Fault Detection in Structures from Changes in Their Modal Parameters

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ABSTRACT

Modal testing has become commonplace in many industries today as a research and development tool. In this capacity, it is used primarily during product prototype development and for trouble-shooting noise and vibration problems in general.

Very little use has been made of this technology to date, though, for detecting faults in mechanical structures. By "faults," we mean any of the following occurrences:

- Failure of the structural material, e.g., cracking or breaking.
- Loosening of assembled parts.
- Flaws, voids, cracks, thin spots, etc. caused during manufacturing.
- Improper assembly of parts during manufacturing.

In this paper, the correlation between a physical change and changes in the structure's modal parameters is investigated. A flat plate structure with a rib stiffener bolted to it is used as the test specimen and modal tests are performed on it using an impact hammer.

This paper not only includes discussion about the advantages of using experimental modal data as a means of detecting structural faults, but also includes demonstrations of the sensitivity of modal parameters to physical changes. Specifically, it is shown how modal parameters can detect variations in the bolt tightness between the plate and the rib.

INTRODUCTION

It is well known that the modal parameters (frequency, damping and mode shapes) of a structure are a function of its physical properties (mass, damping and stiffness). The modal parameters are solutions to the differential equations of motion which are themselves functions of the mass, damping and stiffness of the structure. Therefore, any changes in the physical properties will cause changes in the modal properties.

CURVE FITTING FRF MEASUREMENTS

One of the side benefits that has grown out of the increased use of the transfer function or frequency response function (FRF) approach to modal testing has been the development of a variety of "curve fitting" methods. Curve fitting is a crucial step in the transfer function approach and is required in order to obtain estimates of the modal parameters from FRF measurements.

Curve fitting is a process of matching a complex analytical model to measured data, usually in a least squared error sense. As a result of curve fitting, estimates of modal parameters (frequency, damping and residue) are obtained.

Because an analytical waveform for a linear dynamical system is being matched to a set of measured FRF data over a frequency range, whether or not the measured data matches the linear system waveform is far more important than the frequency resolution, (frequency difference between data points) of the measurement. Hence, the accuracy of the resulting modal parameter estimates depends more on the "shape" of the FRF measurement data and not its frequency resolution.

In fact, in most practical cases, *the frequency and damping of a mode can be estimated with better accuracy than the frequency resolution of the measurement data.* Figure 2 illustrates this point.

In Figure 2, two different FRFs have been synthesized using known modal parameters, which are listed below them. Notice that the only difference between the two sets of parameters is the modal frequency, which is 75 Hz. for one and 72.5 Hz. for the other.

The resolution of the FRFs is 10 Hz. The 75 Hz. mode happens to have a resonance peak which corresponds with one of the FRF frequency points (of frequency lines), and the 72.5 Hz. modal peaks falls in between two frequency lines. This causes the digital form of the two functions to be drastically different, as shown in Figure 2.

If only peak picking were used as a means of identifying modal frequency, then both FRFs would indicate the

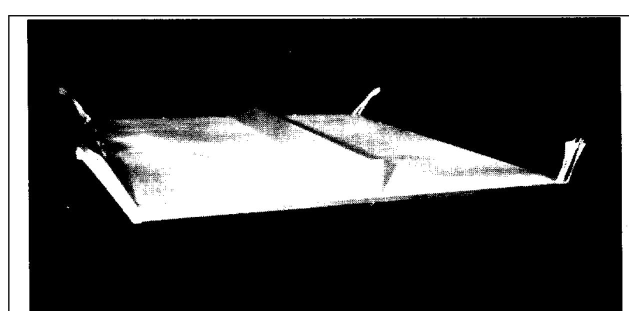
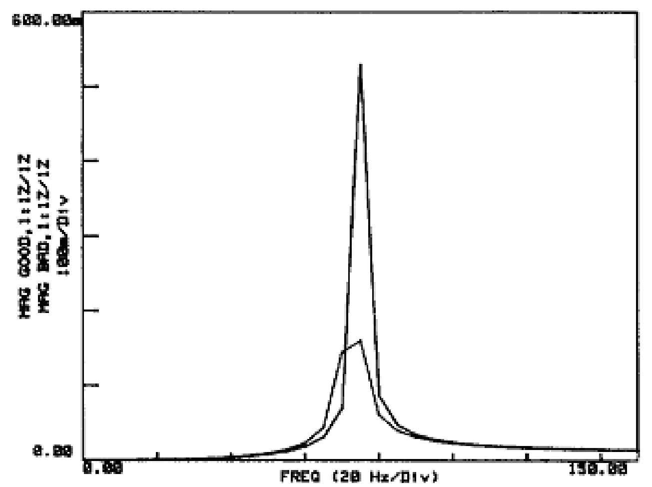


