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Derivation of Mass, Stiffness and Damping Parameters from Experimental Modal Data

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INTRODUCTION

Once a set of modal data has been measured for a vibrating structure, it is possible to compute the full mass, stiffness and damping matrices for the structure by using formulas derived from the relationship between the time domain differential equations of motion and the transfer function matrix model of the structure.

First, the equations for calculating the full mass, stiffness and damping matrices from modal data are derived. This deviation was first presented in [1]. However, these equations are difficult to solve because the first step of their solution requires ***matrix inversion*** of the flexibility matrix to obtain the stiffness matrix. Not only does matrix inversion amplify errors, but the number of linearly independent mode shapes required to yield a non-singular flexibility matrix is beyond the scope of most experimental modal data sets. Further assumptions regarding the modal parameters can simplify the calculations though.

Next it is shown that for ***lightly damped*** structures, where the mode vectors are "almost" real valued (called ***normal modes***), modal mass, stiffness, and damping can be defined directly from the mass, stiffness and damping formulas derived for the complex modal model. These assumptions greatly simply the equations for calculating mass, stiffness, and damping matrices, but a matrix inversion (in this case the mode shape matrix) is still required. Again, a large set of linearly independent mode shapes is still required.

Finally, it is shown that if the modal vectors can also be assumed to be ***orthogonal to one another*** (an assumption that can only be approximated with real world data), then the full mass, stiffness and damping matrices can be computed from modal data without any matrix inversions.

Time Domain Model

These results apply to an vibrating machine or structure, the dynamics of which can be adequately described by ***n***-linear differential equations of motion,

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \qquad (1)$$

where,

$[M] =$ **(n by n)** mass matrix.

$[C] =$ **(n by n)** damping matrix.

$[K] =$ **(n by n)** stiffness matrix.

$\{\ddot{x}(t)\} =$ **n**-dimensional acceleration vector.

$\{\dot{x}(t)\} =$ **n**-dimensional velocity vector.

$\{x(t)\} =$ **n**-dimensional displacement vector.

$\{f(t)\} =$ **n**-dimensional external force vector.

This set of time domain differential equations describes the dynamics between **n**-discrete degrees-of-freedom (DOFs) of the structure.

These equations can be written between as many DOFs on a structure as necessary to adequately describe its dynamic behavior.

Frequency Domain Model

Alternatively, the dynamics of a machine or structure between any pair of DOFs can be equivalently described in the frequency domain by a transfer function. A transfer function matrix model describes the dynamics between n-DOFs of the structure, and contains transfer functions between all combinations of DOF pairs,

$$\{X(s)\}=[H(s)]\{F(s)\} \qquad (2)$$

where,

$s =$ Laplace variable (complex frequency).

$[H(s)] =$ (**n by n**) transfer function matrix.

$\{X(s)\} =$ Laplace transform of displacement **n**-vector.

$\{F(s)\} =$ Laplace transform of external force **n**-vector.

From an experimental point of view, not only can these equations can be written between as many DOFs on a structure as necessary to adequately describe its dynamic behavior, but any element of the transfer function matrix ***can also be measured from a real structure***. Values of the transfer function along the $j\omega$ -axis in the s-plane, called frequency response functions **(FRFs)**, are actually measured.

Transfer Function in Terms of Modes

The **(n by n)** transfer function matrix can also be written in terms of modal parameters as,

$$[H(s)] = \sum_{k=1}^{m} \frac{[r_k]}{2j\,(s-p_k)} - \frac{[r_k^*]}{2j\,(s-p_k^*)} \qquad (3)$$

or,

$$[H(s)] = \sum_{k=1}^{m} \frac{A_k\{u_k\}\{u_k\}^t}{2j\,(s-p_k)} - \frac{A_k^*\{u_k^*\}\{u_k^*\}^t}{2j\,(s-p_k^*)} \qquad (4)$$

where,

$m =$ number of modes of vibration.

