Determining the Accuracy of Modal Parameter Estimation Methods

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Abstract

The most common type of modal testing system today uses an FFT analyzer to measure a set of Frequency Response Functions (FRFs) from a structure, and then uses a parameter estimation (curve fitting) method to determine the structure's modal properties from the FRF measurements. The curve fitting method typically "fits" an analytical model to the FRF data, (or its equivalent Impulse Response data) and estimates of the unknown modal parameters of the model are determined by this process. These parameter estimates are then assumed to be the *correct modal parameters of the structure.*

In this paper, a number of "standard test cases" of synthesized FRFs are presented for testing modal parameter estimation methods. Twelve different FRFs are presented, that are synthesized using the parameters for three modes. Frequency spacings between the modes and modal damping values are varied to make up the different cases, which range from light modal coupling (or modal density) to very heavy coupling. Random noise is also added to the synthesized FRFs to simulate noisy measurements, giving a total of twenty-four different test cases. The advantage of this approach to curve fitter testing is, of course, that the right answers (the modal parameters used to synthesize the FRFs) are known, and can therefore be used as the basis for determining the accuracy of the fitter.

Two different curve fitting methods, an SDOF (single mode-at-a-time) and an MDOF (multiple modes-at-a-time) rational fraction polynomial fitter, were tried on the test case FRFs, and the results are presented. In publishing these "standard test cases" the authors hope to encourage the adoption of a suite of published test cases by the modal testing community which could then be used to qualify the accuracy of commercially available modal testing software.

Introduction

Modal parameters are defined as the eigenvalues and eigenvectors of the linear dynamic model for a vibratory structure. This linear model can be written in terms of FRFs as:

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \qquad (1)$$

where: $\{X(\omega)\}$ = n-vector of Fourier transformed displacement responses
$\{F(\omega)\}$ = n-vector of Fourier transformed force inputs
$[H(\omega)]$ = $(n \text{ by } n)$ matrix of FRFs
n = number of test degrees-of-freedom (DOFs) on the structure
ω = the frequency variable

The FRF matrix can be written in terms of modal parameters as:

$$[H(\omega)] = \sum_{k=1}^{\text{modes}} [R_k]/2j(j\omega - p_k) - [R_k^*]/2j(j\omega - p_k^*) \qquad (2)$$

where: $[R_k]$ = $(n \text{ by } n)$ matrix of residues for mode (k)
$p_k = \sigma_k + j\omega_k$ = complex pole location for mode (k)
σ_k = modal damping for mode (k)
ω_k = modal frequency for mode (k)
modes = the number of modes in the model
$*$ - denotes the complex conjugate

Curve fitting, then amounts to "matching" the analytical expression (2), or an abbreviated, or equivalent form of (2), to experimental FRF data over a chosen frequency range. During the process, some, or all of the modal parameters in the model are determined.

It is straightforward to show that the mode shape can be obtained from *a row or column* of the residue matrix $[R_k]$ for each mode (k), since the residues are related to the mode shape by the formula:

$$[R_k] = A_k \{u_k\}\{u_k\}^t \qquad k=1,\ldots,\text{modes} \qquad (3)$$

where: $\{u_k\}$ = the mode shape for mode (k), an n-vector
A_k = a scaling constant for mode (k)
t - denotes the transpose of the mode shape

Therefore, at least one row or column of FRF measurements are typically made, (from the matrix of n^2 possible measurements), and these measurements are curve fit to obtain the modal pole locations (frequency and damping), and a row or column of modal residues for each mode in the model. Each mode is represented in an FRF by ***two complex parameters***, a complex pole location and a complex residue, or a total of four numbers.

Types of Curve Fitters

During the past 20 years, numerous different curve fitting algorithms have been developed for fitting FRFs. They can all be grouped into four classes:

o Local SDOF
o Local MDOF
o Global
o PolyReference

Local SDOF Fitters: These curve fitters operate on one measurement at a time, and estimate the parameters of one mode at a time. Some curve fitters only estimate one of the four unknowns per mode. For instance, modal frequency can be approximated by simply using the frequency of a resonance peak, if one exists in the FRF data. Local SDOF fitters will usually give satisfactory results on data that contains ***lightly coupled*** modes, i.e. low modal density. This is illustrated in the test cases later in this paper.

Local MDOF Fitters: These fitters also operate on one measurement at a time, but they can simultaneously estimate the parameters of multiple modes at a time. If a set of FRFs contains modes which are ***heavily coupled*** (resulting from the combined effect of heavy damping and small modal frequency separation), then an MDOF fitter is usually required to adequately identify the modal parameters. These fitters typically apply expression (2) to the data in a least squared error sense. That is, a set of parameters for two or more modes is found which minimizes the squared difference between the FRF data and the model, with modes > 1.