Figure 1. Plate with a Rib Stiffener

modal frequency at 75 Hz., with 10 Hz. of uncertainty with respect to the actual value. However, with the use of a curve fitter, the accuracy of the modal frequency estimate is much better.

Both FRFs were curve fit using a polynomial-based frequency domain curve fitter [3]. The resulting modal parameter estimates are listed in Figure 2.

Damping estimates are also improved with the use of a curve fitter. If the "half power point" method were used to estimate damping, the width of the resonance peak at the half power points (or 70.7 percent of the peak magnitude value), would be in great error due to the error in the peak magnitude itself, and the lack of data at the half power points. Notice that the curve



FRF Synthesis Parameters

	Frequency (Hz.)	Damping (%)	Residue Amplitude	Residue Phase
FRF #1	72.5	1.0	5.0	0.0
FRF #2	75.0	1.0	5.0	0.0

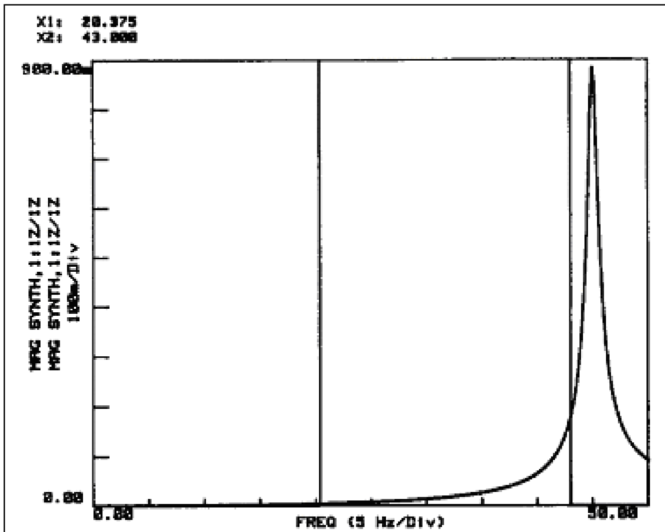
Curve Fit Results

	Frequency (Hz.)	Damping (%)	Residue Amplitude	Residue Phase
FRF #1	72.500	1.000	5.000	1.150
FRF #2	75.000	1.000	5.000	1.150

Figure 2. Curve Fir of Low Resolution FRFs

fitter has no trouble correctly estimating the damping, as shown by the curve fitting estimates in Figure 2.

Figure 3 is another illustration of the accuracy of the polynomial curve fitter. In this case, the frequency of the resonance *lies outside of the band of measurement data* which was used for curve fitting. The data used for curve fitting is indicated by the vertical lines, which are located at about 20 Hz. and 43 Hz.



FRF Synthesis Parameters

Frequency (Hz.)	Damping (%)	Residue Amplitude	Residue Phase
45.0	1.0	5.0	0.0

Curve Fit Results

Frequency (Hz.)	Damping (%)	Residue Amplitude	Residue Phase
45.000	1.000	5.000	1.146

Figure 3. Curve Fit of FRF Away from Resonance

Again, the curve fitter was able to very accurately estimate the modal parameters of the resonance because *the complex form of the measurement data within the measurement band is unique* and only one analytical model can be fit to it.

Clearly then, curve fitting is preferred as a means of more accurately estimating modal frequency and damping. A state-of-the-art curve fitter could be used on FRF measurements taken from successive modal tests and any changes in these parameters would indicate a physical change in the structure.

