CORRELATING MINUTE STRUCTURAL FAULTS WITH CHANGES IN MODAL PARAMETERS

Mark H. Richardson
Vibrant Technology, Inc.
Jamestown, CA

M. A. Mannan
Royal Institute of Technology
Stockholm, Sweden

ABSTRACT

Many types of structural faults, such as cracking, delamination, unbonding loosening of fastened parts, etc., will cause changes in the measured dynamic response of a structure. These changes will, in turn, cause changes in the structure's experimentally derived modal parameters.

Using this premise, a structural monitoring system which measures the vibration of a structure, identifies changes in its modal parameters, and predicts occurrences of structural faults can be hypothesized. Such a system would require a level of accuracy far beyond the traditional peak picking implementations of the past, and should be able to benefit from as much a priori knowledge of the structure's dynamic properties as possible.

In this paper, we examine several important issues associated with the use of experimentally derived modal parameters as a means of structural fault detection. They include measurement techniques, changes in the modal parameters caused by physical changes, fault location and quantification, and the use of a neural network to recognize patterns of change in the modal parameters. Also included are the results of some experiments we conducted to correlate modal parameter changes with the size of a hole drilled in two different metal plates.

NOMENCLATURE

t = time variable (seconds).

ω = frequency variable (radians/second).

n = number of measured DOFs.

m = number of modes.

$[M]$ = $(n \text{ by } n)$ mass matrix (force/unit of acceleration).

$\{x''(t)\}$ = acceleration response n-vector

$[C]$ = $(n \text{ by } n)$ damping matrix (force/unit of velocity).

$\{x'(t)\}$ = velocity response n-vector.

$[K]$ = $(n \text{ by } n)$ stiffness matrix (force/unit of displacement).

$\{x(t)\}$ = displacement response n-vector.

$\{f(t)\}$ = excitation force n-vector.

$\{X(j\omega)\}$ = discrete Fourier transform of the displacement response n-vector

$\{F(j\omega)\}$ = discrete Fourier transform of the excitation force n-vector.

$[H(j\omega)]$ = $(n \text{ by } n)$ Frequency Response Function (FRF) matrix.

$[B(j\omega)]$ = $(n \text{ by } n)$ System matrix.

$[I]$ = $(n \text{ by } n)$ identity matrix.

$\{u_k\}$ = complex mode shape (n-vector) for the k^{th} mode.

p_k = pole location for the k^{th} mode = $-\sigma_k + j\omega_k$

σ_k = damping of the k^{th} mode (radians/second).

ω_k = frequency of the k^{th} mode (radians/second).

A_k = a non-zero scaling constant for the k^{th} mode.

$[R_k]$ = the $(n \text{ by } n)$ residue matrix for the k^{th} mode

tr - denotes the transpose.

$*$ - denotes the complex conjugate.

INTRODUCTION

The physical mass, stiffness, and damping properties of a structure determine how it vibrates. Vibration is caused by an exchange of energy between the mass (inertial) property and the stiffness (restoring) property of the structure. The damping property dissipates vibrational energy, usually as friction heat.

A structure's modal properties are directly related to its physical properties. That is, changes in the structure's mass, stiffness, or damping properties will cause changes in its modal properties (modal frequencies, modal damping and mode shapes). Also, changes in the structure's boundary conditions (mountings) can be viewed as changes in the mass, stiffness, or damping of the structure plus its surroundings, and will change its modal parameters.

If changes in a structure's modal parameters are to used as a reliable means of detecting, and possibly even locating and quantifying structural faults, then a strong correlation between changes in modal parameters and structural faults must be established beforehand. However, the real question is:

- *What is the smallest physical change in a structure that can be detected, located and quantified from changes in its modal parameters?*

Naturally, the best answer to this question is: *"The smaller the better!"* This answer presumes that it is always better to detect the onset of a structural fault as early as possible, when it is still small, so that repairs can be made or other preventative measures taken.