Global Fitters: Expression (2) makes it clear that all of the FRFs of a structure contain the same denominator, hence the same modal pole locations. Only the numerators, or residues, are different from measurement to measurement. Global fitters take advantage of this fact and use all, or a large number of, the FRFs to estimate the poles first, and then estimate the residues during a second pass through the data. This process yields one global estimate of frequency and damping for each mode, and usually provides better mode shape estimates, especially near nodal points where a mode's residues are small and not well defined.

PolyReference Fitters: This class of fitters extends the idea of a global fitter to include multiple references, or multiple rows or columns, of the FRF matrix. Equation (3) shows that ***every row or column*** of the residue matrix contains the mode shape of each mode. PolyReference fitters take advantage of this fact and obtain additional estimates of the mode shape by curve fitting multiple rows or columns of data from the FRF matrix. These multiple estimates are then combined in a manner which favors the references where each mode is more strongly represented, (i.e. its modal participation is greater), to yield a better estimate of each mode shape.

Repeated roots, (i.e. two or more modes at approximately the same frequency but with different mode shapes), can also be found from multiple rows or columns of FRF data. A single row or column is not sufficient for this.

The test cases presented here are only useful for testing the accuracy of Local SDOF and Local MDOF curve fitters. Additional cases are needed to test Global and PolyReference curve fitters.

Sources of Error

When any kind of parameter estimation procedure is applied to a set of experimentally determined data, a number of errors can occur. In particular, when curve fitting a set of FRF measurements, the following problems must be dealt with:

o Insufficient frequency resolution
o Measurement distortion
o Measurement noise
o Determining the model size, or number of modes

Insufficient frequency resolution: This may or may not be a problem depending upon the type of curve fitter used. For instance, an SDOF "circle" fitter [1] estimates the residue of a single mode by fitting the equation of a circle to FRF data in Nyquist (real versus imaginary) format. To use this method, sufficient frequency resolution is required in the vicinity of the FRF resonance peaks to approximate circles.

In general, SDOF curve fitters are applied to FRF data in the vicinity of each of the resonance peaks. Consequently, there must he a sufficient number of data points in the vicinity of each resonance peak to obtain accurate results. How many is enough? The answer depends on the type of curve fitter used, but a general "rule of thumb" is that *five to ten* data points between the half power points (71% of the FRF magnitude at the peak), is sufficient.

MDOF fitters are less sensitive to frequency resolution since they apply the "waveform" generated by expression (2) to the FRF data over a range of frequencies. Therefore, they are more sensitive to the "shape" of the FRF data, and can estimate the modal parameters with *far more accuracy* than the FRF frequency resolution itself, provided that the shape of the data closely matches the shape of expression (2).

Measurement distortion: From a curve fitting standpoint, the most detrimental contaminant of FRF measurements is distortion. Distortion is caused either by non-linear behavior of the structure, or by windowing effects in the analyzer.

Expression (2) creates a waveform for the linear dynamics of a structure. Therefore, only the linear dynamics of the structure can be represented in the FRFs, if they are to be accurately matched to expression (2) over a range of frequencies. Many structures behave non-linearly, however. SO some means must be employed during the measurement process to "filter out" the non-linear part of the structural motion and reserve only the linear part. A common method for doing this is to use random excitation and frequency domain signal averaging to "average away" the nonlinear portion of the motion in the cross and autopower spectrum averages that are used to form the FRFs.

The other common cause of distortion in FRF measurements is due to truncation of the time domain signals by the finite sampling time period of the FRF analyzer. This windowing effect is called "**leakage**". Leakage distorts the FRF waveform, especially in the vicinity of the resonance peaks, where the data is most critical for curve fitting.

Leakage can be eliminated by using periodic signals, or it can be minimized by using specially shaped time domain windows. In the test cases presented here, no attempt is made to simulate distortion. Nevertheless, to successfully apply curve fitting, every effort should be made during the measurement process to eliminate, or at least minimize, distortion.

Measurement noise: Numerous sources of noise can contaminate FRF measurements, thus making it more difficult to estimate modal parameters. The different types of noise and how they are dealt with will not be discussed here, but suffice it to say that every effort should be made during the measurement process to reduce noise to a minimum.

In the test cases presented here, Gaussian random noise is added to the synthesized measurements to simulate measurement noise. Even though least squared error curve fitters, like the ones used here, are designed to estimate parameters in the presence of noise, it will be shown that noise does reduce the accuracy of the parameter estimates.