COMPARISON OF MODE SHAPES

Mechanical resonances are described with the same concepts and mathematics as resonances in electronic networks. A vibrating structure behaves in the same manner as an electronic amplifier. That is, a small amount of input applied at the right frequency can yield a greatly amplified output. This "unpredictable" nature of mechanical resonances under dynamic loads, as compared to the very predictable deflections under static loads, is what causes structures to make noise, vibrate excessively and break.

The one unique characteristic of a mechanical structure, which is not found in an electronic network, however, is the *spatial description* of the strength (amplitude) of each resonance. This is known as the mode shape.

Each mode of vibration or resonance, has a mode shape associated with it, *which describes spatially the predominant motion of the structure at or near the frequency of the mode.*

Just as with modal frequency and damping, if a physical change occurs in the structure, its mode shapes will also change to reflect the change. When a change does occur, all of the mode shapes will be changed differently, depending upon where on the structure the change occurred and what the mode shapes look like in the vicinity of the change.

MODE SHAPE PLOTTING

Given that the mode shapes will change when a physical change occurs, we need a method for detecting the change. One is to simply plot the mode shapes obtained from successive tests super-imposed upon one another. Since mode shapes are not unique in value, but only in "shape," they can always be scaled so that they can be plotted together. This is certainly quick and simple, but is not a quantitative method for comparing mode shapes.

MAC VALUES

A simple quantitative method for comparing mode shapes is the Modal Assurance Criterion (MAC) method [1]. This calculation, which is no more than a DOT product between two complex unit vectors, results in a single number for comparing shapes; one (1) if they are identical shapes and zero (0) if they are orthogonal to and very unlike, one another.

Again, this method could be used on mode shapes taken from successive tests of a structure and if the MAC values of all mode shapes from the two tests are within a prescribed limit (e.g., greater than 0.95), it is assumed that the structure has not changed.

RANK ORDERING OF DIFFERENCES

Another advantage of mode shapes is that they can be used to "localize" the change on the structure. Mode shapes can be sampled (measured) from as many points on the structure as desired. As with time domain sampling of a signal, the more a mode shape is sampled across the span of a structure, the more accurately a change can be pin-pointed to a specific region of the structure when a change in its mode shapes is detected.

Since each component of a mode shape is associated with a specific point on the structure and direction of motion, the *differences between mode shapes* from successive tests could be rank ordered, from the largest difference to the smallest. This type of ordering will point out *where on the structure the greatest change in mode shapes has occurred,* which should localize the physical change also.

COMPARISON OF NODE LINES

In examining a mode shape, one might ask, "Where is the mode shape most sensitive to changes in the structure?" One way to answer this is to consider the *slope* of the mode shape, i.e., its first derivative with respect to the space variable.

The closed form expressions for the mode shapes of a straight beam are all represented by sine, cosine and hyperbolic sine and cosine functions [2]. For example, the closed form solution for the mode shape of the i^{th} mode of a pinned-pinned beam is:

$$\text{Mode Shape}_i(x) = \sin(i\pi x/L)$$

where:

L = Length of the beam
x = Distance along the beam

A node point of a mode is defined as a point where *its mode shape is zero.* A node point can also be defined in a specific direction. For example, all points where the mode shape is zero in a normal direction to the plane of a surface can be considered as node points. A *node line,* then, is defined as *a locus of node points.*

For a pinned-pinned beam, the node points for the i^{th} mode occur at those values of x where:

$$\sin(i\pi x/L) = 0$$

Now, since the derivative of the sine is the cosine, it follows that *the maximum slope or rate of change* of the mode shape *always occurs at the node points.* It can therefore be concluded that any change in the relative amplitudes of a mode shape will have its greatest effect at the node points. Furthermore, any changes in a mode shape should cause the location of the node points to move.

Hence, it is proposed here that the node lines of the modes of a structure be monitored as a means of detecting physical changes. Figure 4 shows the node line plots for the first eight modes of a flat plate structure. The node lines are drawn by connecting node points, which are computed as points where the mode shape is zero in a normal direction to the surface of the structure.