It is relatively straightforward to demonstrate the strong sensitivity of modal parameter changes to induced structural faults. Examples using simple structures and ideal testing conditions are given later in this paper. However, before a reliable structural monitoring system could be implemented, other more difficult questions need to be addressed:

- *How many measurements are necessary to adequately identify modal parameters?*
- *Where is the best place (or places) on the structure to make measurements?*
- *What types of measurements should be made?*
- *How much measurement noise can be tolerated?*

CONTROLLED EXCITATION VERSUS OPERATING DATA

Modal properties are independent of structural excitation. A key difference between operating deflection shapes and mode shapes is that operating deflection shapes change with structural excitation; mode shapes do not. Operating deflection shapes can be obtained directly from operating data; that is, the measured vibration response of the structure under operating conditions. When operating data is acquired, the excitation forces are usually not measured. (See reference [1]). On the other hand, to identify modal properties, it is preferable to artificially excite the structure, and not use operating data.

MODAL PARAMETERS FROM FRFs

Advances in FFT-based test equipment and frequency domain parameter estimation (curve fitting) methods have significantly improved the accuracy and repeatability with which modal parameters can be identified from test data.

Modal properties are typically estimated from Frequency Response Function (FRF) measurements. An FRF is a 2-channel measurement, involving two simultaneously sampled signals; a response signal and an excitation (force) signal. The FRF measurement can be estimated in several ways, but the most common calculation involves dividing the Cross Power Spectrum between the response and excitation signals by the Auto Power Spectrum of the excitation signal, at each frequency. Averaging several Cross and Auto Power Spectra together is commonly done to reduce measurement noise.

An FRF captures the *unique* dynamic characteristics of the structure between two degrees of freedom (DOFs); the response DOF and the excitation DOF. If the force is applied at the same DOF as the response, the measurement is a *driving point* measurement. If the force is applied at a different DOF than the response, the FRF is called a *cross* measurement

Equations of Motion

A brief look at the mathematical representation of the dynamics of a structure reveals that FRFs can be completely represented in terms of modal parameters The equations of motion for a vibrating structure are commonly derived by applying Newton's second law to all of the DOFs of interest on the structure. In an experimental situation, this results in a countable set of equations, one for each measured DOF:

$$[M]\{x''(t)\}+[C]\{x'(t)\}+[K]\{x(t)\}=\{f(t)\} \qquad (1)$$

The excitation forces and responses are functions of time (t), and the coefficient matrices $[M]$, $[C]$ and $[K]$ are constants. This dynamic model describes the vibration response of a linear, time invariant structure.

If initial conditions are ignored, the *equivalent* frequency domain form of the dynamic model can be represented in terms of discrete Fourier transforms, either as,

$$[B(j\omega)]\{X(j\omega)\}=\{F(j\omega)\} \qquad (2)$$

where: $[B(j\omega)]=[M](j\omega)^2+[C](j\omega)+[K]$

or as,

$$\{X(j\omega)\} = [H(j\omega)]\{F(j\omega)\} \qquad (3)$$

These equations are valid for all discrete frequency values for which the discrete Fourier transforms of the excitation and responses are computed. Equation (3) is a definition of the FRF matrix. Each element of this matrix is an FRF measurement between two DOFs of the test structure.

Using the two equations above, it follows that,

$$[B(j\omega)][H(j\omega)] = [I] \qquad (4)$$

Modal Parameters

If it is further assumed that reciprocity is valid for the test structure, (the $[M]$, $[C]$ and $[K]$ matrices are symmetric), then the FRF matrix can be represented completely in terms of the modal parameters of the structure. Using superposition, the FRF matrix can be represented as a summation of terms, each term due to the contribution of a single mode of vibration:

$$[H(j\omega)] = [H_1(j\omega)] + [H_2(j\omega)] + \dots + [H_k(j\omega)] + \dots + [H_m(j\omega)]$$

where:

$$[H_k(j\omega)] = A_k\{u_k\}\{u_k\}^{tr}/(j\omega - p_k) + A_k^*\{u_k^*\}\{u_k^*\}^{tr}/(j\omega - p_k^*) \qquad (5)$$