Model size, or number of modes: The *most critical step* in curve fitting is picking the model size, or equivalently, determining how many modes are represented in the FRF data. The problems already mentioned (frequency resolution, measurement distortion, and measurement noise), together with modal density, and repeated roots (modes at the same frequency with different mode shapes) all have a direct effect on determining the correct model size. The model size, in turn, directly affects the accuracy of the parameter estimates obtained by curve fitting.

Most commercially available curve fitters require that the operator choose the model size. Singular value decomposition (SVD) and error-based iterative methods have been developed recently which can assist the operator in choosing the model size, but noise, distortion, high modal density, and repeated roots can still make if difficult to choose the correct model size.

In this paper, we concentrate on just two of the causes of error in curve fitting, modal coupling (or density) and noise.

Curve Fitting Test Cases

Twelve different FRFs were synthesized using expression (2) and parameters for three modes. The residues of the modes remained fixed at the following values:

Mode	...................residue	....
No.	Magnitude	Phase
1	100	0
2	100	180
3	100	0

Modal damping was made the same for each mode in each case, but varied from case to case.

case 1: frequencies = 50, 100, 150 Hz
damping, = 0.5 Hz
case 2: frequencies = 50, 100, 150 Hz
damping = 1 Hz
case 3: frequencies = 50, 100, 150 Hz
damping = 5 Hz
case 4: frequencies = 50, 100, 150 Hz
damping = 10 Hz
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case 5: frequencies = 95, 100, 105 Hz
damping = 0.5 Hz
case 6: frequencies = 95, 100, 105 Hz
damping = 1 Hz
case 7: frequencies = 95, 100, 105 Hz
damping = 5 Hz
case 8: frequencies = 95, 100, 105 Hz
damping = 10 Hz
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case 9: frequencies = 99, 100, 101 Hz
damping = 0.5 Hz
case 10: frequencies = 99, 100, 101 Hz
damping = 1 Hz
case 11: frequencies = 99, 100, 101 Hz
damping = 5 Hz
case 12: frequencies = 99, 100, 101 Hz
damping = 10 Hz

The FRFs are synthesized over the frequency range (0 Hz to 200 Hz) using 3201 spectral lines. This gives a frequency resolution of Δf = 0.0625 Hz. As a percentage of critical damping the modal damping varies from approximately 0.3% to 20%, a realistic range of damping for the majority of structures. The FRFs for test cases 1 through 12 are shown in Figure 1.

Test cases 13 through 24 are generated by adding 2.5% random noise to each of the cases 1 through 12. The block of random noise that is added to the 12 FRFs shown in Figure 2. The FRFs for test cases 13 through 24 are shown in Figure 3.

Curve Fitting Results

SDOF Fitter: First, an SDOF rational fraction polynomial fitter [2] was applied to the twelve test cases, to identify the parameters of the center (100 Hz) mode. The SDOF fitter was restricted to a frequency band of data in the vicinity of the 100 Hz mode, to minimize the influence of the other two (higher and lower frequency) modes. The following curve fitting bands were used for SDOF fitting:

SDOF Curve Fitting Frequency Bands
cases (1 to 4) & (13 to 16): 75 Hz to 125 Hz
cases (5 to 8) & (17 to 20): 97.5 Hz to 102.5 Hz
cases (9 & 10) & (21 to 22): 99.5 Hz to 100 5 Hz
cases (11 & 12) &. (23 to 24): 94 Hz to 106 Hz

The SDOF curve fitting results are given in Figure 4 (no noise), and Figure 5 (noise added). The errors are the *magnitude of the differences* between the correct values for each of the modal parameters and the SDOF fitter estimates.

MDOF Fitter: An MDOF rational fraction polynomial fitter [2] was also applied to the 24 test case FRFs, to simultaneously estimate the parameters if all three modes. The MDOF fitter was also restricted to frequency bands of FRF data in the vicinity of the three resonance peaks, which, generally speaking, gives better results. The following curve fitting bands were used for MDOF fitting:

MDOF Curve Fitting Frequency Bands
cases (1 to 4) & (13 to 16): 25 Hz to 175 Hz
cases (5 to 8) & (17 to 20): 75.5 Hz to 124.5 Hz
cases (9, 10 & 12) & (21, 22 & 24): 92 Hz to 108 Hz
cases 11 & 23: 70.5 Hz to 129.5 Hz

Even though the parameters of all three modes were estimated, only the errors of the parameters of the center (100 Hz) mode are given in Figure 6 (no noise) and Figure 7 (noise added).

Conclusions

Both the SMS StarModal SDOF and MDOF polynomial curve fitters were applied to 12 different synthesized FRFs. with and without additive noise. The curve fitting estimates of the modal parameters of these FRFs were then compared with the known answers. The *magnitudes of the differences* between the estimates and the correct answers are given in Figures 4 to 7.

