EXPERIMENTAL MODAL ANALYSIS

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ABSTRACT

Experimental modal analysis has grown steadily in popularity since the advent of the digital FFT spectrum analyzer in the early 1970's. Today, impact testing (or bump testing) has become widespread as a fast and economical means of finding the modes of vibration of a machine or structure.

In this paper, we review all of the main topics associated with experimental modal analysis (or modal testing), including making FRF measurements with a FFT analyzer, modal excitation techniques, and modal parameter estimation from a set of FRFs (curve fitting).

INTRODUCTION

Modes are used as a simple and efficient means of characterizing resonant vibration. The majority of structures can be made to resonate. That is, under the proper conditions, a structure can be made to vibrate with ***excessive, sustained, oscillatory motion***.

Resonant vibration is caused by an interaction between the ***inertial*** and ***elastic*** properties of the materials within a structure. Resonant vibration is often the cause of, or at least a contributing factor to many of the vibration related problems that occur in structures and operating machinery.

To better understand any structural vibration problem, the resonances of a structure need to be identified and quantified. A common way of doing this is to define the structure's modal parameters.

TWO TYPES OF VIBRATION

All vibration is a combination of both forced and resonant vibration. Forced vibration can be due to,

- Internally generated forces.
- Unbalances.
- External loads.
- Ambient excitation.

Resonant vibration occurs when one or more of the ***resonances or natural modes of vibration*** of a machine or structure is excited. Resonant vibration typically ***amplifies the vibration response*** far beyond the level deflection, stress, and strain caused by static loading.

What are Modes?

Modes (or resonances) are inherent properties of a structure. Resonances are determined by the material properties (mass, stiffness, and damping properties), and boundary conditions of the structure. Each mode is defined by a ***natural (modal or resonant) frequency***, ***modal damping***, and a ***mode shape***. If either the material properties or the boundary conditions of a structure change, its modes will change. For instance, if mass is added to a vertical pump, it will vibrate differently because its modes have changed.

At or near the natural frequency of a mode, the overall vibration shape (operating deflection shape) of a machine or structure will tend to be ***dominated by*** the mode shape of the resonance.

What is an Operating Deflection Shape?

An operating deflection shape (ODS) is defined as ***any forced motion of two or more points*** on a structure. Specifying the motion of two or more points defines a shape. Stated differently, a shape is the motion of one point relative to all others. Motion is a ***vector quantity***, which means that it has both a ***location*** and a ***direction*** associated with it. Motion at a point in a direction is also called a Degree Of Freedom, or DOF.

"All experimental modal parameters are obtained from measured ODS's."

That is, experimental modal parameters are obtained by artificially exciting a machine or structure, measuring its operating deflection shapes (motion at two or more DOFs), and post-processing the vibration data.

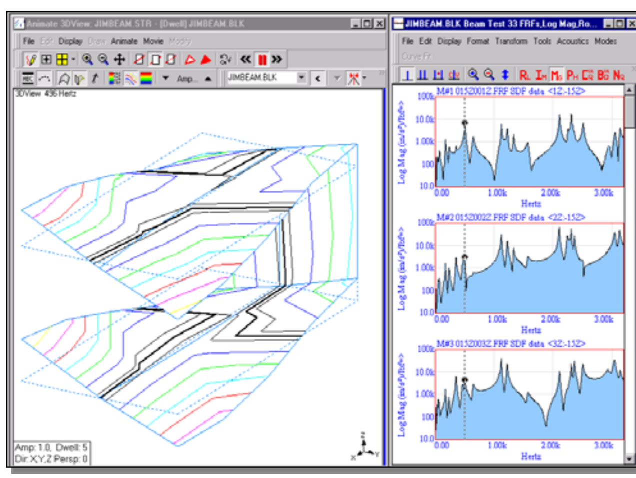

Figure 1. Frequency Domain ODS From a Set of FRFs

Figure 1 shows an ODS being displayed from a set of FRF measurements with the cursor located at a resonance peak. In this case, ***the ODS is being dominated by a mode*** and therefore is a close approximation to the mode shape.

Two Kinds of Modes

Modes are further characterized as either ***rigid body*** or ***flexible body*** modes. All structures can have up to ***six rigid body*** modes, three translational modes and three rotational modes. If the structure merely bounces on some soft springs, its motion approximates a rigid body mode.

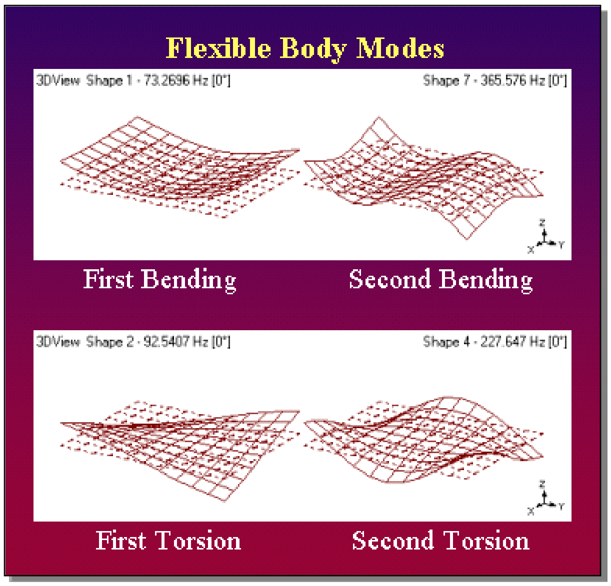

Figure 2. Flexible Body Modes.

Many vibration problems are caused, or at least amplified by the excitation of one or more flexible body modes. Figure 2 shows some of the common fundamental (low frequency) modes of a plate. The fundamental modes are given names like those shown in Figure 2. The higher frequency mode shapes are usually more complex in appearance, and therefore don't have common names.

FRF MEASUREMENTS

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of a mechanical structure. Experimental modal parameters (frequency, damping, and mode shape) are also obtained from a set of FRF measurements.

The FRF describes the input-output relationship between two points on a structure as a function of frequency, as shown in Figure 3. Since both force and motion are vector quantities, they have directions associated with them. Therefore, an FRF is actually defined between a single input DOF (point & direction), and a single output DOF.

An FRF is a measure of ***how much displacement, velocity, or acceleration response*** a structure has ***at an output DOF***, ***per unit of excitation force at an input DOF***.

Figure 3 also indicates that an FRF is defined as the ***ratio of the Fourier transform of an output response*** ($X(\omega)$) ***divided by the Fourier transform of the input force*** ($F(\omega)$) that caused the output.