Notice that each term of the FRF matrix is represented in terms of a pole location and a mode shape. Notice also that all the numerators are simply constants, and that only the denominators are functions of frequency. The numerators are called **residues.** Each term of the FRF matrix can also be represented in terms of poles and residues:

$$[H_k(j\omega)] = [R_k]/(j\omega - p_k) + [R_k^*]/(j\omega - p_k^*) \qquad (6)$$

where:

$[R_k]$ = the $(n \text{ by } n)$ residue matrix for the k^{th} mode
= $A_k\{u_k\}\{u_k\}^{tr}$

Note that each element of the residue matrix for mode (k) is a function of the product between the mode shape component at DOF (i) and the mode shape component at DOF (j),

$$r_k(i,j) = A_k u_k(i) u_k(j)$$

Curve fitting

Parameter estimation (or curve fitting) is the process of numerically applying equation (6) to one or more FRF measurements. The result is an estimate of the *residue* and *pole location* for each mode in the frequency band of the measurements. In a monitoring system, *these modal parameter estimates would be monitored for any significant changes.*

SDOF System

For a single DOF (SDOF), the FRF merely becomes:

$$H(j\omega) = 1/\left(M(j\omega)^2 + C(j\omega) + K\right) \qquad (7)$$

Since an SDOF system has only one mode, its FRF can also be written,

$$H(j\omega) = (1/M)/\left((j\omega)^2 + 2\sigma(j\omega) + \sigma^2 + \omega^2\right) \qquad (8)$$

WHERE SHOULD MEASUREMENTS BE MADE?

Our objective, in monitoring the modes of a structure, is to accurately identify changes in its modal parameters from as few FRF measurements as possible. This means that only those FRF measurements where the modes are *well represented* should be made. In general, a mode is well represented if its residue is large. For lightly damped structures, this means that the modal resonance peak is prominent in the FRF.

If it is assumed that measurement noise adds uniformly to an FRF measurement over all frequencies, then the signal to noise ratio function of an FRF has the same shape as the magnitude of the FRF itself. This means that the data with the best signal to noise ratio is in the vicinity of the modal resonance peaks. Hence, the modal peaks with the largest peaks (largest residues), will yield the most accurate curve fitting results.

It was shown above that the residue between two DOFs is a function of the product of the mode shape components for each of the two DOFs. Therefore, a mode's residue will be large in any measurement made between two DOFs *on or near the anti-nodes* of the mode shape. The anti-nodes are those DOFs for which the mode shape is maximum relative to other components. Reference [2] shows how to locate an excitation DOF (driving point) where the residues of all of the modes are maximized.

This idea can be extended to the entire residue matrix, not just its diagonal elements (driving points). The FRF that best represents modes satisfies the following two criteria.

- It *maximizes the sum* of the magnitudes of the residues for all modes.
- It *minimizes the difference* between the maximum and minimum residue magnitudes among all the modes.

Given a set of mode shapes for a structure, these two rules can be applied to the data to determine the best DOF pairs between which to make FRF measurements. The mode shapes could be obtained from a prior modal survey of the structure, or from a finite element analysis. (A finite element model that has been validated with experimental modal data, can potentially yield shapes with many more DOFs than the experimental shapes.)

FAULT DETECTION

The simplest, and perhaps most common type of structural fault is one where the structure loses stiffness only. This, of course, would cause some or all on the modal frequencies to shift to lower values. However, more complex situations could also arise:

- What happens to the modes if the fault causes a loss in both mass and stiffness?
- What happens to the modes if the fault causes a loss in stiffness and an increase in damping?