The SDOF results in figures 4 & 5 make it clear that the .SDOF fitter obtained sufficiently accurate results for cases (1 to 6) and (13 to 18). (Those cases for which all four modal parameters had small errors were considered sufficiently accurate). The curve fitter estimates of the additive noise cases (13 to 18) allow slightly more error than the non-noisy cases (1 to 6), as expected.

Examining the FRFs in Figure 1. it is clear that the modal peaks of all three modes we clearly discernible in the first 6 cases, but not for the remaining cases. This indicates a “rule of thumb” for applying SDOF fitters, namely, “only use an SDOF fitter where a resonance peak is clearly evident.”

The MDOF fitter yielded accurate results for more cases than the SDOF fitter, as expected. As shown in Figures 6 & 7, the MDOF fitter yielded accurate results for cases (1 to 10), (13 to 18), and case 21. 0ne noticeable attribute of the MDOF; fitter is that for the non-noisy cases (Figure 6), when the modal density became too great, (cases 11 & 12), all four of the modal parameters simultaneously incurred large errors.

The synthesized FRF test cases used here had sufficient frequency resolution, (Δf = 0.0625 Hz), and yet the MDOF fitter could not accurately identity the parameters for cases 11 & 12. These two are both cases of very high modal density, or they could also be classified as repeated root cases. Typically, a Global or a PolyReference fitter is needed to successfully handle such cases

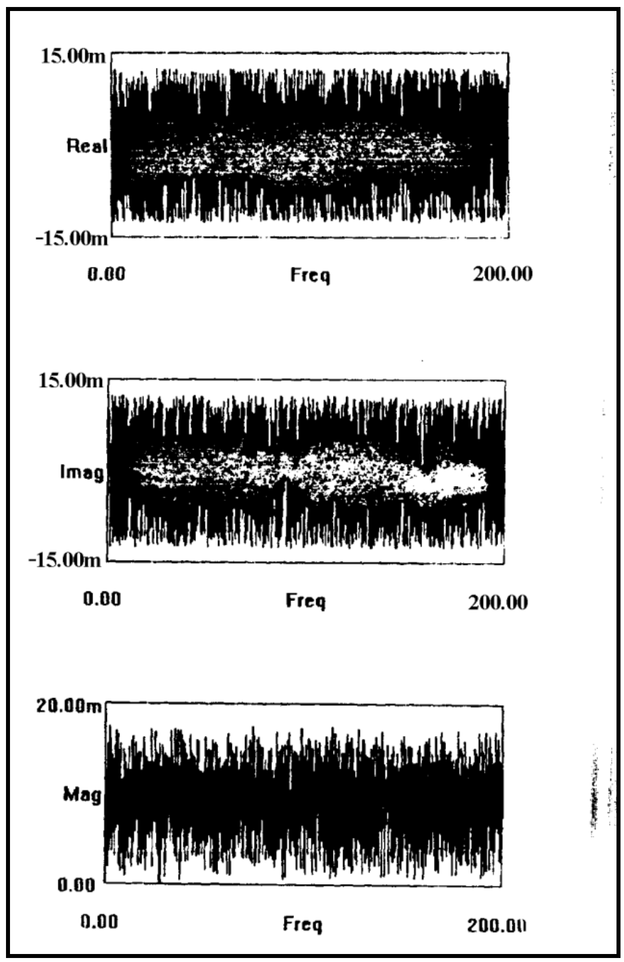

Figure 2. 2.5% Random Noise Block

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[4] Vold, H. & Leuridan, J. “A Generalized Frequency Domain Matrix Estimation Method for Structural Parameter Identification” 7th International Seminar on Modal Analysis, Leuven University, Belgium, 1982.

[5] Zhang, L. et.al. “A Polyreference Frequency Domain Method for Modal Parameter Identification” ASME Design Engineering Conference, Cincinnati, Ohio, Sept. 1985.