$[r_k] =$ (**n by n**) residue matrix for the k^{th} mode.

$p_k = -\sigma_k + j\omega_k =$ pole location for the k^{th} mode.

$\sigma_k =$ damping coefficient of the k^{th} mode.

$\omega_k =$ damped natural frequency of the k^{th} mode.

$\{u_k\} =$ **n**-dimensional complex mode shape vector for the k^{th} mode.

$A_k =$ a scaling constant for the k^{th} mode.

The following assumptions were made in order to derive equations (3) & (4).

1. **Linearity:** The structural dynamics are linear and are adequately described by either equation (1) or (2).
2. **Maxwell's Reciprocity:** The matrices in equations (1) or (2) are assumed to be symmetric. Reciprocity means that the dynamic response at DOF A due to an applied force at DOF B are the same as the response at DOF B due to an applied force at DOF A.
3. **Distinct Pole Locations:** Each mode of resonant frequency is adequately described by the a pair of distinct poles, and the transfer function matrix can be written in the partial fraction form of either equation (3) or (4).

TRANSFER, MASS, STIFFNESS, & DAMPING MATRIX RELATIONSHIPS

Next, relationships between the mass, stiffness, damping and transfer matrices are explored. Taking Laplace transforms of equation (1) and ignoring initial conditions yields,

$$[B(s)]\{X(s)\} = \{F(s)\} \qquad (5)$$

where,

$[B(s)] = [M]s^2 + [C]s + [K] =$ (**n by n**) system matrix

By comparing equations (2) and (5), it is clear that the transfer function and system matrices are ***inverses of one another***. This relationship can be written as,

$$[H(s)][B(s)] = [B(s)][H(s)] = [I] \qquad (6)$$

where,

$[I] =$ an (**n by n**) identity matrix.

Stiffness Matrix

The stiffness matrix can be related to the transfer matrix by evaluating equation (6) at the origin ($s = 0$) in the complex s-plane. By evaluating both matrices in equation (6) at $s = 0$ and applying the definition of the system matrix,

$$[H(0)][B(0)] = [H(0)][K] = [I]$$

or,

$$[K] = [H(0)]^{-1} \qquad (7)$$

Equation (7) says that the stiffness matrix is simply the inverse of the transfer function matrix evaluated as $s = 0$. $[H(0)]$ is called the **flexibility matrix**.

Damping Matrix

Next, the damping matrix can be related to the transfer function matrix by first taking derivatives of the terms in equation (6), and then evaluating at $s = 0$,

$$[\dot{B}(0)][H(0)] + [\dot{H}(0)][B(0)] = [0] \qquad (8)$$

or,

$$[\dot{B}(0)] = -[H(0)]^{-1}[\dot{H}(0)][B(0)] \qquad (9)$$

Substituting the system matrix and stiffness matrix definitions into equation (9) gives,

$$[C] = -[K][\dot{H}(0)][K] \qquad (10)$$

Equation (8) says that the damping matrix is equal to the first derivative of the transfer function matrix evaluated at $s = 0$, pre- and post-multiplied by the stiffness matrix.

Mass Matrix

Finally, the mass matrix can be related to the transfer function matrix by taking second derivatives of the terms in equation (6) and evaluating at $s = 0$,

$$[\ddot{B}(0)][H(0)] + 2[\dot{H}(0)][\dot{B}(0)] + [\ddot{H}(0)][B(0)] = [0] \qquad (11)$$

or,

$$\frac{[\ddot{\mathbf{B}}(\mathbf{0})]}{\mathbf{2}} = -[\mathbf{H}(\mathbf{0})]^{-1}\left[\dot{\mathbf{H}}(\mathbf{0})\dot{\mathbf{B}}(\mathbf{0}) + \frac{\ddot{\mathbf{H}}(\mathbf{0})}{\mathbf{2}}\mathbf{B}(\mathbf{0})\right] \quad (12)$$