Some insight can be gained into these more complex situations by examining the equations of motion. Comparing the two forms of the FRF for an SDOF system gives the following relationships;

$$2\sigma = C/M \tag{9}$$

$$\sigma^2 + \omega^2 = K/M \tag{10}$$

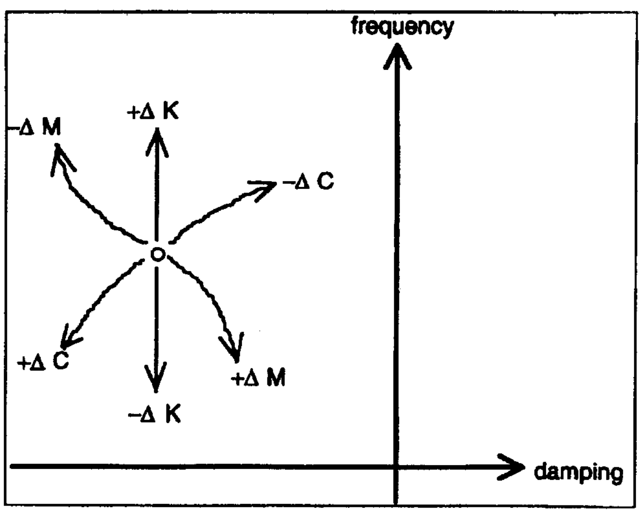

Figure 1. Movement of a Pole Due to Mass, Stiffness, & Damping Changes

From these equations, we can conclude that:

- Stiffness changes only change modal frequency (ω).
- Damping changes affect both modal frequency (ω) and damping (σ).
- Mass changes affect both modal frequency (ω) and damping (σ).

Figure 1. shows how the poles will move in the complex plane (s-plane) due to mass, stiffness, and damping changes. From this, the two previous questions can be answered, in part;

- A *decrease* in modal frequency (ω) combined with an *increase* in the damping (σ) of a mode means that a loss of stiffness, an increase of damping, and possibly a decrease in mass occurred in the structure.
- A *decrease* in modal frequency (ω) combined with a *decrease* in the damping (σ) of a mode means that a loss of stiffness and damping, and possibly an increase in mass occurred in the structure.

These are the two common types of faults. Others, involving, increases in modal frequency, can be hypothesized, but are not generally expected from material failure in a structure.

DETECTING HOLES IN PLATES

To demonstrate the sensitivity of modal parameters to minute structural changes, several holes of different diameters were drilled in both an aluminum and a steel plate. Figure 2 shows the size of the plate and the holes, drawn to scale. The thickness of the aluminum plate was 10mm, and the thickness of the steel plate was 3mm.

FRF measurements were made on the plates before and after each of the holes was made in them. Five measurements were made for each case. Figure 3 shows a Modal Peaks Function for the Aluminum plate with no hole in it. (A Modal Peaks Function is the average of the *imaginary part squared* of the 5 FRFs.)

Figure 4 shows expanded views of the Modal Peaks Functions in the frequency range of just two modes (1.92 kHz to 2.04 kHz). The three graphs superimpose the Peaks Function of the plate with no hole on the Peaks Function of the plate with three different sized holes: 2mm, 7mm, and 12mm.

There are about 40 modes in the frequency range of the FRFs. The expanded views reveal that the 2 modes chosen clearly indicate the presence of the 12mm hole, by the frequency shift of the modes (Figure 4.C). These two modes partially detect the 7mm hole (Figure 4.B), and don't detect the 2mm hole at all (Figure 4.A).