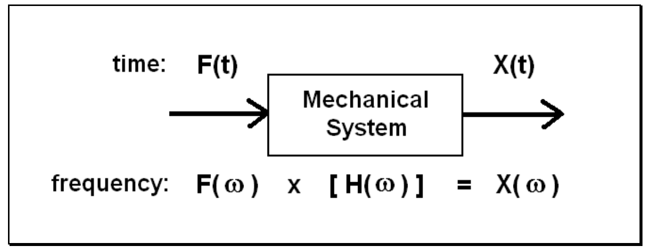

Figure 3. Block Diagram of an FRF.

Depending on whether the response motion is measured as displacement, velocity, or acceleration, the FRF and its inverse can have a variety of names,

- **Compliance** ⇔ (displacement / force)
- **Mobility** ⇔ (velocity / force)
- **Inertance or Receptance** ⇔ (acceleration / force)
- **Dynamic Stiffness** ⇔ (1 / Compliance)
- **Impedance** ⇔ (1 / Mobility)
- **Dynamic Mass** ⇔ (1 / Inertance)

An FRF is a complex valued function of frequency that is displayed in various formats, as shown in Figure 4.

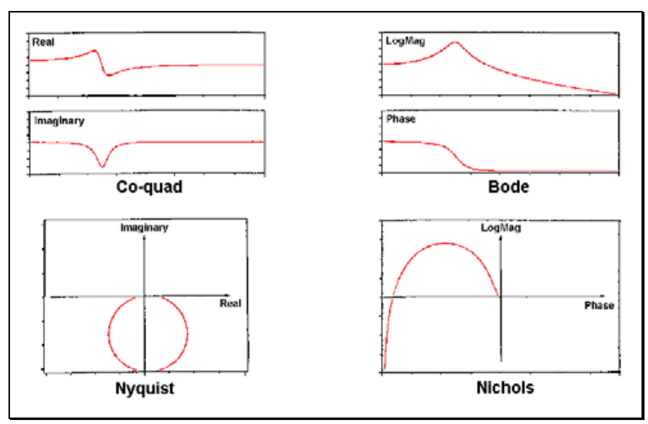

Figure 4. Alternate Formats of the FRF.

VIBRATION IS EASIER TO UNDERSTAND IN TERMS OF MODES

Figure 5 points out another reason why vibration is easier to understand in terms of modes of vibration. It is a plot of the Log Magnitude of an FRF measurement (the solid curve), but several ***resonance curves*** are also plotted as dotted lines below the FRF magnitude. Each of these resonance curves is the structural response due to a single mode of vibration.

The overall structural response (the solid curve) is in fact, the ***summation of resonance curves***. In other words, the overall response of a structure at any frequency is a ***summation of responses due to each of its modes***. It is also evident that close to the frequency of one of the resonance peaks, the response of ***one mode will dominate the frequency response***.

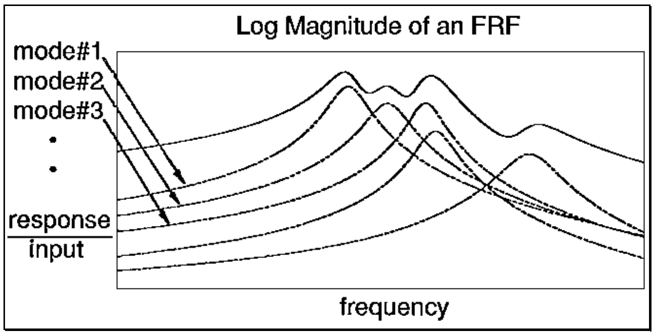

Figure 5. Response as Summation of Modal Responses.

WHY ARE MODES DANGEROUS?

Figure 6 shows why modes cause structures to act as ***"mechanical amplifiers"***. At certain natural frequencies of the structure (its modal frequencies), a small amount of input force can cause a ***very large response***. This is clearly evident from the narrow peaks in the FRF. (When a peak is very narrow and high in value, it is said to be a ***high Q resonance***.)

If the structure is excited at or near one of the peak frequencies, the response of the structure per unit of input force will be large. On the other hand, if the structure is excited at or near one of the ***anti-resonances*** (zeros or inverted peaks), the structural response will be very small per unit of input force.

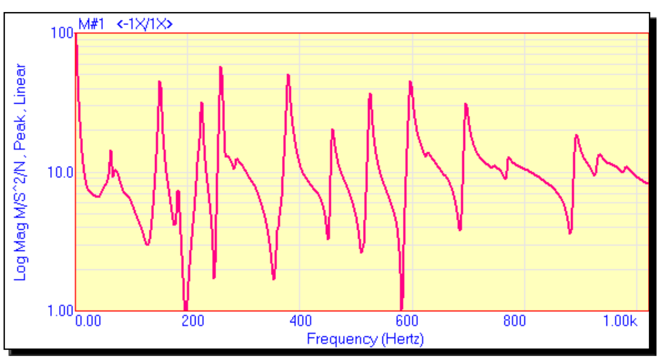

Figure 6. FRF With High Q Resonance Peaks.

TESTING REAL STRUCTURES

Real continuous structures have an infinite number of DOFs, and an infinite number of modes. From a testing point of view, a real structure can be ***sampled spatially*** at as many DOFs as we like. There is no limit to the number of unique DOFs between which FRF measurements can be made.

However, because of time and cost constraints, we only measure a small subset of the FRFs that could be measured on a structure. This is depicted in Figure 7.

Yet, from this small subset of FRFs, we can accurately define the modes that are within the frequency range of the measurements. Of course, the more we ***spatially sample*** the surface of the structure by taking more measurements, the more definition we will give to its mode shapes.

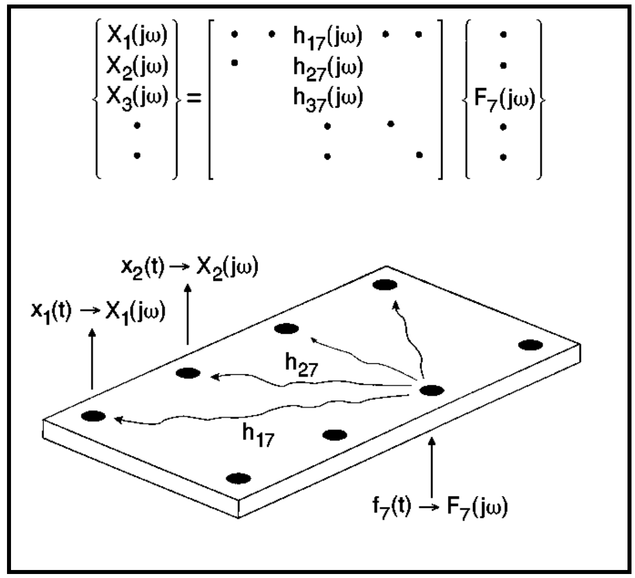

Figure 7. Measuring FRFs on a Structure

FRF CALCULATION

Although the FRF was previously defined as a ratio of the Fourier transforms of an output and input signal, is it actually computed differently in all modern FFT analyzers. This is done to ***remove random noise*** and ***non-linearity's*** (distortion) from the FRF estimates.