Substituting equation (9), plus the system matrix and stiffness matrix definitions into equation (12) gives,

$$[\mathbf{M}] = [\mathbf{K}]\left[\dot{\mathbf{H}}(\mathbf{0})\,\mathbf{K}\,\dot{\mathbf{H}}(\mathbf{0}) - \frac{\ddot{\mathbf{H}}(\mathbf{0})}{\mathbf{2}}\right][\mathbf{K}] \quad (13)$$

To summarize, the mass, stiffness, and damping matrices can be calculated from the transfer function matrix and its derivatives by using equations (7), (10), and (13).

The flexibility matrix $[\mathbf{H}(\mathbf{0})]$ and the first and second derivative matrices $[\dot{\mathbf{H}}(\mathbf{0})]$ and $[\ddot{\mathbf{H}}(\mathbf{0})]$ can each be written in terms of modal parameters by using either equation (3) or (4). Hence, full mass, stiffness, and damping matrices can calculated once a set of modal data is obtained.

Inverting the Flexibility Matrix

The greatest limitation of this computational process is the first step, that is, computing the stiffness matrix as the inverse of the flexibility matrix $[\mathbf{H}(\mathbf{0})]^{-1}$. An (**n by n)** matrix must have a **rank of n** in order to compute its inverse. This means that **n** linearly independent mode shapes are needed in order to insure that the flexibility matrix has a rank of **n**. For example, 1000 linearly independent mode shapes are required to compute a stiffness matrix with a 1000 DOFs. This is impossible with experimental data.

MODAL MASS, STIFFNESS AND DAMPING FOR LIGHTLY DAMPED STRUCTURES

So far, we have derived formulas for computing the mass, stiffness, and damping matrices of a dynamical system from modal parameters. The formulas are impractical, though, since sufficient modal parameters are usually not obtainable for computing realistic mass, stiffness, and damping matrices.

In this section, modal mass, stiffness, and damping will be defined, and their use will lead to computationally feasible formulas for computing the mass, stiffness, and damping matrices.

In addition to the three assumptions already made regarding the dynamic model, the following further assumptions are now made:

4. **Light Damping:** The damping coefficient (σ_k) of each mode (**k**) is much less than the damped natural frequency (ω_k). That is,

$$\sigma_k \ll \omega_k$$

5. **Normal Mode Shapes:** The imaginary part of each mode shape vector $\{\mathbf{u}_k\}$ is much less than the real part. That is,

$$\mathbf{Im}\left(\{\mathbf{u}_k\}\right) \ll \mathbf{Re}\left(\{\mathbf{u}_k\}\right)$$

where

$$\{\mathbf{u}_k\} = \mathbf{Re}\left(\{\mathbf{u}_k\}\right) + \mathbf{j}\,\mathbf{Im}\left(\{\mathbf{u}_k\}\right)$$

When these assumptions are applied to the stiffness, damping, and mass equations (7), (10), and (13) it will be shown that modal mass, modal stiffness and modal damping can be defined in a straight forward manner.

Modal Stiffness

Evaluating equation (3) at $\mathbf{s} = \mathbf{0}$ gives,

$$[\mathbf{H}(\mathbf{0})] = \sum_{k=1}^{m} -\mathbf{I}_m\left(\frac{[\mathbf{r}_k]}{\mathbf{p}_k}\right) \quad (14)$$

Since the residue matrix is complex in general, equation (14) can be rewritten as,

$$[\mathbf{H}(\mathbf{0})] = \sum_{k=1}^{m} \frac{\omega_k\,\mathbf{Re}\left([\mathbf{r}_k]\right) + \sigma_k\,\mathbf{Im}\left([\mathbf{r}_k]\right)}{\sigma_k^2 + \omega_k^2} \quad (15)$$