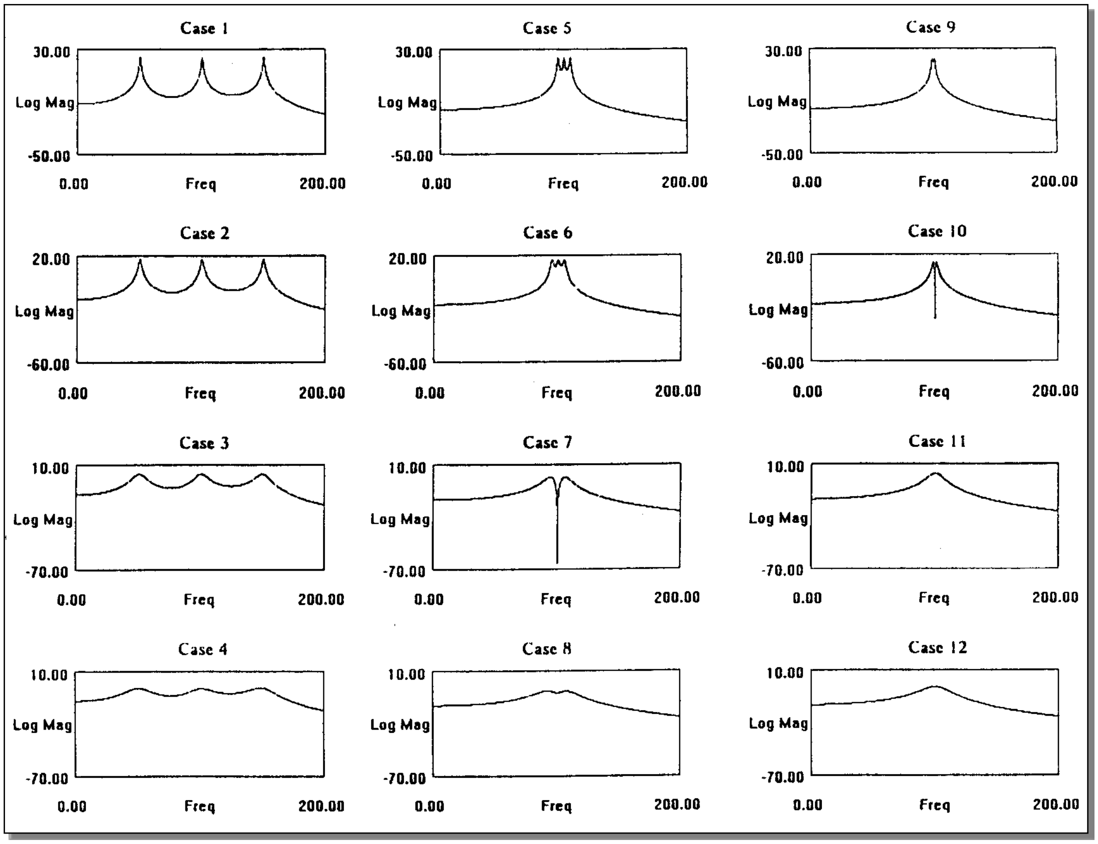

Figure 1. FRFs for Test Cases 1-12

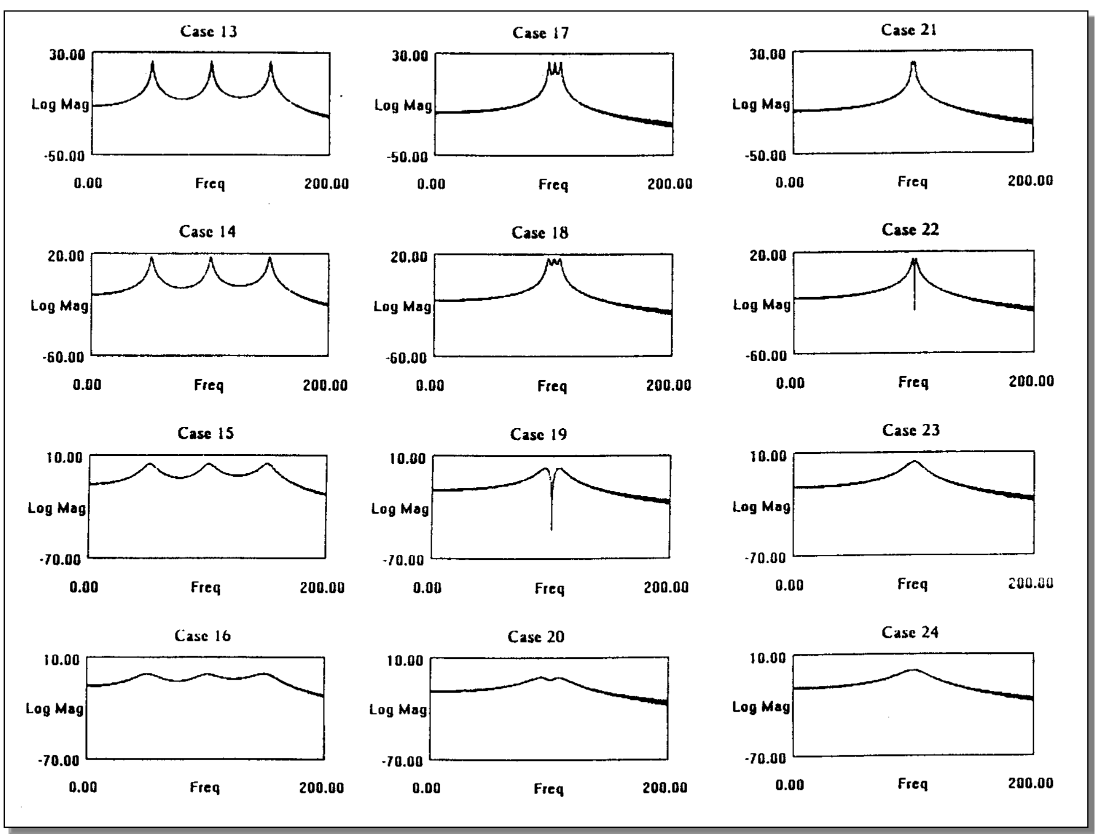

