INTRODUCTION TO OPERATING DEFLECTION SHAPES

By
Brian J. Schwarz & Mark H. Richardson
Vibrant Technology, Inc.
Jamestown, California 95327

ABSTRACT

Mode shapes and operating deflection shapes (ODS's) are related to one another. In fact, ODS's are always measured in order to obtain mode shapes. Yet, they are quite different from one another in a number of ways. In this paper, we will discuss ODS measurements, and their relationship to experimental modal parameters.

INTRODUCTION

In another article on operating deflection shapes [2], the author made the following statement,

"Operational deflection shapes (ODS's) can be measured directly by relatively simple means. They provide very useful information for understanding and evaluating the absolute dynamic behavior of a machine, component or an entire structure."

What is an Operating Deflection Shape?

Traditionally, an ODS has been defined as the deflection of a structure at a particular frequency. However, an ODS can be defined more generally as ***any forced motion of two or more points*** on a structure. Specifying the motion of two or more points defines a shape. Stated differently, a shape is the motion of one point relative to all others. Motion is a vector quantity, which means that it has location and direction associated with it. This is also called a Degree Of Freedom, or DOF.

Why Measure ODS's?

Measuring ODS's can help answer the following vibration related questions,

- ***How Much*** is a machine moving?
- ***Where*** is it moving the most, and in what direction?
- What is the motion of one point relative to another (***Operating Deflection Shape***)?
- Is a ***resonance*** being excited? What does its mode shape look like?
- Is there ***structure-born noise***?
- Do corrective actions ***reduce noise or vibration*** levels?

TWO TYPES OF VIBRATION

All vibration is a combination of both forced and resonant vibration. Forced vibration can be due to,

- Internally generated forces.
- Unbalances.
- External loads.
- Ambient excitation.

An operating deflection shape contains the overall vibration for two or more DOFs on a machine or structure. That is, the ODS contains both forced and resonant vibration components. Other the other hand, a mode shape characterizes only the resonant vibration at two or more DOFs.

Resonant vibration typically ***amplifies the vibration response*** of a machine or structure far beyond the design levels for static loading. Resonant vibration is the cause of, or at least a contributing factor to many of the vibration related problems that occur in structures and operating machinery.

To understand any structural vibration problem, the resonances of a structure need to be identified. A common way of doing this is to define its modes of vibration. Each mode is defined by a ***natural (modal) frequency***, ***modal damping***, and a ***mode shape***.

UNDERSTANDING RESONANT VIBRATION

The majority of structures can be made to resonate. That is, under the proper conditions, a structure can be made to vibrate with ***excessive, sustained, oscillatory motion***. Resonant vibration is caused by an interaction between the inertial and elastic properties of the materials within a structure. Striking a bell with a hammer causes it to resonate. Striking a sandbag, however, will not cause it to resonate.

Trapped Energy Principle

One of the most useful ways of understanding resonant vibration is with the trapped energy principle. When energy enters a structure due to dynamic loading of any kind, resonant vibration occurs when the energy becomes trapped within the structural boundaries, travels freely within those boundaries, and cannot readily escape. This trapped energy is manifested in the form of traveling waves of deformation that also have a characteristic frequency associated with them. Waves traveling within the structure, being reflected off of its boundaries, ***sum together to form a standing wave*** of deformation. This standing wave is called a mode shape, and its frequency is a resonant or ***natural*** frequency of the structure.

Another way of saying this is that structures ***readily absorb*** energy at their resonant frequencies, and retain this energy in the form of a deformation wave called a mode shape. They are said to be ***compliant*** at their natural frequencies.

Why then, won't a sandbag resonate when it is struck with a hammer? Because energy doesn't travel freely within its boundaries. The sand particles don't transmit energy efficiently enough between themselves in order to produce standing waves of deformation. Nevertheless, a sandbag can still be made to vibrate. Simply shaking it with a sinusoidal force will cause it to vibrate. Sandbags can have operating deflection shapes, but don't have resonances or mode shapes.