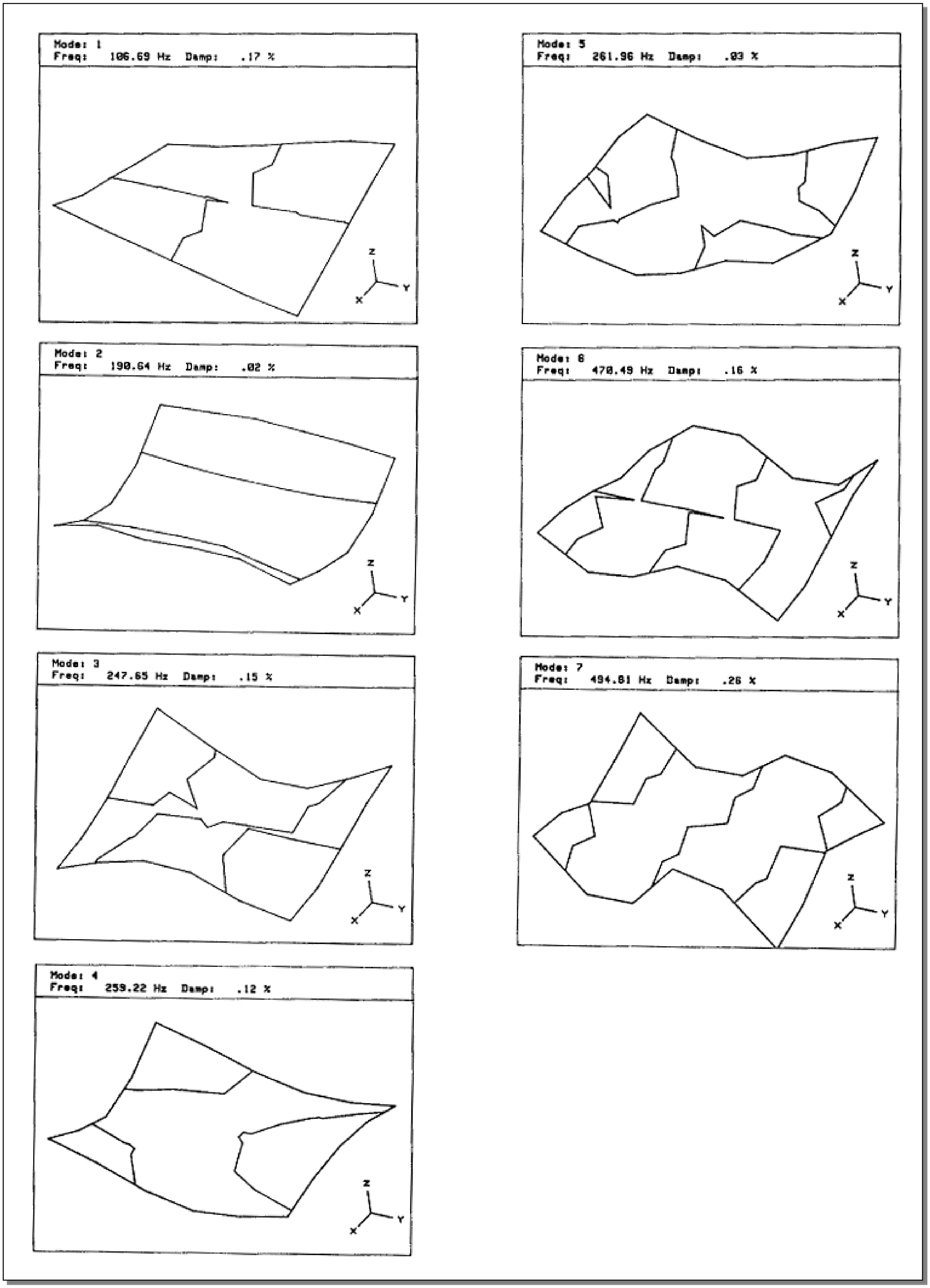


Figure 4. Mode Shapes with Node Lines

NODE LINE MAC

As a quantitative means of determining when a node line has moved, a MAC calculation can be done on the node points. The coordinates of each set of node points for a mode can be considered as a vector quantity. The MAC calculation is done, then, on the same set of node points from two successive modal tests of the structure. A MAC value of less than 1 would indicate that a change had occurred, but would give no information about where it had occurred.