Figure 3. FRFs for Test Cases 11-24

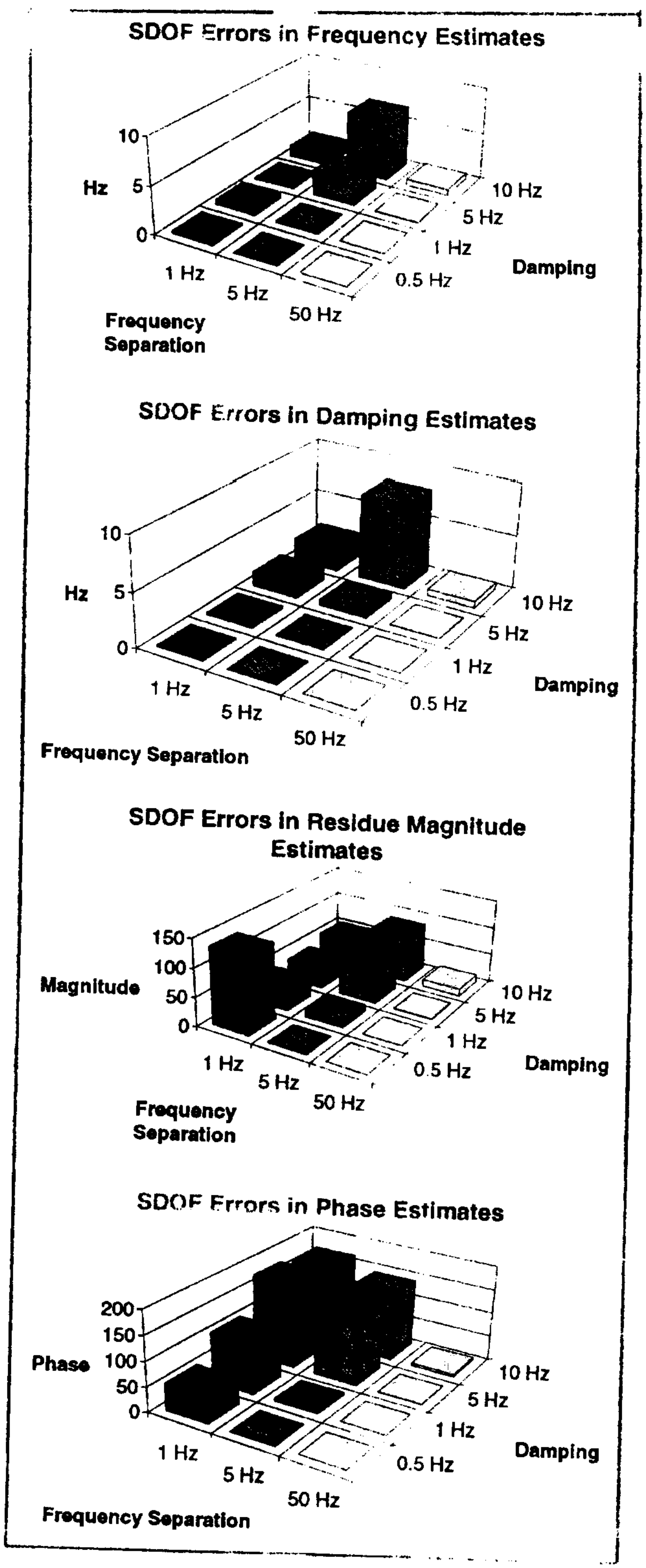

Figure 4. SDOF Curve Fitting Errors: Cases 1-12