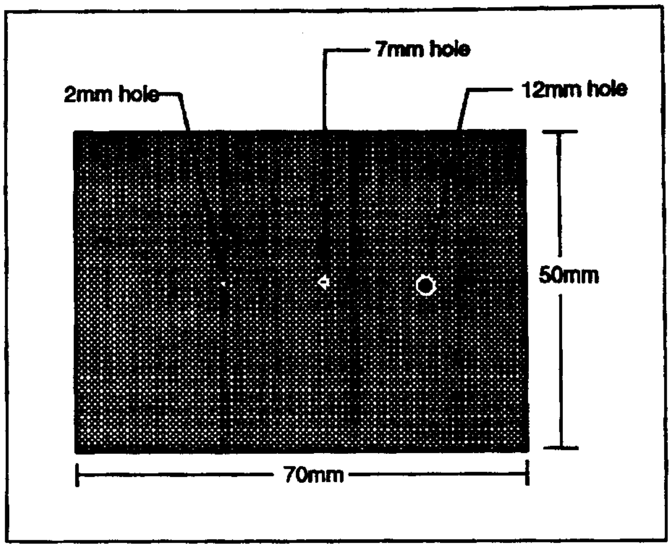

Figure 2. Plate Structure Showing Holes to Scale

Figures 5 and 6 show the same kind of results for the steel plate as those in Figures 3 and 4. Again, the expanded views of the Modal Peaks Functions reveal that the 12mm hole is easily detected, the 7mm hole is marginally detected, and the 2mm hole is not detected.

FAULT LOCATION AND QUANTIFICATION

Fault detection is relatively straightforward if based only on modal frequency and damping changes, since changes in these parameters can be determined from practically any FRF measurement taken from a structure. Locating and quantifying a fault is a much more complex matter, however.

As a minimum requirement, to locate and quantify a fault, the mode shapes of all of the dominant modes of the undamaged structure must be known.

Maximum Stiffness Changes

In previous publications (references [4] and [5]) it was shown that faults that are predominantly stiffness losses can be located by finding the region of maximum negative stiffness change on the structure. However, this approach relies on the solution of an underdetermined (rank deficient) set of equations to find the stiffness changes.

With underdetermined equations, a larger set of modes, and mode shapes from a validated finite element model of the undamaged structure, definitely improve the results. Although this approach worked satisfactorily for simple cases, the amount and required accuracy of the modal data makes it difficult to envision this as a useful technique for on-line monitoring of complex structures.

Neural Networks

It is known from SDM theory (see reference [3]) that the modes with the largest residues at the modification endpoints (DOFs) are affected most by mass, stiffness, or damping modifications. When a fault occurs, this simple fact can be used to focus in on those modes whose poles moved the most, to localize the fault to areas where their anti-nodes are largest.

Furthermore, in order to locate minute faults, modal data for the higher frequency local modes is required. If the fault is very localized, then the local modes with non-zero mode shapes in the vicinity of the fault will be affected most.

To summarize, faults can be localized according to the following rule;

- A fault will be located in the vicinity of the anti-nodes of those modes whose poles move the most.

In any realistic monitoring situation, a pattern recognition scheme will be needed to decipher the complex pattern of modal parameter changes that occurs due to a fault. Neural networks are proving to be an effective tool for pattern recognition in a variety of applications.

Neural networks were developed to mimic the pattern recognition capabilities of the human brain. Recently, they have been successfully implemented in Optical Character Recognition (OCR) software with a success rate in the high 90 percents, far exceeding previously tried statistical methods.

SDM and Neural Network Training

A key requirement of the use of a neural network is that it be “trained” beforehand. In this application, training the network would involve feeding it sets of modal parameter changes along with the mass, stiffness, and damping changes that caused them. The neural network, in turn, evolves (computes) a set of internal weights that allow it to predict the mass, stiffness, and damping changes that caused the modal parameter changes.

The SDM algorithm can compute the new modal parameters due to mass, stiffness, or damping modifications very rapidly, compared, for instance, to the eigensolution process used in finite element analysis. SDM can therefore be used to generate the numerous sets of data (mass, stiffness, damping changes / modal parameter changes) required to train a neural network. Using SDM, only a set of modal parameters for the undamaged structure is required. Random mass, stiffness, and damping modifications could be fed to SDM to generate the resulting modal parameter changes.

Once a network has been trained for a particular structure, it can be implemented in an on-line monitoring system that will predict the *location* and *severity* of any fault that causes changes in the structure's measured modal parameters.

CONCLUSIONS

All of the tools necessary to implement an accurate and sensitive on-line structural health monitoring system are available in current day technology. In this paper, we have reviewed the theory which shows that modal parameters are global properties of a structure that only change if its physical properties change. We also showed that changes in modal parameters can be used to detect, locate, and quantify structural faults.