RANK ORDERING OF NODE POINT DIFFERENCES

Just as with the mode shapes themselves, movement of node lines will indicate not only that a physical change has occurred, but also *where it has occurred.*

A quantitative indication of where a change has occurred can be done by rank ordering the geometric difference between node points from two successive tests of the structure. This is a straightforward calculation which is simply the square root of the squared differences between the node points in the X, Y and Z directions.

AN ILLUSTRATIVE EXAMPLE

To illustrate all of the above measures of change in the modal parameters of a structure, an aluminum flat plate with a rib stiffener bolted along its centerline was tested as shown in Figure 1. The rib was bolted on with six equally spaced bolts. The plate structure was tested with all of the bolts tightened and then with certain bolts removed.

Case #1: Center Bolt Removed

In this case, one of the bolts on the center of the plate was removed to simulate a physical change in the plate-rib assembly. The frequencies of the first seven modes before and after the bolt was removed are shown in Figure 5. Clearly, the higher frequency modes have dropped in frequency due to the removal of the bolt.

Figure 6 shows the MAC values for the modes shapes before and after the bolt was removed. These values indicate that all of the mode shapes, except those for modes 4 and 5, did not change as a result of the bolt removal.

	With Bolt	Without Bolt	
MODE	FREQ (Hz)	FREQ (Hz)	DIFFERENCE (Hz)
1	106.687	105.635	-1.052
2	190.636	190.186	-0.450
3	247.650	242.994	-4.656
4	259.222	254.200	-5.022
5	261.955	260.137	-1.818
6	470.489	466.324	-4.165
7	494.810	484.482	-10.328

Figure 5. Modal Frequencies Before and After Center Bolt Removal

		With Bolt						
	Mode	1	2	3	4	5	6	7
Without Bolt	1	0.98	0.00	0.01	0.00	0.01	0.06	0.00
	2	0.00	0.97	0.00	0.00	0.00	0.00	0.01
	3	0.00	0.00	0.96	0.01	0.03	0.02	0.00
	4	0.00	0.02	0.00	0.62	0.33	0.00	0.01
	5	0.00	0.01	0.01	0.35	0.54	0.00	0.01
	6	0.61	0.00	0.00	0.00	0.00	0.95	0.00
	7	0.01	0.00	0.00	0.02	0.01	0.01	0.94

Figure 6. Mode Shape MAC Values Before and After Center Bolt Removal

Figure 7 shows the mode shapes for modes 4 and 5 before and after the bolt was removed. The shapes show apparently little change, but the low MAC values between them indicate the strong sensitivity of the MAC calculation for determining mode shape changes