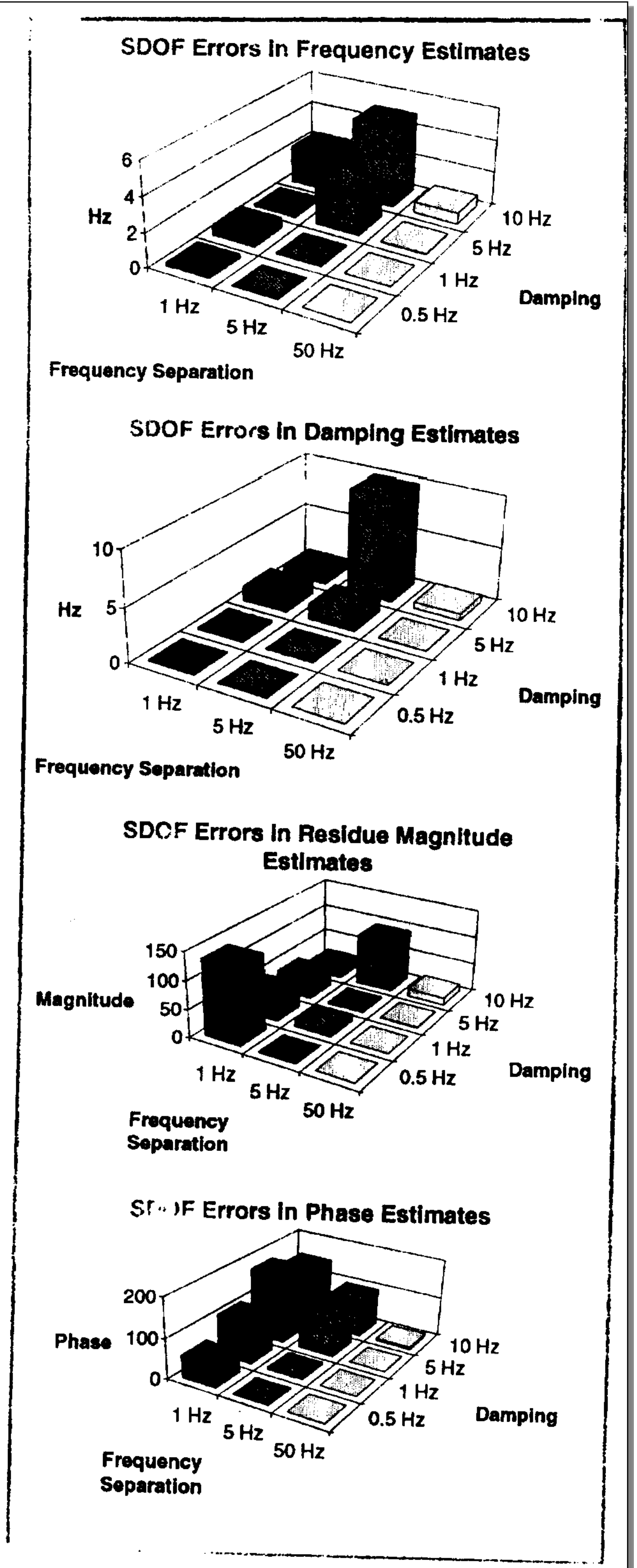

Figure 5. SDOF Curve Fitting Errors: Cases 13-24

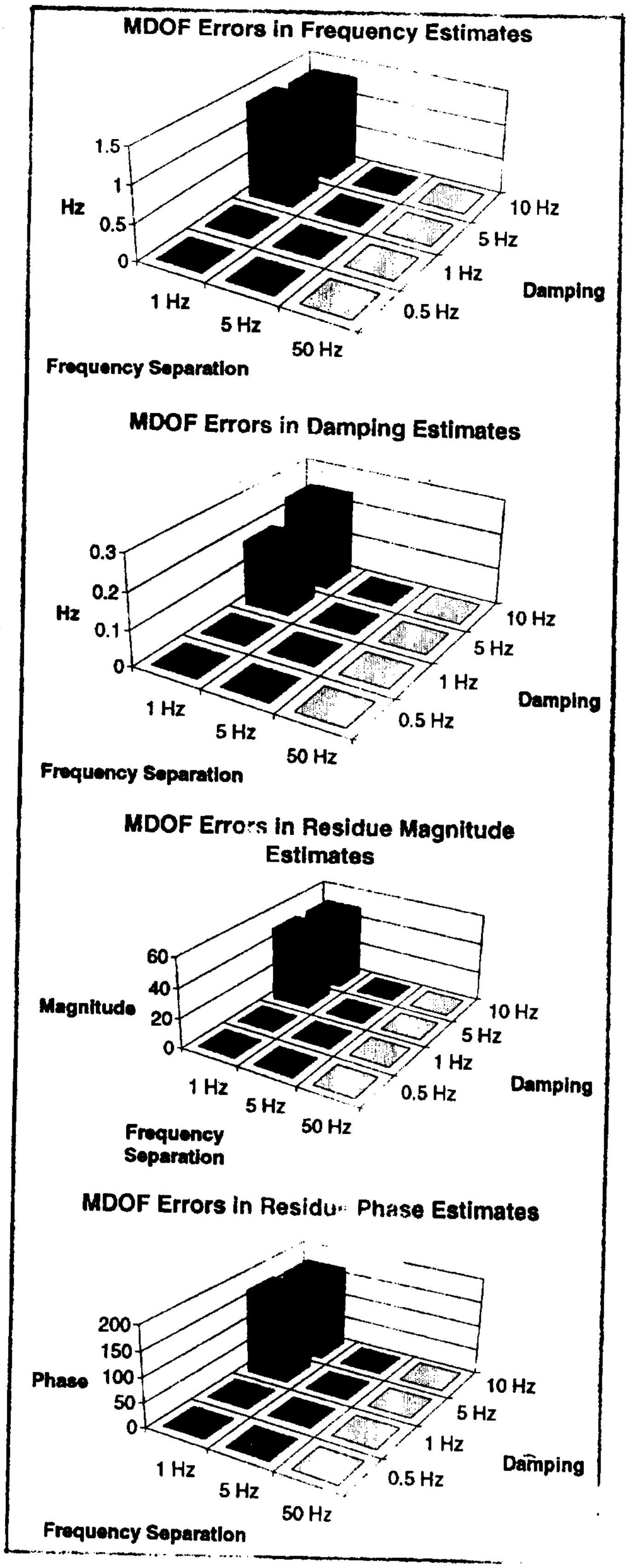