Tri-Spectrum Averaging

The measurement capability of all multi-channel FFT analyzers is built around a tri-spectrum averaging loop, as shown in Figure 8. This loop assumes that two or more time domain signals are simultaneously sampled. Three spectral estimates, an Auto Power Spectrum (**APS**) for each channel, and the Cross Power Spectrum (**XPS**) between the two channels, are calculated in the tri-spectrum averaging loop. After the loop has completed, a variety of other cross channel measurements (including the FRF), are calculated from these three basic spectral estimates.

In a multi-channel analyzer, tri-spectrum averaging can be applied to as many signal pairs as desired. Tri-spectrum averaging removes random noise and randomly excited non-linearity's from the XPS of each signal pair. This ***low noise***

measurement of the effective linear vibration of a structure is particularly useful for experimental modal analysis.

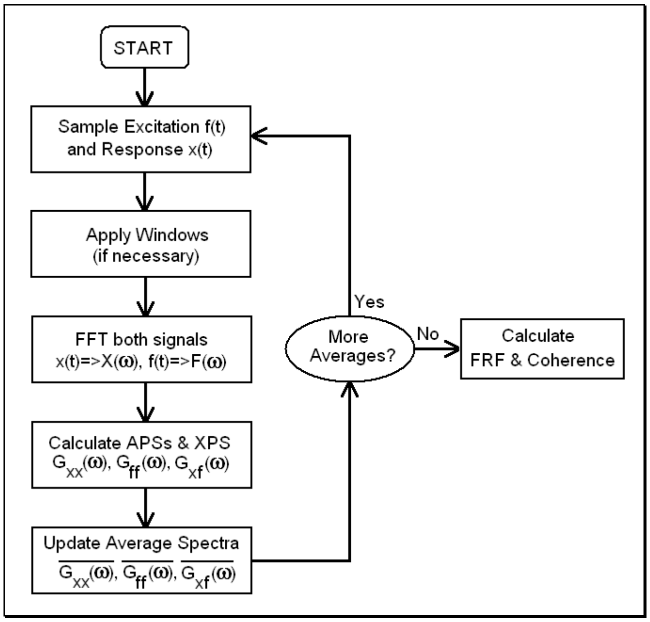

Figure 8. Tri-Spectrum Averaging Loop

Following tri-spectrum averaging, FRFs can be calculated in several different ways.

Noise on the Output (H_1)
This FRF estimate assumes that random noise and distortion are summing into the output, but not the input of the structure and measurement system. In this case, the FRF is calculated as,

$$H_1 = \frac{\text{XPS}}{\text{Input APS}}$$

where **XPS** denotes the ***cross power spectrum*** estimate between the input and output signals, and **Input APS** denotes the ***auto power spectrum*** of the input signal.

It can be shown that H_1 is a ***least squared error estimate*** of the FRF when extraneous noise and randomly excited non-linearity's are modeled as Gaussian ***noise added to the output*** [2].

Noise on the Input (H_2)
This FRF estimator assumes that random noise and distortion are summing into the input, but not the output of the structure and measurement system. For this model, the FRF is calculated as,

$$H_2 = \frac{\text{Output APS}}{\text{XPS}}$$

Likewise, it can be shown that H_2 is a least squared error estimate for the FRF when extraneous noise and randomly excited non-linearity's are modeled as Gaussian ***noise added to the input***. [2].

Noise on the Input & Output (H_V)
This FRF estimator assumes that random noise and distortion are summing into both the input but and output of the system. The calculation of H_V requires more steps, and is detailed in [2].

THE FRF MATRIX MODEL

Structural dynamics measurement involves measuring elements of an FRF matrix model for the structure, as shown in Figure 7. This model represents the dynamics of the structure between all pairs of input and output DOFs.

The FRF matrix model is a frequency domain representation of a structure's linear dynamics, where linear spectra (FFTs) of multiple inputs are multiplied by elements of the FRF matrix to yield linear spectra (FFTs) of multiple outputs.

FRF matrix ***columns correspond to inputs***, and ***rows correspond to outputs***. Each input and output corresponds to a measurement DOF of the test structure.

Modal Testing
In modal testing, FRF measurements are usually made under controlled conditions, where the test structure is artificially excited by using either an impact hammer, or one or more shakers driven by broadband signals. A multi-channel FFT analyzer is then used to make FRF measurements between input and output DOF pairs on the test structure.

Measuring FRF Matrix Rows or Columns
Modal testing requires that FRFs be measured from ***at least one row or column*** of the FRF matrix. Modal frequency & damping are global properties of a structure, and can be estimated from any or all of the FRFs in a row or column of the FRF matrix. On the other hand, each mode shape is obtained by assembling together FRF numerator terms (called ***residues***) from at least one row or column of the FRF matrix.

Impact Testing
When the output is fixed and FRFs are measured for multiple inputs, this corresponds to measuring elements from a ***single row*** of the FRF matrix. This is typical of a roving hammer impact test.

Shaker Testing
When the input is fixed and FRFs are measured for multiple outputs, this corresponds to measuring elements from a ***single column*** of the FRF matrix. This is typical of a shaker test.

Single Reference (or SIMO) Testing
The most common type of modal testing is done with either a single fixed input or a single fixed output. A roving hammer impact test using a single fixed motion transducer is a common example of single reference testing. The single fixed output is called the reference in this case.

When a single fixed input (such as a shaker) is used, this is called SIMO (Single Input Multiple Output) testing. In this case, the single fixed input is called the reference.

Multiple Reference (or MIMO) Testing
When ***two or more fixed inputs*** are used, and FRFs are calculated between each of the inputs and multiple outputs, then FRFs from multiple columns of the FRF matrix are obtained. This is called Multiple Reference or MIMO (Multiple Input Multiple Output) testing. In this case, the inputs are the references.

Likewise, when ***two or more fixed outputs*** are used, and FRFs are calculated between each output and multiple inputs, this is also multiple reference testing, and the outputs are the references.

