.
- Non-linear motion (distortion) of the structure can be removed by using random excitation and averaging.
- Acceleration responses, which are typically measured, are easily converted to displacement responses without approximations.
- A variety of single and multiple reference estimation techniques are available for obtaining modal parameters from FRFs.

The Computational Method

The equations used for the computation of the $[M]$, $[C]$, and $[K]$ matrices are derived from the orthogonalitiy properties of classically, or lightly, damped systems. They also utilize the pseudo-inverse of the mode shape matrix, which is easily obtained once a singular value decomposition (SVD) has been performed on the mode shape matrix.

The full mass, damping, and stiffness matrices are then computed with the formulas:

$$
\begin{aligned}
[M] &= [U^t]^+ [-m-][U]^+ \qquad (n \text{ by } n) \\
[C] &= [U^t]^+ [-c-][U]^+ \qquad (n \text{ by } n) \\
[K] &= [U^t]^+ [-k-][U]^+ \qquad (n \text{ by } n)
\end{aligned}
\tag{1}
$$

where:

$[U]^+$ = pseudo-inverse of the mode shape matrix

$[-m-]$ = modal mass matrix $(m \text{ by } m)$

$[-c-]$ = modal damping matrix $(m \text{ by } m)$

$[-k-]$ = modal stiffness matrix $(m \text{ by } m)$

n = number of DOFs

m = number of modes

t -denotes the transpose

We have found this procedure to be very stable computationally, and to yield matrices from which the original modal parameters can still be recovered by eigensolution. This is shown in [3].

The real benefit of this method, though, is that if the mode shapes contain n-DOFs, then the full $(n \text{ by } n)$ $[M]$, $[C]$, and $[K]$ matrices can be computed, regardless of the number of modes used. However, if a sufficient number of modes is not used which adequately represents the structure's dynamics, then the $[M]$, $[C]$, and $[K]$ estimates will, of course, be incorrect. This modal truncation effect will be illustrated in the following example.

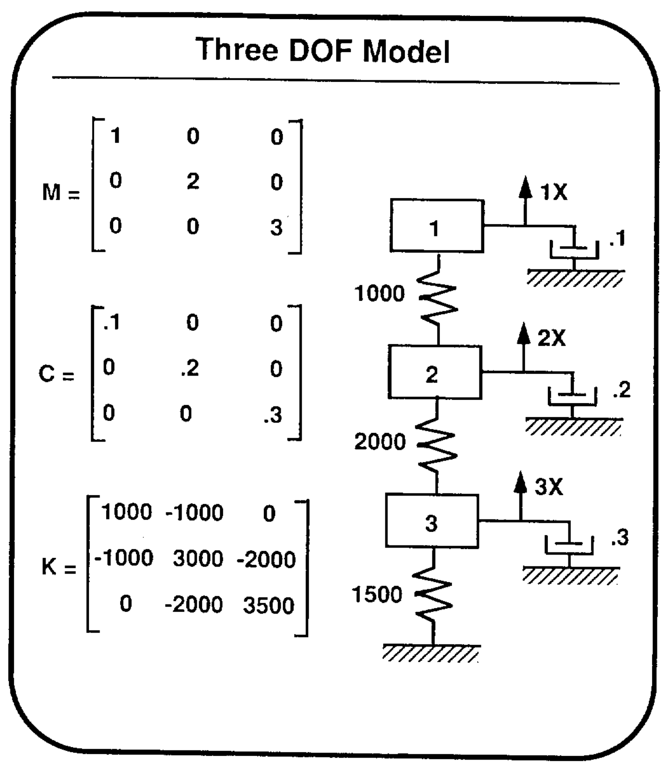

Figure 4

Illustrative Example

To simulate a structural fault, a stiffness change was made to the 3-DOF structure shown in Figure 4. To make the fault rather "insignificant", a 5% reduction in the stiffness between masses 2 and 3, (from 2000 force/displacement units to 1900 force/displacement units), was made. The modal parameters of the structure before and after the change was made are shown in Figure 5. Notice that all of the modal frequencies did shift downward. The mode shapes, however, remained virtually unchanged.