Figure 6. MDOF Curve Fitting Errors: Cases 1-12

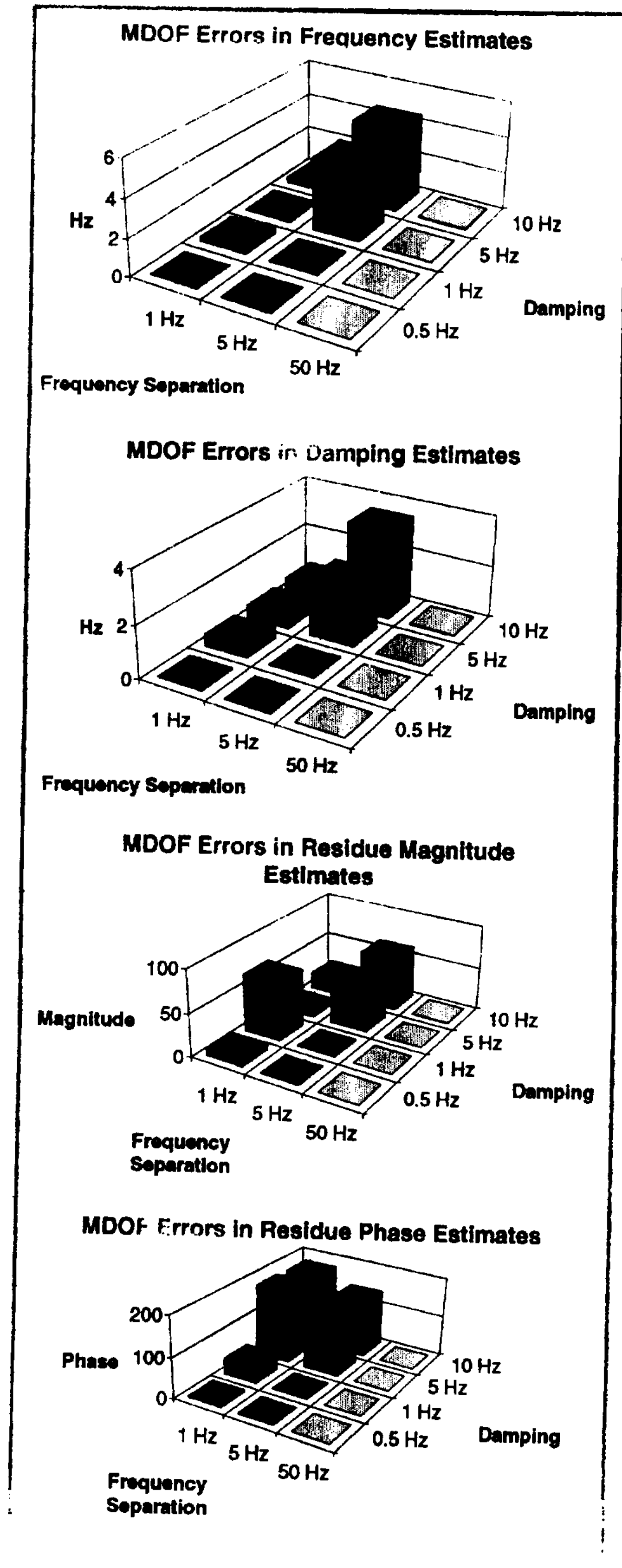

Figure 7. MDOF Curve Fitting Errors: Cases 13-24