Local Modes

Energy can also become trapped in local regions of a structure, and cannot readily travel beyond the boundaries of those regions. In the case of an instrument card cage, at a resonant frequency of one of its PC cards, energy becomes trapped within a card, causing it to resonate. The surrounding card cage is not compliant enough at the resonant frequency of the card to absorb energy, so the energy is reflected back and stays trapped within the card. The card vibrates but the cage does not.

Many structures have local modes; that is, resonances that are confined to local regions of the structure. Local modes will occur whenever part of the structure is compliant with the energy at a particular frequency, but other parts are not.

VIBRATION MEASUREMENTS

The vibration parameters of a machine or structure are typically derived from acquired time domain signals, or from frequency domain functions that are computed from acquired time signals. Using a modern multi-channel FFT analyzer, the vibration response of a machine is measured for multiple points and directions (DOFs) with motion sensing transducers. Signals from the sensors are then amplified, digitized, and stored in the analyzer's memory as blocks of data, one data block for each measured DOF.

ODS MEASUREMENTS

An ODS can be defined from any forced motion, either at a moment in time, or at a specific frequency. Having acquired either a set of sampled time domain responses, or computed (via the FFT) a set of frequency domain responses, an operating deflection shape is defined as:

Operating Deflection Shape: The values of a set of time domain responses at a specific time, or the values of a set of frequency domain responses at a specific frequency.

Time Domain ODS

An ODS can be obtained from a set of measured ***time domain responses***,

- Random.
- Impulsive.
- Sinusoidal.
- Ambient.

Figure 1 shows the display of an ODS from a set of impulse response measurements.

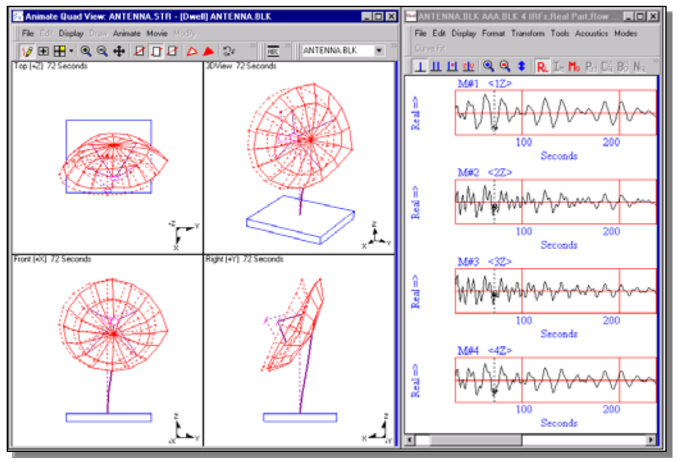

Figure 1. Time Domain ODS From Impulse Responses.

Frequency Domain ODS

An ODS can also be obtained from a set of computed ***frequency domain measurements***,

- Linear spectra (FFTs).
- Auto power spectra (APS's).
- Cross power spectra (XPS's)
- FRFs (Frequency Response Functions).
- ODS FRFs.

Figure 2 shows the display of an ODS from a set of FRF measurements.

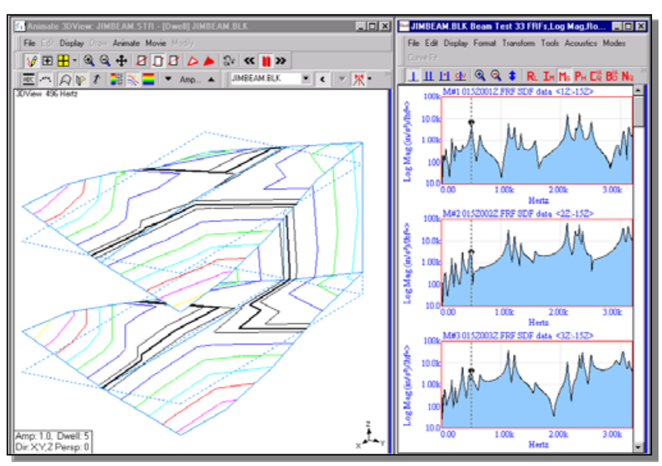

Figure 2. Frequency Domain ODS From a Set of FRFs

FRF values at a frequency, taken from two or more FRFs, is an ODS.