Multi-reference testing is done for the following reasons,

- The structure cannot be adequately excited from one reference.
- All modes of interest cannot be excited from one reference.
- The structure has ***repeated roots***, modes that are so closely coupled that more than one reference is needed to identify them.

EXCITING MODES WITH IMPACT TESTING

With the ability to compute FRF measurements in an FFT analyzer, impact testing was developed during the late 1970's, and has become the most popular modal testing method used today. Impact testing is a fast, convenient, and low cost way of finding the modes of machines and structures.

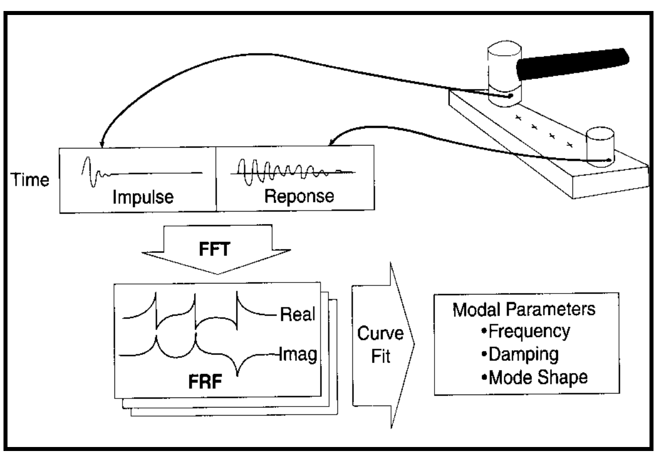

Figure 9. Impact Testing.

Impact testing is depicted in Figure 9. The following equipment is required to perform an impact test,

1. An ***impact hammer*** with a load cell attached to its head to measure the input force.
2. An ***accelerometer*** to measure the response acceleration at a fixed point & direction.
3. A 2 or 4 channel ***FFT analyzer*** to compute FRFs.
4. ***Post-processing modal software*** for identifying modal parameters and displaying the mode shapes in animation.

A wide variety of structures and machines can be impact tested. Of course, different sized hammers are required to provide the appropriate impact force, depending on the size of the structure; small hammers for small structures, large hammers for large structures. Realistic signals from a typical impact test are shown in Figure 10.

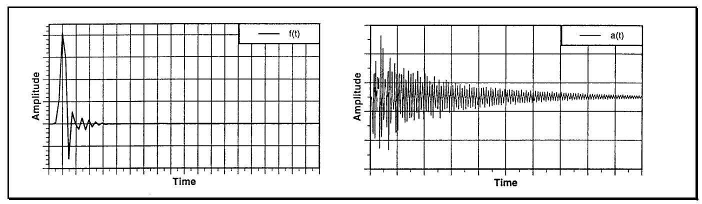

Figure 10A. Impact Force and Response Signals

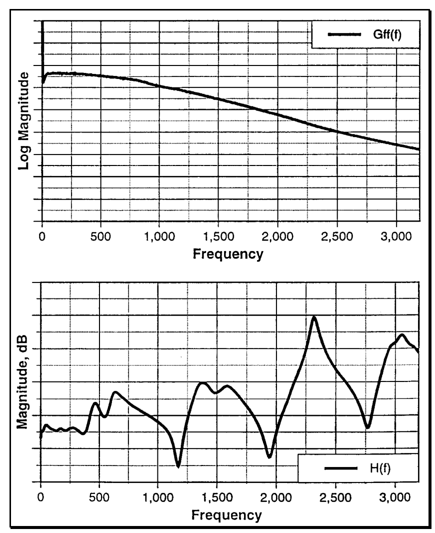

Figure 10B. Impact APS and FRF.

Roving Hammer Test
A roving hammer test is the most common type of impact test. In this test, the accelerometer is fixed at a single DOF, and the structure is impacted at as many DOFs as desired to define the mode shapes of the structure. Using a 2-channel FFT analyzer, FRFs are computed one at a time, between each impact DOF and the fixed response DOF.

Roving Tri-axial Accelerometer Test
The only drawback to a roving hammer test is that ***all of the points*** on most structures cannot be impacted in all three directions, so 3D motion cannot be measured at all points. When 3D motion at each test point is desired in the resulting mode shapes, a roving tri-axial accelerometer is used and the structure is impacted at a fixed DOF with the hammer. Since

the tri-axial accelerometer must be simultaneously sampled together with the force data, a 4-channel FFT analyzer is required instead of a 2-channel analyzer.

Impact Testing Requirements

Even though impact testing is fast and convenient, there are several important considerations that must be taken into account in order to obtain accurate results. They include, pre-trigger delay, force and exponential windowing, and accept/reject capability

Pre-Trigger Delay

Because the impulse signal exists for such a short period of time, it is important to capture all of it in the sampling window of the FFT analyzer. To insure that the entire signal is captured, the analyzer must be able to capture the impulse and impulse response signals ***prior to the occurrence of the impulse***. In other words, the analyzer must begin sampling data before the trigger point occurs, which is usually set to a small percentage of the peak value of the impulse. This is called a pre-trigger delay.

Force & Exponential Windows

Two common time domain windows that are used in impact testing are the force and exponential windows. These windows are applied to the signals after they are sampled, but before the FFT is applied to them in the analyzer.

The ***force window is used to remove noise*** from the impulse (force) signal. Ideally, an impulse signal is non-zero for a small portion of the sampling window, and zero for the remainder of the window time period. Any non-zero data following the impulse signal in the sampling window is assumed to be measurement noise. The force window preserves the samples in the vicinity of the impulse, and removes the noise from all of the other samples in the force signal by making them zero.

The exponential window is applied to the impulse response signal. The ***exponential window is used to reduce leakage*** in the spectrum of the response.

What Is Leakage?

The FFT assumes that the signal to be transforming is ***periodic in the transform window***. (The transform window is the samples of data used by the FFT). To be periodic in the transform window, the waveform must have no discontinuities at its beginning or end, if it were repeated outside the window. Signals that are always periodic in the transform window are,

1. Signals that are completely contained within the transform window.
2. Cyclic signals that complete an integer number of cycles within the transform window.

If a time signal is not periodic in the transform window, when it is transformed to the frequency domain, a ***smearing of its spectrum*** will occur. This is called ***leakage***. Leakage ***distorts the spectrum and makes it inaccurate***.

Therefore, if the response signal in an impact test decays to zero (or near zero) before the end of the sampling window, there will be no leakage, and no special windowing is required.