Improvements in the accuracy of FFT-based signal analyzers, frequency domain modal parameter estimation, and recent successes in the application of neural networks to real world pattern recognition problems make the implementation of an on-line monitoring system a practical reality

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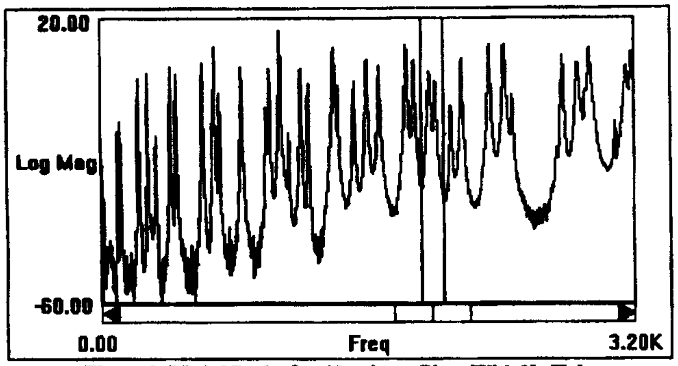

Figure 3. Modal Peaks for Aluminum Plate With No Hole.

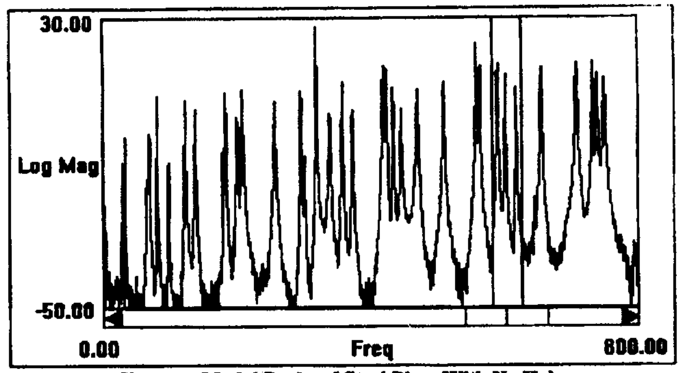

Figure 5. Modal Peaks of Steel Plate With No Hole.

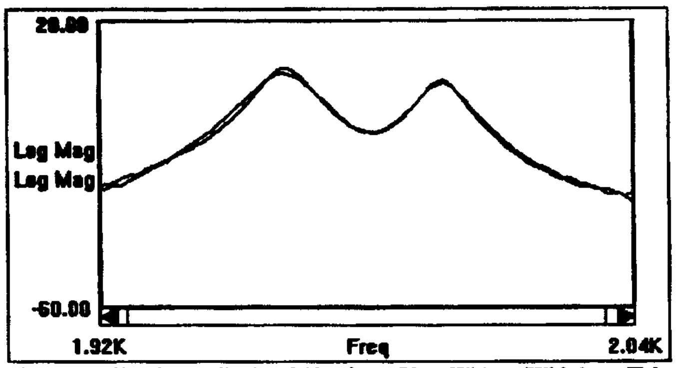

Figure 4.A. Two Modal Peaks of Aluminum Plate Without/With 2mm Hole.

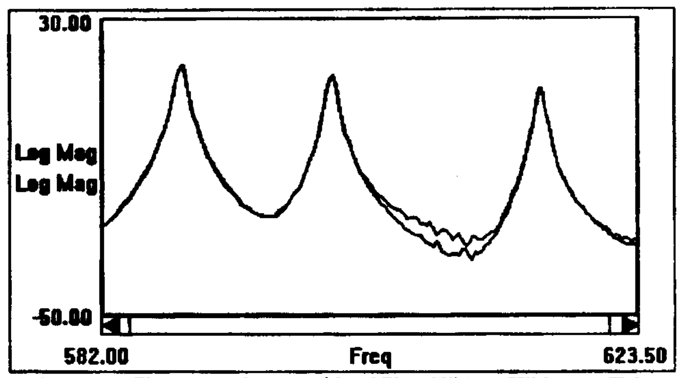

Figure 6.A. Three Modal Peaks of Steel Plate Without/With 2mm Hole.