TESTING REAL STRUCTURES

Real continuous structures have an infinite number of DOFs, and an infinite number of modes. From a testing point of view, a real structure can be ***sampled spatially*** at as many DOFs as we like. There is no limit to the number of unique DOFs at which we can make measurements.

Because of time and cost constraints, we only measure a small subset of the measurements that could be made on a structure. Yet, from this small subset of measurements, we can accurately define the resonances that are within the frequency range of the measurements. Of course, the more we spatially sample the surface of the structure by taking more measurements, the more definition we will give to its ODS's and mode shapes.

DIFFICULTY WITH ODS MEASUREMENTS

In general, an ODS is defined with a ***magnitude*** and ***phase*** value at each point on a machine or structure. To define an ODS vector properly, at least the ***relative magnitude*** and ***relative phase*** are needed at all response points.

In a time domain ODS, magnitude and phase are implicitly assumed. This means that either all of the responses have to be ***measured simultaneously***, or they have to be measured under conditions which guarantee their correct magnitudes and phases relative to one another.

Simultaneous measurement of all responses means that a multi-channel acquisition system, that can simultaneously sample all of the response signals, must be used. This requires lots of transducers and signal conditioning equipment, which is expensive.

Repeatable Operation

If the structure or machine is undergoing, or can be made to undergo, repeatable operation, then response data can be acquired one channel at a time. To be repeatable, data acquisition must occur so that ***exactly the same time waveform*** *is* obtained in the sampling window, every time one is acquired. Figure 3 depicts repeatable operation. For repeatable operation, the magnitude and phase of each response signal is unique and repeatable, so ODS data can be acquired using a single channel analyzer. An ***external trigger*** is usually required to capture the repeatable event in the sampling window.

Order Related Data

A single channel analyzer can also be used to acquire ODS's using a tachometer pulse as the trigger. In this case, spectral magnitude & phase data at any fixed multiple of the rotational speed (order) of the machine can acquired with a single channel analyzer, but only if the operation is repeatable.

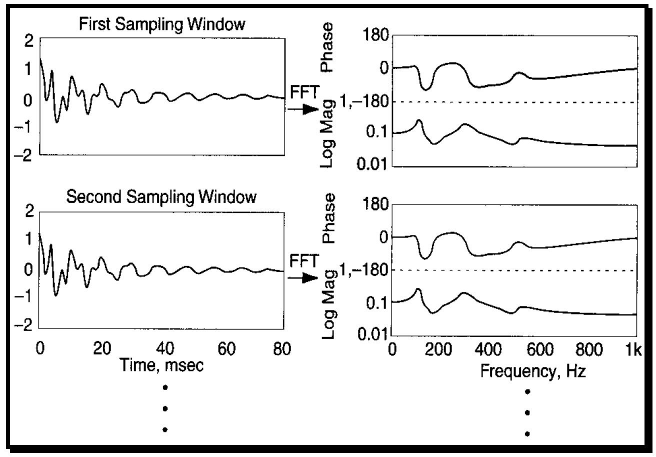

Figure 3. Repeatable Operation.

Steady State (Stationary) Operation

Steady state, or stationary operation can be achieved in many situations where repeatable operation cannot. Steady state operation is achieved when the ***auto power spectrum (APS) of a response signal does not change*** over time, or from measurement to measurement. Figure 4 shows a steady state operation. Notice that the time domain waveform can be different during each sampling window time interval, but its auto power spectrum does not change.