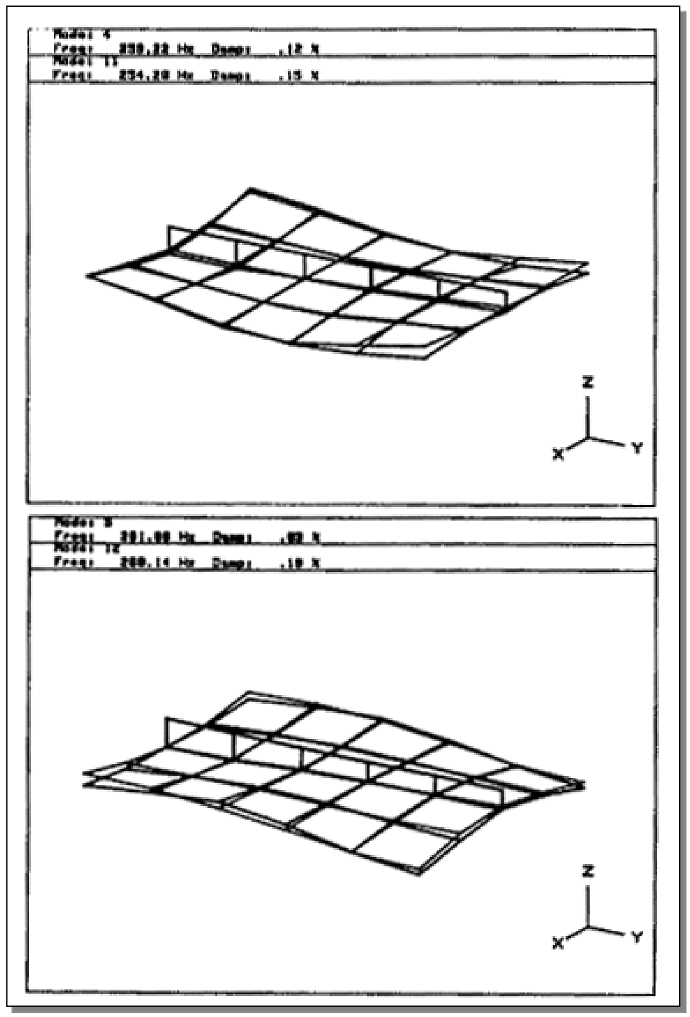


Figure 7. Mode Shapes of Modes 4 and 5 Before and After Center Bolt Removal

Case #2 End Bolt Removal

In this case, only one of the bolts on the end of the rib was removed from the plate-rib assembly and a modal test performed on it.

Figure 8 shows the frequencies of the first seven modes before and after the end bolt was removed. This caused even greater frequency shifts than removal of the center bolt.

	With Bolt	Without Bolt	
Mode	FREQ (Hz)	FREQ (Hz)	DIFFERENCE (Hz)
1	106.687	103.796	-2.891
2	190.636	188.184	-2.452
3	247.650	233.385	-14.265
4	259.222	242.108	-17.114
5	261.955	259.559	-2.396
6	470.489	442.153	-28.336
7	494.810	464.330	-30.480

Figure 8. Modal Frequencies Before and After End Bolt Removal

The mode shape MAC values are given in Figure 9 and indicate that all of the mode shapes (except the first one) have also changed substantially.

		With Bolt						
	Mode	1	2	3	4	5	6	7
Without Bolt	1	0.98	0.00	0.00	0.00	0.01	0.05	0.00
	2	0.01	0.79	0.00	0.02	0.01	0.01	0.01
	3	0.00	0.01	0.13	0.45	0.42	0.00	0.00
	4	0.00	0.00	0.81	0.04	0.08	0.01	0.00
	5	0.00	0.00	0.01	0.45	0.46	0.00	0.01
	6	0.00	0.00	0.00	0.01	0.02	0.01	0.82
	7	0.06	0.00	0.00	0.00	0.00	0.96	0.03

Figure 9. Mode Shape MAC Values Before and After End Bolt Removal

Figure 10 shows plots of the difference between the mode shapes from before and after the end bolt removal. Clearly, there are large differences in the shapes, but they *do not pinpoint* the location of the fault. One explanation for this is that all of these modes are "global" in nature (which is true for most simple structures) and hence will change globally even due to a "local" change such as the end bolt removal.

Figure 11 shows the mode shape node lines from before and after the end bolt removal. Again, the movement of the node lines clearly reflects the effect of the bolt removal.

CONCLUSIONS

We have introduced and demonstrated the use of several new quantitative methods for measuring changes in the modal parameters of a structure. It was also assumed at the outset of course, that changes in the modal parameters of a structure are sensitive indicators of changes that have occurred in its physical properties. This is readily apparent from an examination of the dynamical equations of motion of an elastic structure.

The methods demonstrated here are based upon the curve fitting of experimental FRF data, which could be acquired with any modern multi-channel fast Fourier transfer analyzer and processed automatically in an on-line computer-based monitoring system. The core of the monitoring scheme, then, is to detect "significant" changes in the modal parameters of the structure.

Any set of measurements that are repeatedly made over time will exhibit variations. These variations are caused either by the "natural" statistical variation in the measurement process, due to numerous sources of measurement error or they are caused by a physical change in the structure, i.e., an "assignable cause."