Applying assumption **5. Normal Mode Shapes**, we can conclude that,

$$\mathbf{Im}\left([\mathbf{r}_k]\right) \ll \mathbf{Re}\left([\mathbf{r}_k]\right)$$

or that the residue matrix is essentially real.

$$[\mathbf{r}_k] \approx \mathbf{Re}\left([\mathbf{r}_k]\right) \quad (16)$$

Comparing equations (3) and (4), it is clear that the residue matrix is defined in terms of mode shapes as,

$$[\mathbf{r}_k] = \mathbf{A}_k\{\mathbf{u}_k\}\{\mathbf{u}_k\}^t$$

Therefore, equation (15) can be rewritten as,

$$[\mathbf{H}(\mathbf{0})] = \sum_{k=1}^{m} \frac{\omega_k[\mathbf{r}_k]}{\sigma_k^2 + \omega_k^2} = \sum_{k=1}^{m} \frac{\mathbf{A}_k\omega_k\{\mathbf{u}_k\}\{\mathbf{u}_k\}^t}{\sigma_k^2 + \omega_k^2} \quad (17)$$

If all of the mode shapes are collected together as columns of an (**n by m**) mode shape matrix,

$$[\phi] = \left[\{\mathbf{u}_1\},\{\mathbf{u}_2\},\ldots\{\mathbf{u}_m\}\right] \quad (\mathbf{n\ by\ m})$$

then equation (17) can be written in matrix form as,

$$[\mathbf{H(0)}] = [\phi]\begin{bmatrix} \ddots & & \\ & \frac{1}{\mathbf{k}} & \\ & & \ddots \end{bmatrix}[\phi]^t \qquad (18)$$

where the matrix in the middle is a diagonal matrix. Expanding this diagonal matrix in detail gives,

$$\begin{bmatrix} \ddots & & \\ & \frac{1}{\mathbf{k}} & \\ & & \ddots \end{bmatrix} = \begin{bmatrix} \frac{A_1\omega_1}{\sigma_1^2+\omega_1^2} & & 0 \\ & \ddots & \\ 0 & & \frac{A_m\omega_m}{\sigma_m^2+\omega_m^2} \end{bmatrix} = \begin{bmatrix} \ddots & & \\ & \frac{A\omega}{\sigma^2+\omega^2} & \\ & & \ddots \end{bmatrix}$$

Hence the stiffness matrix can be written in terms of modal parameters as,

$$[\mathbf{K}] = [\mathbf{H(0)}]^{-1} = \left[[\phi]\begin{bmatrix} \ddots & & \\ & \frac{1}{\mathbf{k}} & \\ & & \ddots \end{bmatrix}[\phi]^t \right]^{-1}$$

or,

$$[\mathbf{K}] = [\phi^t]^{-1}\begin{bmatrix} \ddots & & \\ & \mathbf{k} & \\ & & \ddots \end{bmatrix}[\phi]^{-1} \qquad (19)$$

Now, by pre- and post-multiplying equation (19) by the inverses of the mode shape matrix and its transpose, a definition of modal stiffness is obtained,

$$[\phi]^t[\mathbf{K}][\phi] = \begin{bmatrix} \ddots & & \\ & \mathbf{k} & \\ & & \ddots \end{bmatrix} = \begin{bmatrix} \ddots & & \\ & \frac{\sigma^2+\omega^2}{A\omega} & \\ & & \ddots \end{bmatrix} \qquad (20)$$

Equation (19) is a formula for computing the full stiffness matrix [**K**] from modal parameters, provided that the inverse of the mode shape matrix $[\phi]^{-1}$ exists.