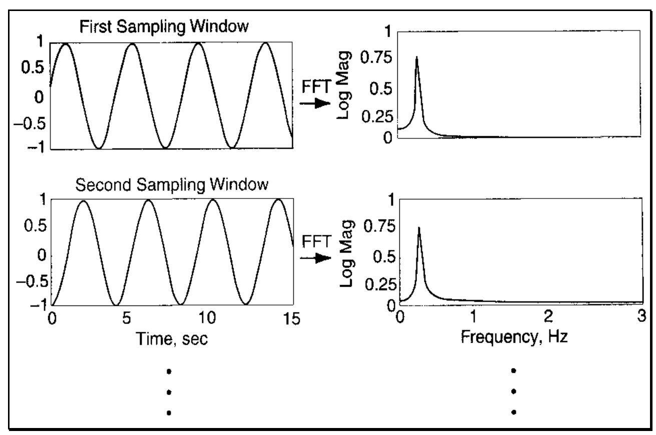

Figure 4. Steady State Operation.

For steady state operation, ODS data can be measured with a 2 channel FFT analyzer or acquisition system. The cross spectrum measurement (XPS) contains the ***relative phase*** between two responses, and the auto power spectrum (APS) of each response contains the correct magnitude of the response. Since the 2 response signals are simultaneously acquired, the relative phase between them is always maintained. No special triggering is required for steady state operation.

ODS's FROM FREQUENCY DOMAIN MEASUREMENTS

Any set of vibration data taken from a structure is the result of applied excitation forces. Whether it be operating data, caused by self-excitation, or data taken during a modal test under tightly controlled excitation conditions, the operating deflection shapes are always subject to both the ***amount*** and ***location*** of the excitation.

Linear Spectrum

This frequency domain function is simply the FFT of a sampled time domain function. Phase is preserved in the Linear Spectrum, so in order to obtain operating deflection shapes from a set of Linear Spectra, either the measurement process ***must be repeatable***, or the time domain signals ***must be simultaneously sampled***. Since the Linear Spectrum is complex valued (contains both magnitude and phase information), the resulting operating deflection shapes will also contain magnitude and phase information.

Auto Power Spectrum

The APS is derived by taking the FFT of a sampled time domain function, and multiplying the resulting Linear Spectrum by the complex conjugate of the Linear Spectrum at each frequency. ***Phase is not preserved*** in the APS, so a set of these measurements need not be obtained by simultaneously sampling all of the time domain responses. Since phase is not retained in these measurements, operating deflection shapes derived from them will contain only magnitude, and no phase information.

FRFs

The FRF is a 2-channel measurement, involving a response and an excitation signal. It can be estimated in several ways, depending on whether the excitation or the response has more measurement noise associated with it.

The most common calculation involves ***dividing*** an estimate of the cross power spectrum (XPS) between the response and excitation signals by an estimate of the auto power spectrum (APS) of the excitation, at each frequency. Averaging together of several XPS's and APS's is commonly done to reduce noise in these estimates.

Since a set of FRFs contains both magnitude and phase at each frequency, the operating deflection shapes derived from a set of FRFs will also contain both magnitude and phase information. The units of the operating deflection shapes are ***acceleration, velocity, or displacement per unit of excitation force at the reference DOF***.

Difficulty with FRF Measurements

FRF measurement requires that ***all of the excitation forces causing a response must be measured simultaneously with the response***. Measuring all of the excitation forces can be difficult, if not impossible in many situations. FRFs cannot be measured on operating machinery or equipment where internally generated forces, acoustic excitation, and other forms of excitation are either unmeasured or un-measurable. On the other hand, ODS's can always be measured, no matter what forces are causing the vibration.

Transmissibility

Transmissibility measurements are made when ***the excitation force(s) cannot be measured***. Transmissibility is a 2-channel measurement like the FRF. It is estimated in the same way as the FRF, but the response is ***divided by a reference response signal*** instead of an excitation force. Phase is also preserved in Transmissibility's, and a set of them need not be obtained by simultaneously sampling all of the time domain responses. Each response & reference response pair must be simultaneously sampled, however.