On the other hand, if the response does not decay to zero before the end of the sampling window, an exponential window must be used to reduce the leakage effects in the response spectrum. The exponential window ***adds artificial damping to all of the modes of the structure in a known manner***. This artificial damping can be subtracted from the modal damping estimates after curve fitting. But more importantly, a properly applied exponential window will cause the impulse response to be completely contained within the sampling window, thus leakage will be reduced to a minimum in its spectrum.

Accept/Reject

Because accurate impact testing results depend on the skill of the one doing the impacting, FRF measurements should be made with ***spectrum averaging***, a standard capability in all modern FFT analyzers. FRFs should be measured using 3 to 5 impacts per measurement.

Since one or two of the impacts during the measurement process may be bad hits, an FFT analyzer designed for impact testing should have the ability to accept or reject the result of each impact. An accept/reject capability saves a lot of time during impact testing since you don't have to restart the measurement process after each bad hit.

SHAKER MEASUREMENTS

Not all structures can be impact tested, however. For instance, structure with ***delicate surfaces*** cannot be impact tested. Or because of its ***limited frequency range*** or ***low energy density over a wide spectrum***, the impacting force is not be sufficient to adequately excite the modes of interest.

When impact testing cannot be used, FRF measurements must be made by providing artificial excitation with one or more shakers, attached to the structure. Common types of shakers are electro-dynamic and hydraulic shakers. A typical shaker test is depicted in Figure 11.

A shaker is usually attached to the structure using a ***stinger*** (long slender rod), so that the shaker will only impart force to the structure along the axis of the stinger, the axis of force measurement. A ***load cell*** is then attached between the structure and the stinger to measure the excitation force.

At least a 2-channel FFT analyzer and a single axis accelerometer are required to make FRF measurements using a shaker. If an analyzer with 4 or more channels is used, then a tri-axial accelerometer can be used and 3D motion of the structure measured at each test point.

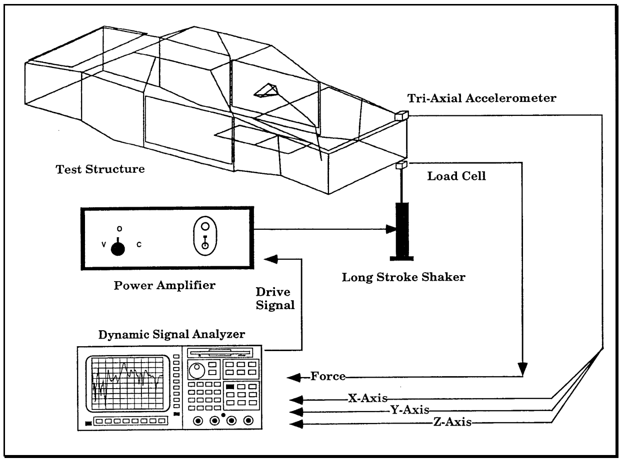

Figure 11. Shaker Test Setup.

In a SIMO test, one shaker is used and the shaker is the (fixed) reference. In a MIMO test, multiple shakers are used, and the shakers are the multiple references. When multiple shakers are used, care must be taken to insure that the shaker signals are not completely correlated (the same signal). Furthermore, special matrix processing software is required to calculate the FRFs from the multiple input APSs and XPSs resulting from a MIMO test.

Broad Band Excitation Signals

A variety of broadband excitation signals have been developed for making shaker measurements with FFT analyzers. These signals include,

- Transient
- True Random
- Pseudo Random
- Burst Random
- Fast Sine Sweep (Chirp)
- Burst Chirp

Since the FFT provides a spectrum over a broad band of frequencies, using a broadband excitation signal makes the measurement of broadband spectral measurements much faster than using a stepped or slowly sweeping sine wave.

Transient Signals

Using a transient signal in shaker testing provides the same leakage free measurements as impact testing, but with more controllability over the test. Application of the force is more repeatable than impacting with a hand held hammer. However, this one advantage is usually outweighed by the disadvantages of using an impulsive force, when compared to the other broadband signals.

True Random

Probably the most popular excitation signal used for shaker testing with an FFT analyzer is the true random signal. When used in combination with spectrum averaging, random excitation randomly excites the non-linearity's in a structure, which are then removed by spectrum averaging. Obtaining a set of noise free FRF estimates with no distortion in them is very important for obtaining accurate modal parameters.

A true random signal is synthesized with a random number generator, and is an unending (non-repeating) random sequence. The main disadvantage of a true random signal is that it is always ***non-periodic in the sampling window***. Therefore, a special time domain window (a Hanning window or one like it), ***must always be used*** with true random testing to minimize leakage. Typical true random signals are shown in Figure 12.

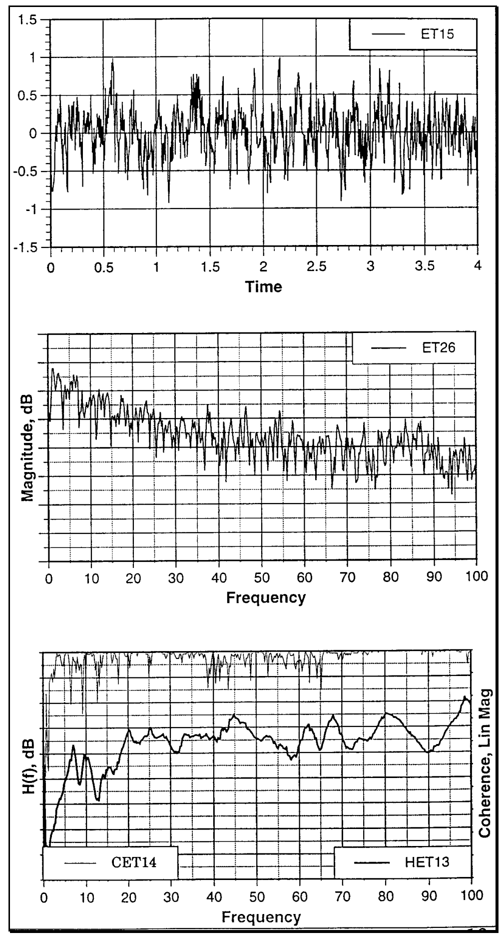

Figure 12. True Random Excitation (Time waveform, APS, FRF & Coherence).

Pseudo Random

A pseudo random signal is specially synthesized within an FFT analyzer to coincide with the FRF measurement window parameters. A typical pseudo random signal starts as a uniform (or shaped) magnitude and random phase signal, synthesized over the same frequency range and samples as the intended FRF measurement. It is then inverse FFT'd to obtain a random time domain signal, which is subsequently output through a digital-to-analog converter (DAC) as the shaker excitation signal.