Figure 6 shows the driving point FRFs at DOF 1X, for before and after the simulated fault. This makes it clear that the frequency shift for the higher frequency (7.8 Hz) mode was greater than for the other two.

Equations (1) were then used to compute the $[M]$, $[C]$, and $[K]$ matrices for the structure using its modal parameters. If all three modes are used, then the pseudo-inverse of the mode shape matrix will equal the ordinary inverse, and the formulas in (1) will yield *exactly* the same values as those given in Figure 4. If we then took the difference between the stiffness matrices from before and after the stiffness change, we would clearly see that stiffness between DOFs 2X and 3X was different by exactly 100 force/displacement units, while all the other differences would be zero. This non-zero difference of stiffness between DOF 2X and 3X could then be used for locating the fault.

However, a more realistic simulation is to use only modes 1 and 2 to compute $[M]$, $[C]$, and $[K]$, since we can never measure all of the higher frequency modes of real structures. The differences in $[M]$, $[C]$, and $[K]$, where only modes 1 and 2 are used, are shown in Figure 7. It is clear from this that using only these two modes is not sufficient for locating the fault. Apparently, the higher frequency mode is needed.

Also shown in Figure 7 are the differences in $[M]$, $[C]$, and $[K]$, where only modes 2 and 3 were used. This result clearly shows the location of the fault.

Modal Parameters for Three DOF Model

Without Simulated Fault		Mode 1	Mode 2	Mode 3
Frequency (Hz)		2.217	5.216	7.795
Damping (%)		0.359	0.153	0.102
Mode Shape	1X	0.557	0.733	-0.388
	2X	0.449	-0.054	0.543
	3X	0.308	-0.389	-0.294
With Simulated Fault $(\delta K_{23} = -100)$		Mode 1	Mode 2	Mode 3
Frequency (Hz)		2.205	5.187	7.681
Damping (%)		0.361	0.153	0.103
Mode Shape	1X	0.559	0.721	-0.407
	2X	0.452	-0.045	0.541
	3X	0.304	-0.398	-0.287

Figure 5

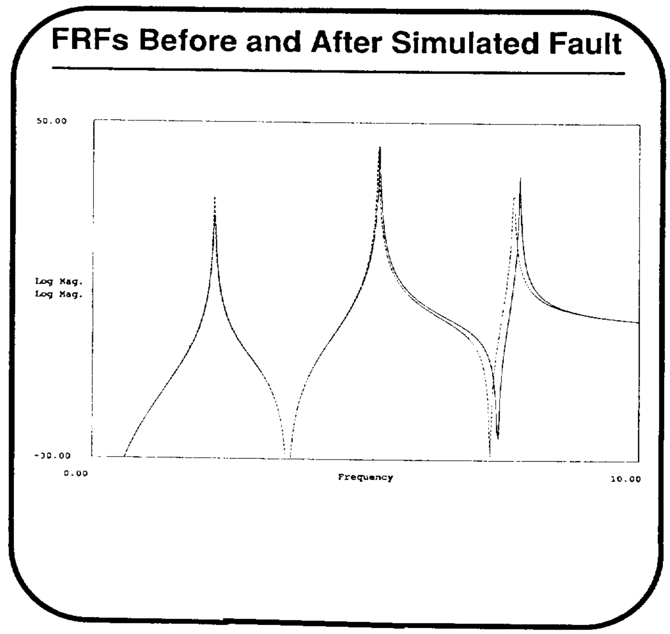

Figure 6

[M], [C], and [K] Differences

Using modes 1 & 2

mass
$$\begin{bmatrix} .001 & .013 & .014 \\ & .019 & .007 \\ (sym) & & .040 \end{bmatrix}$$

damping
$$\begin{bmatrix} .000 & .001 & .001 \\ & .002 & .001 \\ (sym) & & .004 \end{bmatrix}$$

stiffness
$$\begin{bmatrix} 13.58 & 15.25 & 9.90 \\ & 16.13 & 7.85 \\ (sym) & & 34.70 \end{bmatrix}$$