As with FRFs, a set of Transmissibility's contain both magnitude and phase at each frequency, so ODS's obtained from a set of Transmissibility's will also contain correct magnitude and phase information. The units of the operating deflection shapes are ***response units per unit of response at the reference DOF***.

An unexpected drawback of Transmissibility measurements however, is that each resonance is represented by a ***"flat spot"*** in the data instead of a peak. This is shown in Figure 5. The top curve in Figure 7 is a response APS showing 4 resonance peaks. The Transmissibility below has ***"flat spots"*** (no peak) in the frequency range where a resonance peak occurs.

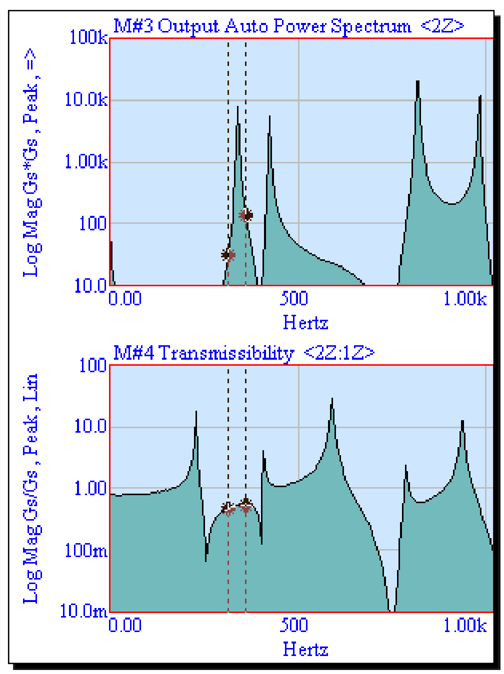

Figure 5. APS & Transmissibility.

ODS FRF

An ODS FRF is a different 2-channel measurement that can also be used when ***excitation forces cannot be measured***. The advantage of the ODS FRF over the Transmissibility is that the ODS FRF has peaks at resonances, thus making it easy for locating resonances.

Like Transmissibility, an ODS FRF also requires a **reference (fixed) response** measurement along with each response measurement. Each ODS FRF is formed by ***replacing the magnitude*** of each XPS between a response and the reference response with the APS of the response. The phase of the XPS is retained as the phase of the ODS FRF.

This new measurement contains the correct magnitude of the response at each point, and the correct phase relative to the reference response. Evaluating a set of ODS FRF measurements at any frequency yields the frequency domain ODS for that frequency. Figure 6 shows the display of an ODS from a set of ODS FRF measurements.

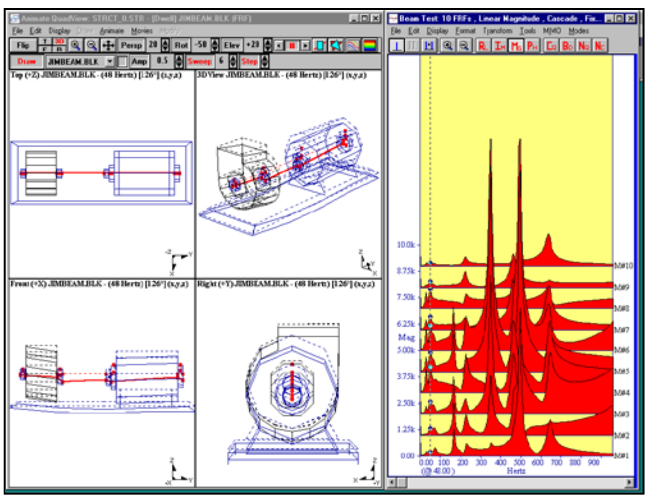

Figure 6. ODS Displayed Directly From ODS FRF Data.