During the measurement process, the measured force and response signals are sampled over the same sampling time window as the excitation signal. Since the excitation signal is completely contained in the sampling window, this insures that the acquired signals are ***periodic in the sampling window***. Therefore, the acquired signals are leakage free.

However, pseudo random excitation doesn't excite non-linearity's differently between spectrum averages. Therefore spectrum averaging of pseudo random signals will not remove non-linearity's from FRF measurements.

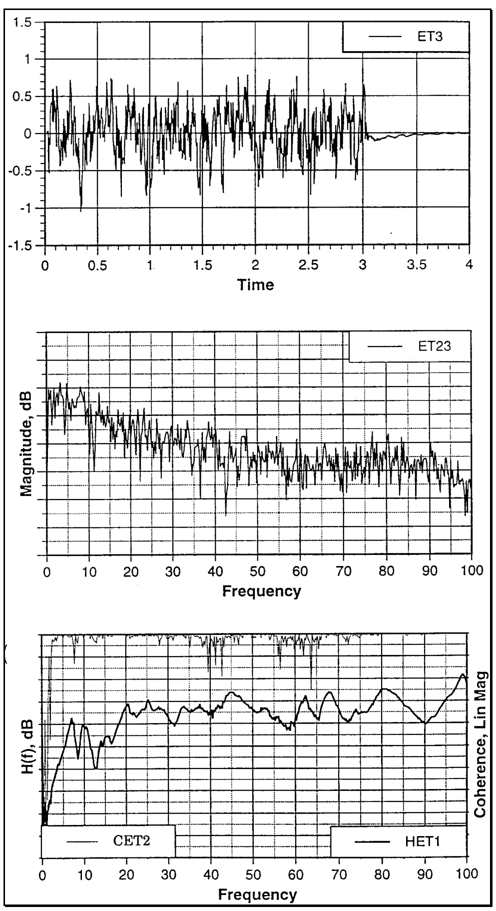

Figure 13. Burst Random Excitation (Time waveform, APS, FRF & Coherence).

Burst Random

Burst random excitation has the combined advantages of both pure random and pseudo random testing. That is, its signals are leakage free and when using with spectrum averaging, will remove non-linearity's from the FRFs.

In burst random testing, either a true random or time varying pseudo random signal can be used, but it is ***turned off prior to the end of the sampling window*** time period. This is done in order to allow the structural response to decay within the sampling window. This insures that both the excitation and response signals are completely contained within the sampling window. Hence, they are periodic in the window and leakage free.

Figure 13 shows a typical burst random signal. The random signal generator must be turned off early enough to allow the structural response to decay to zero (or nearly zero) before the end of the sampling window. The length of the decay period depends on the damping in the test structure. Therefore, a burst random test must be setup interactively on the FFT analyzer, after observing the free decay of the structural response following the removal of random excitation.

Chirp & Burst Chirp

A swept sine excitation signal can also be synthesized in an FFT analyzer to coincide with the parameters of the sampling window, in a manner similar to the way a pseudo random signal is synthesized. Since the sine waves must sweep from the lowest to the highest frequency in the spectrum, over the relatively short sampling window time period, this fast sine sweep often makes the test equipment sound like a bird chirping, hence the name chirp signal.

A burst chirp signal is the same as a chirp, except that it is ***turned off prior to the end of the sampling window***, just like burst random. This is done to insure that the measured signals are periodic in the window. A typical burst chirp signal is shown in Figure 14.

The advantage of burst chirp over chirp is that the structure has returned to rest before the next average of data is taken. This insures that the measured response is only caused by the measured excitation, an important requirement for FRF measurement.

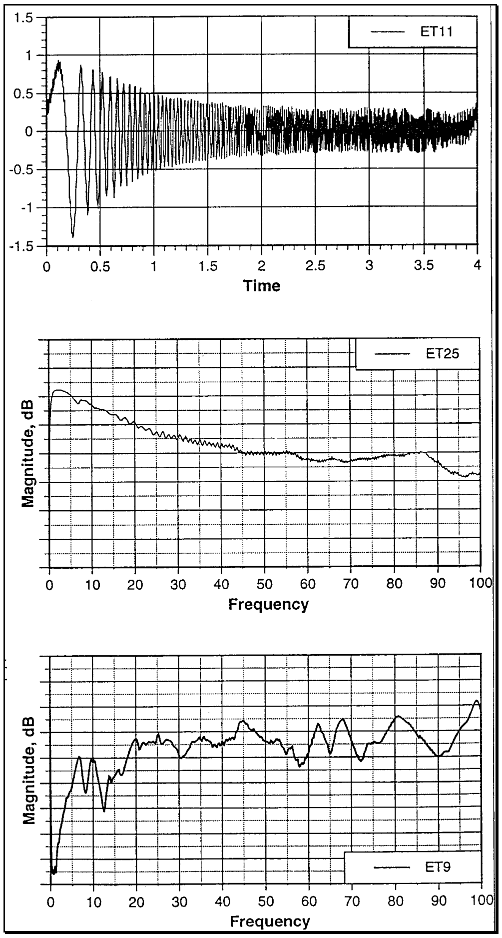

Figure 14. Burst Chirp Excitation (Time waveform, APS, FRF & Coherence)

Comparison of Excitation Signals

Ideally, all of the shaker signals that are leakage free (periodic in the window) should yield the same results. Figure 15 shows an overlay of two FRF magnitudes, one measured with a ***burst random*** test and the other with a ***burst chirp*** test. The two FRFs match very well at low frequencies, but show some disparity at high frequencies. This could possibly be due to a small amount of non-linear behavior in the structure, which burst chirp signal processing cannot remove through averaging.

HOW ARE MODAL PARAMETERS OBTAINED?

Figure 16 shows the different ways in which modal parameters can be obtained, both analytically and experimentally. A growing amount of finite element modeling, with extraction of modal parameters from the finite element model, is being done in an effort to understand and solve structural dynamics problems. Experimental modal analysis is also done for this same purpose.