The Statistical Process Control (SPC) method, popularized by Demming [4] for use in manufacturing quality control, is a way of determining whether or not a variation is "statistical" or due to an assignable cause. SPC charts could also be used in the proposed monitoring environment to show whether the modal parameter estimates are within their statistical limits or are developing a trend which is caused by a structural fault.

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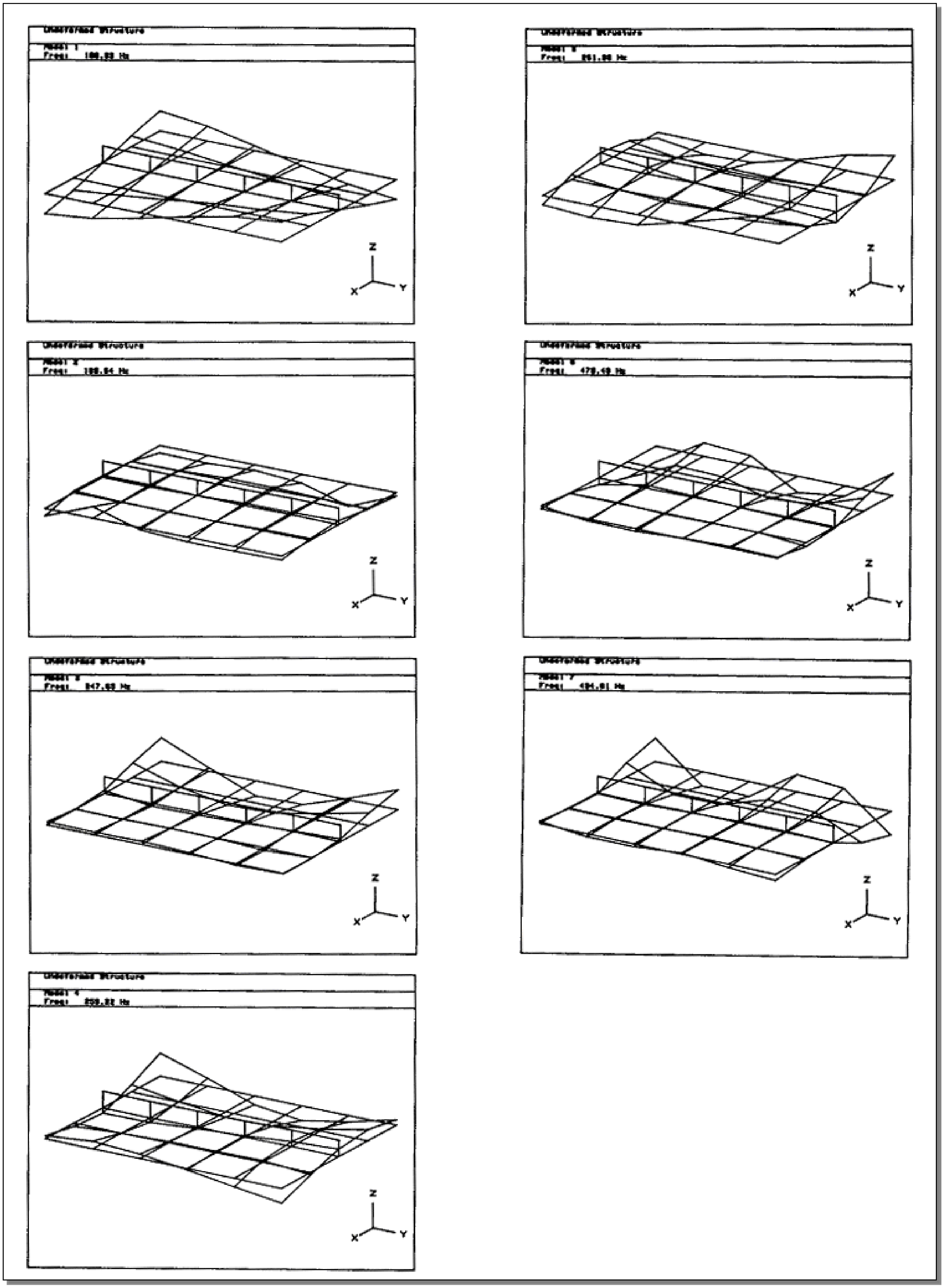