MODE SHAPES FROM ODS's

We have already seen that ODS's are obtained either from a set of time domain responses, or from a set of frequency domain functions that are computed from time domain responses. In addition, modal parameters (natural frequency, damping, & mode shape) can be obtained from a set of FRF measurements. In general, the following statement can be made,

"All experimental modal parameters are obtained from measured ODS's."

Stated differently, modal parameters are obtained by post-processing (curve fitting) a set of ODS data. In other words, a set of FRFs can be thought of as a set of ODS's over a frequency range. At or near one a resonance peak, the ODS is ***dominated by a mode***. Therefore, the ODS is ***approximately equal*** to the mode shape. This concept becomes clearer when sine wave excitation is considered.

USING A SINUSOIDAL ODS AS A MODE SHAPE

If a single sinusoidal force excites the structure, its steady state response will also be sinusoidal, regardless of the frequency of excitation. However, the ODS that is measured also depends on whether or not a resonance is excited. In order to excite a resonance, two conditions must be met:

Condition 1: The excitation force must be applied at a DOF, which is ***not on a nodal line*** of the mode shape.

Condition 2: The excitation frequency must be ***close*** to the resonance peak frequency.

If both of these conditions are met, and the resonance is "lightly" damped, it will act as a ***mechanical amplifier*** and greatly increase the amplitude of response, or the ODS. Conversely, if either condition is not met, the mode will not participate significantly in the ODS.

All single frequency sine wave modal testing is based upon achieving the two conditions above, plus a third,

Condition 3: At a resonant frequency, if the ODS is ***dominated*** by one mode, then the ODS will closely approximate the mode shape.

If Condition 3 is not met, then two or more modes are contributing significantly to the ODS, and the ODS is a linear combination of their mode shapes.

EXCITING RESONANCES WITH IMPACT TESTING

With the ability to compute FRF measurements in an FFT analyzer, impact testing became popular during the late 1970s as a fast, convenient, and relatively low cost way of finding the mode shapes of machines and structures.

To perform an impact test, all that is needed is an impact hammer with a load cell attached to its head to measure the input force, a single accelerometer to measure the response at a single fixed point, a two channel FFT analyzer to compute FRFs, and post processing software for identifying and displaying the mode shapes in animation.

In a typical impact test, the accelerometer is attached to a single point on the structure, and the hammer is used to impact it at as many points and as many directions as required to define its mode shapes. FRFs are computed one at a time, between each impact point and the fixed response point. Modal parameters are defined by ***curve fitting*** the resulting set of FRFs. Figure 7 depicts the impact testing process.

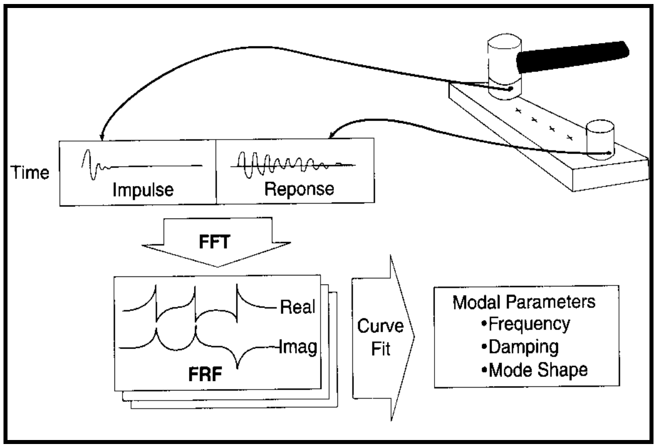

Figure 7. Impact Testing.

Curve Fitting

In general, curve fitting is a process of matching an analytical function or mathematical expression to some empirical data. This is commonly done by minimizing the squared error (or difference) between the function values and the data. In statistics, fitting a straight line through empirical data is called regression analysis. This is a form of curve fitting.