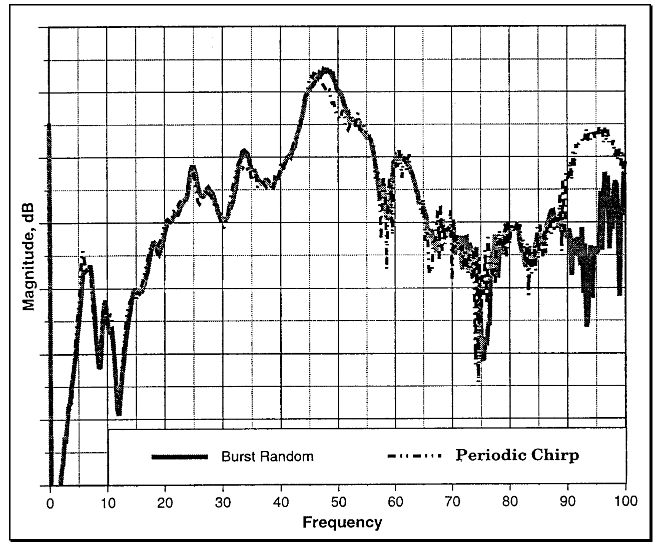

Figure 15. Burst Random Versus Burst Chirp FRF.

The majority of modern experimental modal analysis relies upon the application of a modal parameter estimation (curve fitting) technique to a set of FRF measurements. As indicated in Figure 16, the FRFs can also be ***inverse FFT'd*** and curve fitting techniques applied to their equivalent Impulse Response Functions (IRFs).

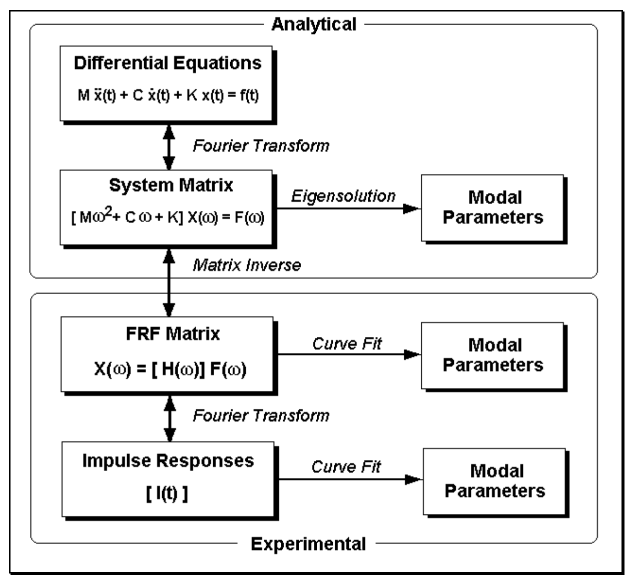

Figure 16. Sources of Modal Parameters.

MODAL PARAMETERS FROM CURVE FITTING

Modal parameters are most commonly identified by curve fitting a set of FRFs. (They can also be identified by curve fitting an equivalent set of Impulse Responses, or IRFs). In general, curve fitting is a process of matching a mathematical expression to a set of empirical data points. This is done by ***minimizing the squared error*** (or squared difference) between the analytical function and the measured data. An example of FRF curve fitting is shown in Figure 17.

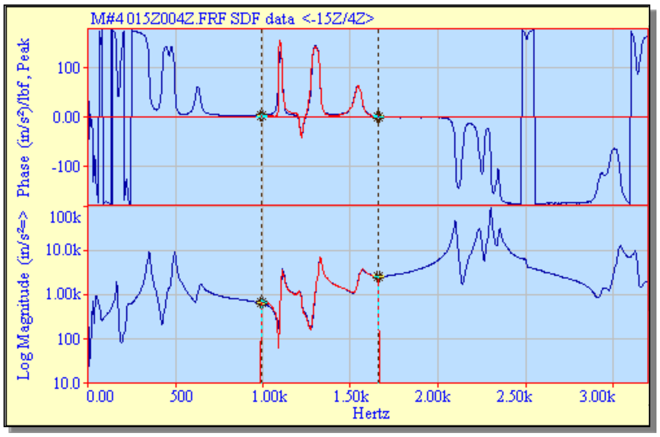

Figure 17. A Curve Fitting Example.

CURVE FITTING METHODS

All curve fitting methods fall into one of the following categories,

- **Local SDOF**
- **Local MDOF**
- **Global**
- **Multi-Reference (Poly Reference)**

In general, the methods are listed in order of increasing complexity. SDOF is short for a Single Degree Of Freedom, or single mode method. Similarly, MDOF is short for a Multiple Degree Of Freedom, or multiple mode method.

SDOF methods estimate modal parameters ***one mode at a time***. MDOF, Global, and Multi-Reference methods can simultaneously estimate modal parameters for ***two or more modes at a tim***e.

Local methods are applied to ***one FRF at a time***. Global and Multi-Reference methods are applied to an ***entire set of FRFs*** at once.

Local SDOF methods are the easiest to use, and should be used whenever possible. SDOF methods can be applied to most FRF data sets with ***light modal density (coupling)***, as depicted in Figure 19. MDOF methods must be used in cases of ***high modal density***.

Global methods work much better than MDOF methods for cases with ***local modes***. Multi-Reference methods can find ***repeated roots*** (very closely coupled modes) where the other methods cannot.

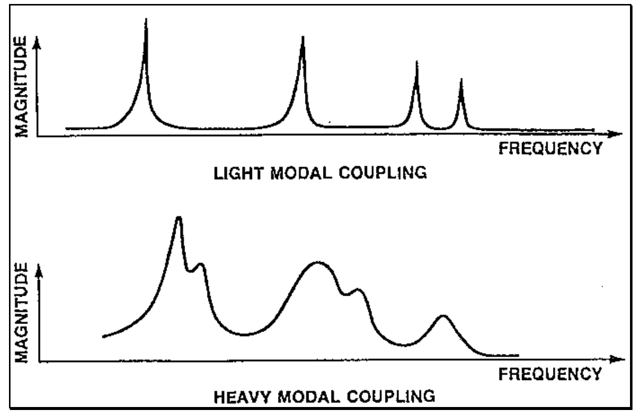

Figure 19. Light Versus Heavy Modal Density (Coupling).

Local SDOF Methods

Figure 20 depicts the three most commonly used curve-fitting methods for obtaining modal parameters. These are referred to as SDOF (single degree of freedom, or single mode) methods. Even though they don't look like curve fitting methods (in the sense of fitting a curve to empirical data), all three of these methods are based on applying an analytical expression for the FRF to measured data [3].

Modal Frequency as Peak Frequency

The ***frequency of a resonance peak*** in the FRF is used as the modal frequency. This peak frequency, which is also dependent on the frequency resolution of the measurements, is not exactly equal to the modal frequency but is a close approximation, especially for lightly damped structures. The resonance peak should appear ***at the same frequency in almost every FRF measurement***. It won't appear in those measurements corresponding to nodal lines (zero magnitude) of the mode shape.