Figure 10. Mode Shape Differences Before and After End Bolt Removed

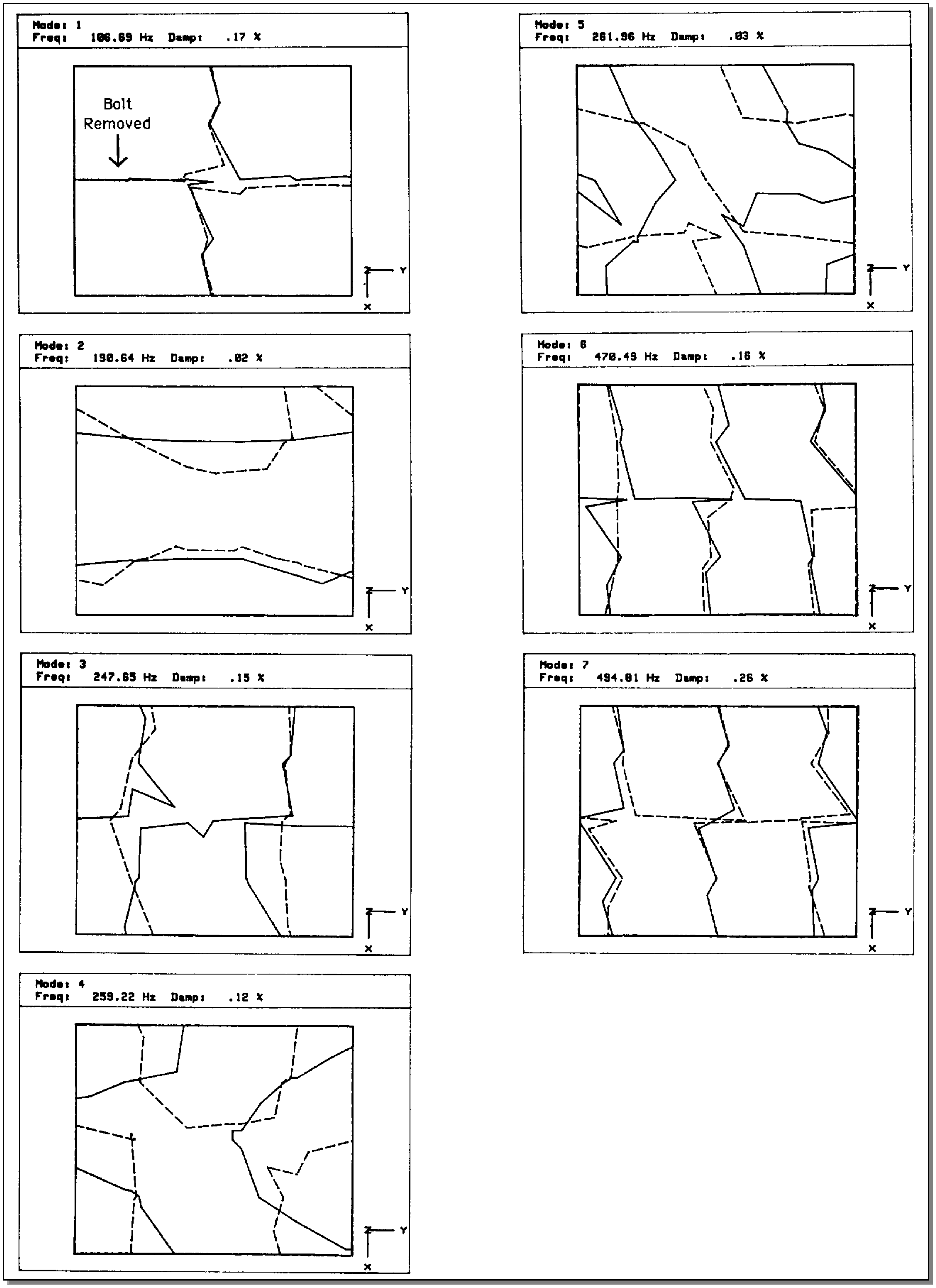

Figure 11. Mode Shape Node Lines Before and After End Bolt Removed