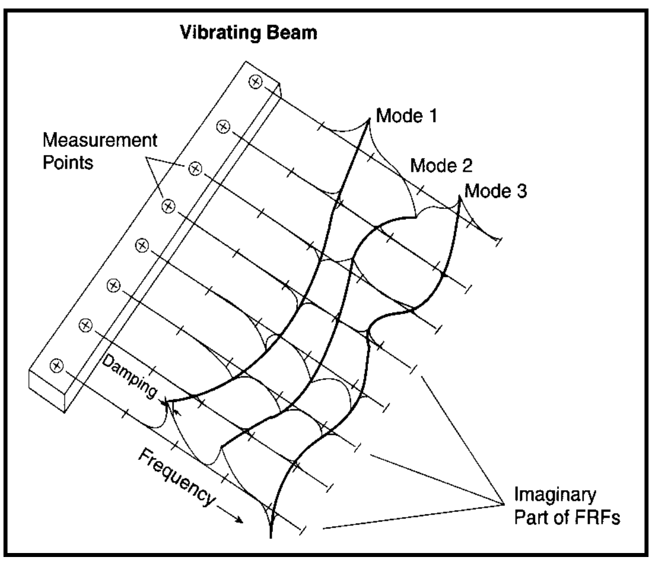

Figure 8. Curve Fitting FRF Measurements.

Estimates of modal parameters are obtained by curve fitting FRF data. Figure 8 depicts the three most commonly used curve-fitting methods used to obtain modal parameters. The frequency of a ***resonance peak*** in the FRF is taken as the modal frequency. This peak should appear at the same frequency in every FRF measurement.

The ***width of the resonance peak*** is a measure of modal damping. The resonance peak width should also be the same for all FRF measurements. The ***peak values of the imaginary part*** of the FRFs are taken as the mode shape, for ***displacement or acceleration*** responses. (The peak values of the ***real part*** are used for ***velocity*** responses.) All of these very simple curve-fitting methods are based on an analytical expression for the FRF, written in terms of modal parameters [3].

ODS's AND MODE SHAPES CONTRASTED

Even though all experimental mode shapes are obtained from measured ODS's, modes are different from ODS's in the following ways,

1. Each mode is defined for a specific ***natural frequency.*** An ODS can be defined at ***any frequency***.
2. Modes are only defined for ***linear, stationary*** structures. ODS's can be defined for ***non-linear*** and ***non-stationary*** structures.
3. Modes are used to characterize ***resonant vibration***. ODS's can characterize resonant as well as ***non-resonant vibration***.
4. Modes ***don't depend*** on forces or loads. They are inherent properties of the structure. ODS's ***depend on*** forces or loads. They will change if the loads change.
5. Modes only change if the ***material properties*** or ***boundary conditions*** change. ODS's will change if either the modes or the loads change.
6. Mode shapes ***don't have unique values or units***. ODS's do have unique values and units.
7. Mode shapes can answer the question, "What is the ***relative motion*** of one DOF versus another?" ODS's can answer the question, "What is the ***actual motion*** of one DOF versus another?"

CONCLUSIONS

Operating deflection shapes were defined for both time and frequency domain functions. We saw that ODS's can be obtained from a variety of both time and frequency domain functions, but restrictive assumptions must also made with each measurement type.

We also discussed ODS's and modes shapes, and made the statement that, ***"All experimental modal parameters are obtained from measured ODS's."*** In spite of this close relationship, we contrasted ODS's with mode shapes and pointed out seven ways in which the two are different from one another.

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[2] Døssing, Ole "Structural Stroboscopy-Measurement of Operational Deflection Shapes" Sound and Vibration Magazine, August 1988.

[3] Richardson, M. H., "Modal Analysis Using Digital Test Systems," Seminar on Understanding Digital Control and Analysis in Vibration Test Systems, Shock and Vibration Information Center Publication, Naval Research Laboratory, Washington D.C. May, 1975.