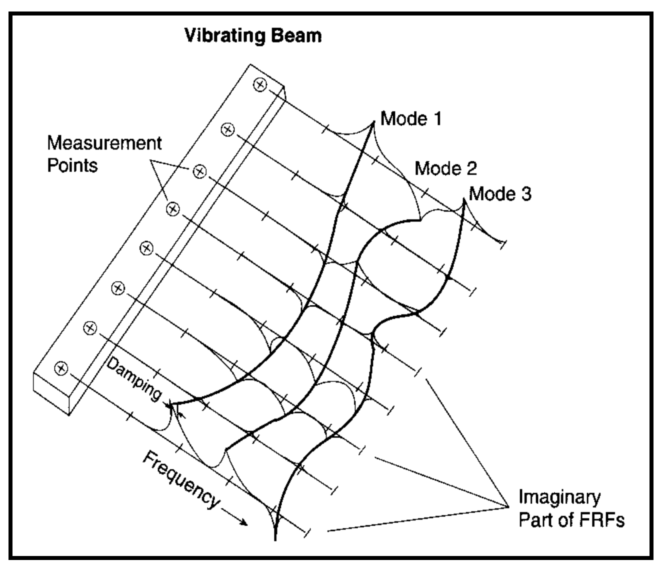

Figure 20. Curve Fitting FRF Measurements.

Modal Damping as Peak Width

The ***width of the resonance peak*** is a measure of modal damping. The resonance peak width should also be the same for all FRF measurements, meaning that ***modal damping is the same in every FRF measurement***. The width is actually measured at the so-called ***half power point***, and is approximately equal to twice the modal damping (in Hz).

Mode Shape From Quadrature Peaks

From (displacement/force) or (acceleration/force) FRFs, the ***peak values of the imaginary part*** of the FRFs are taken as components of the mode shape. This is called the ***Quadrature*** method of curve fitting. From (velocity/force) FRFs, the ***peak values of the real part*** are used as mode shape components.

Hence, using the simplest Local SDOF curve fitting methods, all three modal parameters (frequency, damping, and mode shape) can be extracted directly from a set of FRF measurements.

Local MDOF Methods

The Complex Exponential and the Rational Fraction Polynomial methods are two of the most popular Local MDOF curve fitting methods.

Complex Exponential (CE)

This algorithm curve fits and analytical expression for a structural impulse response to experimental impulse response data. A set of impulse response data is normally obtained by applying the Inverse FFT to a set of FRF measurements, as shown in Figure 16.

Figure 21 shows the analytical expression used by Complex Exponential curve fitting. Also pointed out in Figure 20 is the leakage (wrap around error) caused by the inverse FFT, which distorts the impulse response data. This portion of the data cannot be used because of this error.

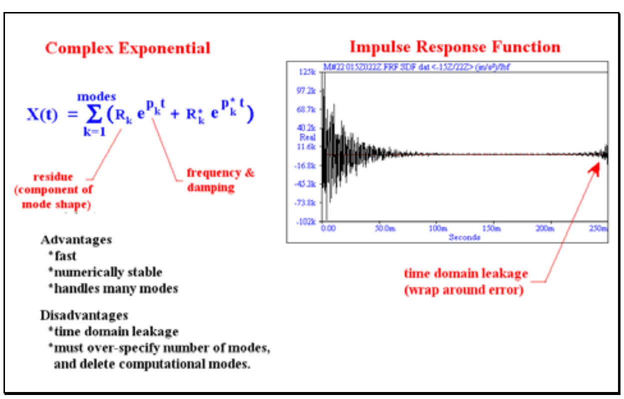

Figure 21. CE Curve Fitting.

Rational Fraction Polynomial (RFP)

This method applies the rational fraction polynomial expression shown in Figure 22 directly to an FRF measurement. Its advantage is that it can be applied over any frequency range of data, and particularly in the vicinity of a resonance peak.

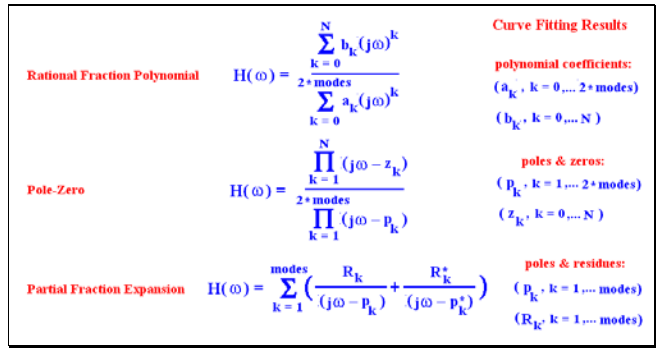

Figure 22. Alternate Curve Fitting Forms of the FRF.

As shown in Figure 23, not only can the RFP method be used to estimate modal parameters, but it also yields the ***numerator & denominator polynomial coefficients***, as well as the ***poles & zeros*** of the FRF.

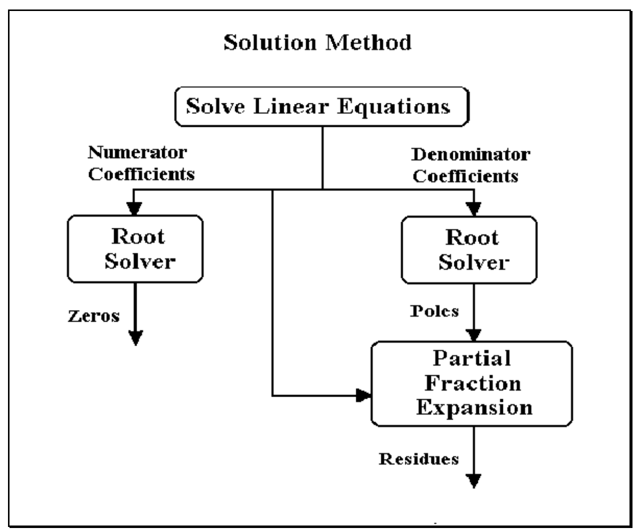

Figure 23. RFP Solution Method.

Global and Multi-Reference Methods

Both the CE and RFP algorithms have been implemented as Global and Multi-Reference methods also. The details of these methods are given in references [4] through [6].

CONCLUSIONS

Modern experimental modal analysis techniques have been reviewed in this paper. The three main topics pertaining to modal testing; FRF measurement techniques, excitation techniques, and modal parameter estimation (curve fitting) methods were covered.

FRF based modal testing started in the early 1970's with the commercial availability of the digital FFT analyzer, and has grown steadily in popularity since then. The modern modal testing techniques presented here are just a brief summary of the accumulation of the past 30 years of progress.

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