Using modes 2 & 3

mass
$$\begin{bmatrix} .001 & .005 & .001 \\ & .016 & .002 \\ (sym) & & .028 \end{bmatrix}$$

damping
$$\begin{bmatrix} .000 & .001 & .000 \\ & .016 & .002 \\ (sym) & & .003 \end{bmatrix}$$

stiffness
$$\begin{bmatrix} 2.94 & 8.89 & 7.93 \\ & 138.4 & 94.5 \\ (sym) & & 52.3 \end{bmatrix}$$

Figure 7

We can conclude from this example that, at least for faults which primarily cause stiffness modifications, changes in the lower frequency modes do not provide enough information to locate the fault, whereas changes in the higher frequency modes do allow us to locate the fault. We will find this same result in the following second example.

Testing of a Aluminum Plate Structure

To explore the practical usefulness of this technique, an aluminum plate structure was tested before and after a saw-cut was made in the structure to simulate a fault. The test points grid for the structure is shown in Figure 8. This structure measures 500 mm by 300 mm and has a thickness of 6 mm. As shown in Figure 8, a crack was simulated by making a 50 mm saw-cut between points 12 and 18 on the structure.

During the modal tests, the structure was supported by rubber bands which were attached to its four corners. Impact hammer tests were carried out to obtain the required FRF measurements, and all impacts and responses were measured only in the Z-direction, normal to the plate surface.

A two channel FFT analyzer together with the SMS STAR-Struct structural analysis software package running on a PC-AT were used for testing the structure. A total of 17 modes were identified in the measured frequency band from 18.75 Hz to 1580 Hz. Figure 9 shows typical FRF measurements, one from before and the other from after the saw-cut, with the modes indicated

The two sets of FRF measurements were curve fit, and the resulting modal data sets were then used in equations (1) to compute the structure's stiffness matrices for before and after the saw-cut. The difference between some of these stiffness values, (when only the three highest frequency modes were used), is shown in Figure 10. The columns of the stiffness matrix difference corresponding to DOFs 12Z, 18Z, and 24Z are shown. It is clear from this that the stiffness differences between those DOF pairs which "crossed over" the saw-cut were among the largest. There are, however, also some other "erroneous" large stiffness differences between DOFs along the edges of the plate.