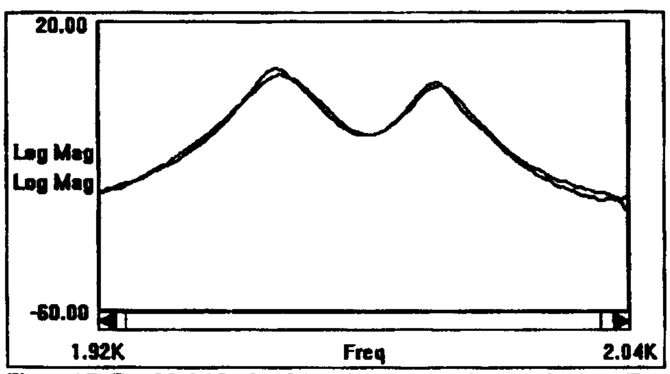

Figure 4.B. Two Modal Peaks of Aluminum Plate Without/With 7mm Hole.

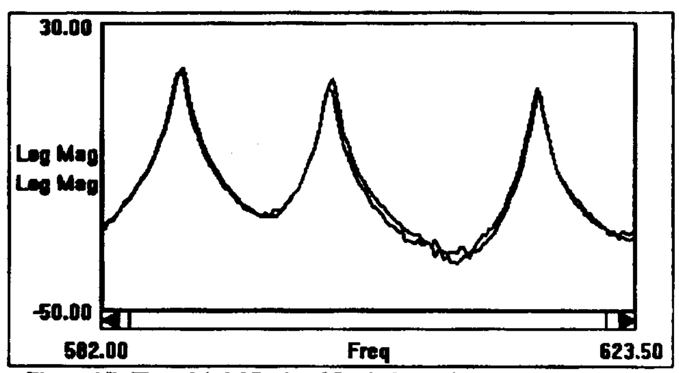

Figure 6.B. Three Modal Peaks of Steel Plate Without/With 7mm Hole.

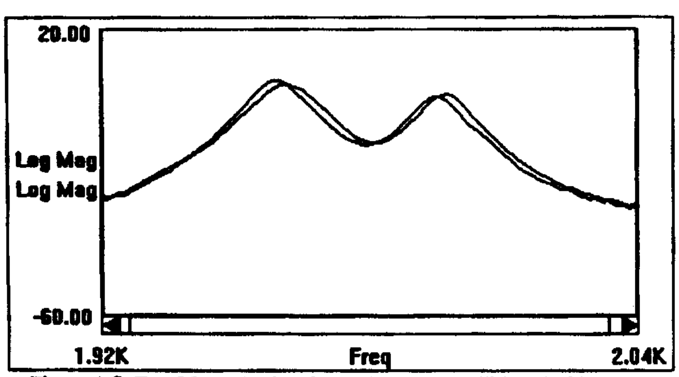

Figure 4.C. Two Modal Peaks of Aluminum Plate Without/With 12mm Hole.

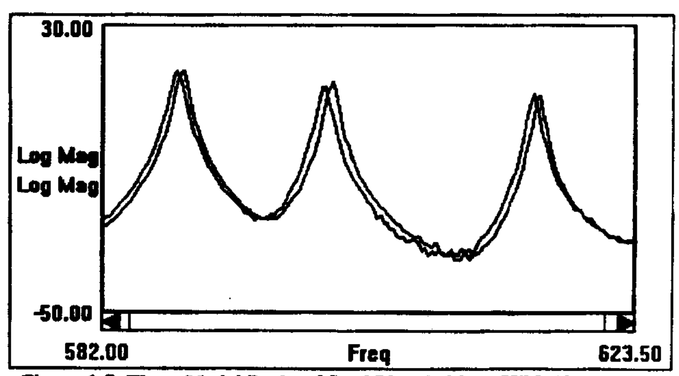

Figure 6.C. Three Modal Peaks of Steel Plate Without/With 12mm Hole.