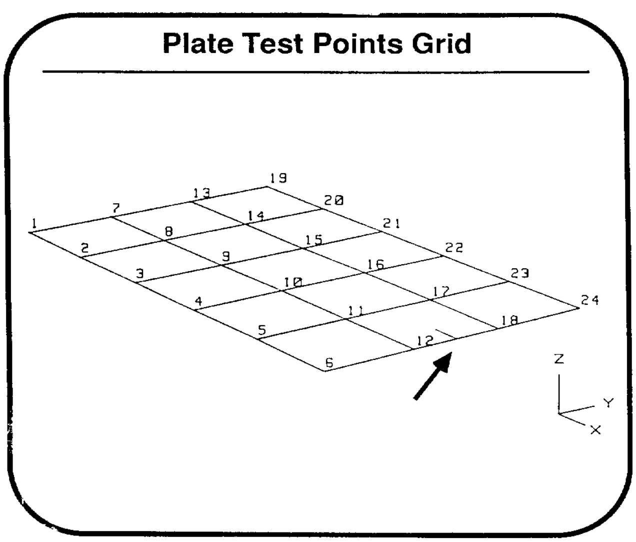

Figure 8

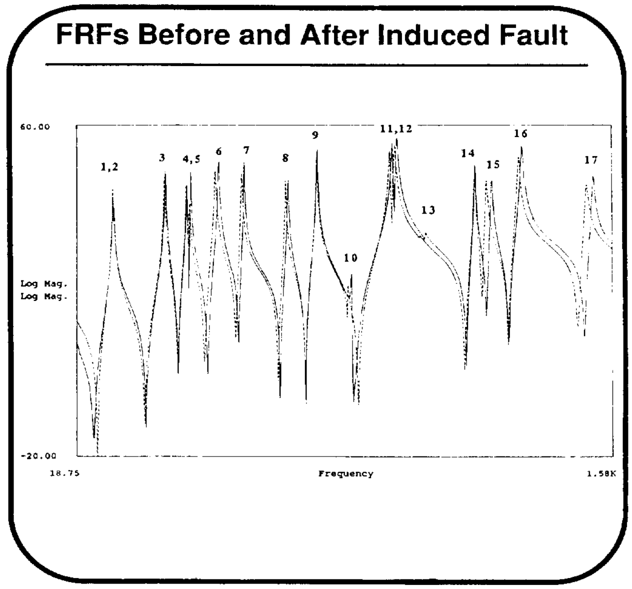

Figure 9

Stiffness Differences for the Plate

	DOF 12Z	DOF 18Z	DOF 24Z
1Z	5.056E+004	7.447L+004	6.401E+004
2Z	5.567E+002	4.206E+004	7.526E+004
3z	2.386E+004	6.535E+004	7.070E+004
4Z	1.621E+005	1.410E+005	1.345E+004
5z	6.043E+004	2.683E+004	7.068E+004
6Z	8.881E+003	4.392E+004	2.053E+005
7z	6.815E+004	3.837E+004	4.474E+004
8Z	8.671E+004	3.293E+004	8.058E+004
9Z	1.634E+004	8.864E+003	1.441E+004
10Z	5.553E+004	2.744E+004	9.964E+003
11Z	1.528E+003	2.744E+004	3.402E+004
12Z	1.209E+005	1.138E+005	5.632E+004
13Z	7.186E+004	9.313E+004	5.703E+004
14Z	6.287E+004	8.268E+004	6.053E+004
15Z	3.431E+004	3.904E+004	4.137E+004
16Z	5.048E+004	3.651E+004	1.645E+004
17Z	4.201E+004	7.405E+003	3.181E+004
18Z	1.138E+005	5.910E+004	4.872E+004
19Z	3.183E+004	5.645E+003	7.093E+004
20Z	7.205E+004	7.546E+003	1.245E+005
21Z	1.723E+004	4.467E+004	1.823E+004
22Z	1.504E+005	1.615E+005	9.945E+004
23Z	1.038E+005	6.072E+004	1.272E+005
24Z	5.632E+004	4.872E+004	1.049E+005

Figure 10

Conclusions

We have introduced here the concept of using not only changes in the modal parameters, but also in the distributed mass, damping, and stiffness parameters of a structure are a means of detecting and locating structural faults. We demonstrated in two simplified cases that the concept did point toward the fault area.

We discovered that for faults which cause stiffness changes, the higher frequency modes are most important. It is, of course, well known that stiffness is governed by the higher frequency modes in a structure.

We used a method for computing the $\left[M\right]$, $\left[C\right]$, and $\left[K\right]$ matrices which uses only modal data. This approach has the advantage of allowing the use of only those modes which are most influenced by the fault, i.e. have the largest frequency shifts or mode shape changes. However, it does not in all cases provide a "fool proof" answer which correctly pinpoints the fault area.

A different approach to estimating the $\left[M\right]$, $\left[C\right]$, and $\left[K\right]$ matrices which involves curve fitting the FRFs directly is being investigated. This may yield more accurate stiffness matrix estimates, even with FRF measurements which are made over lower frequency bands.

References

[1] Wolff, T. & Richardson, M. " Fault Detection in Structures from Changes in their Modal Parameters" Proceedings of the 7th IMAC Conference, Las Vegas, Nevada, January 30, 1989.

[2] West, W. "Structural Fault Detection of a Light Aircraft Structure using Modal Technology" NASA Lyndon B. Johnson Space Center, Loads and Structural Dynamics Branch Report, April, 1988.

[3] Luk, Yiu Wah "Identification of Physical Mass, Stiffness, and Damping Matrices using Pseudo-Inverses" Proceedings of the 5th IMAC Conference, London, England, April 6-9,1987.