When the stiffness matrix in equation (19) is pre- and post-multiplied by a mode shape matrix of normal modes (assumption 5.), the result is a diagonal matrix, shown in equation (20). ***This is a definition of modal stiffness.***

Modal Damping

Starting with equation (10) and evaluating the first derivative of the transfer function matrix at $s = 0$ gives,

$$[\mathbf{C}] = -[\mathbf{K}][\dot{\mathbf{H}}(0)[\mathbf{K}] = [\mathbf{K}]\left[\sum_{k=1}^{m} \mathrm{Im}\left(\frac{[r_k]}{(p_k)^2}\right)\right][\mathbf{K}]$$

Substituting for the residue matrix and the pole definitions,

$$[\mathbf{C}] = [\mathbf{K}]\left[\sum_{k=1}^{m} \frac{2\sigma_k\omega_k A_k\{u_k\}\{u_k\}^t}{\left(\sigma_k^2-\omega_k^2\right)^2+\left(2\sigma_k\omega_k\right)^2}\right][\mathbf{K}] \qquad (21)$$

Applying assumption **4. Light Damping:** $\sigma_k \ll \omega_k$ reduces equation (21) to,

$$[\mathbf{C}] = [\mathbf{K}]\left[\sum_{k=1}^{m} \frac{2\sigma_k A_k\{u_k\}\{u_k\}^t}{\omega_k^3}\right][\mathbf{K}] \qquad (22)$$

Again, if the mode shapes are collected together as columns into an (**n by m**) mode shape matrix, equation (22) can be written in matrix form as,

$$[\mathbf{C}] = [\mathbf{K}]\,[\phi\,]\begin{bmatrix} \ddots & & \\ & \frac{2\sigma A}{\omega^3} & \\ & & \ddots \end{bmatrix}[\phi]^t[\mathbf{K}]$$

Substituting for the stiffness matrices gives,

$$[\mathbf{C}] = \left([\phi]^t\right)^{-1}\begin{bmatrix} \ddots & & \\ & \frac{2\sigma}{A\omega} & \\ & & \ddots \end{bmatrix}[\phi]^{-1} \qquad (23)$$

Equation (23) is a formula for computing the full damping matrix [**C**] from modal parameters, provided that the inverse of the mode shape matrix $[\phi]^{-1}$ exists.

$$[\phi]^t[\mathbf{C}]\,[\phi] = \begin{bmatrix} \ddots & & \\ & \mathbf{c} & \\ & & \ddots \end{bmatrix} = \begin{bmatrix} \ddots & & \\ & \frac{2\sigma}{A\omega} & \\ & & \ddots \end{bmatrix} \qquad (24)$$

When the damping matrix in equation (23) is pre- and post-multiplied by a mode shape matrix of normal modes, the result is a diagonal matrix, shown in equation (24). ***This is a definition of modal damping.***

Modal Mass

Starting with equation (13), a definition of modal mass can be obtained by substituting in some of the previous results.

$$[\mathbf{M}] = [\mathbf{K}]\left[\dot{\mathbf{H}}(0)\,\mathbf{K}\,\dot{\mathbf{H}}(0) - \frac{\ddot{\mathbf{H}}(0)}{2}\right][\,\mathbf{K}]$$

Substituting equation (10) into this equation gives,

$$[\mathbf{M}] = [\mathbf{C}]\,[\mathbf{H(0)}]\,[\mathbf{C}] - [\mathbf{K}]\left[\frac{\ddot{\mathbf{H}}(o)}{2}\right][\mathbf{K}] \qquad (25)$$

Evaluating the second derivative of the transfer function matrix at $s = 0$ gives,

$$\left[\frac{\ddot{H}(0)}{2}\right] = \sum_{k=1}^{m} -\mathrm{Im}\left(\frac{[r_k]}{p_k^3}\right)$$

or,

$$\left[\frac{\ddot{H}(0)}{2}\right] = \sum_{k=1}^{m} \frac{\left(\omega_k\left(3\sigma_k^2 - \omega_k^2\right)\right)[r_k]}{\left(\sigma_k\left(\sigma_k^2 - 3\omega_k^2\right)\right)^2 + \left(\omega_k\left(3\sigma_k^2 - \omega_k^2\right)\right)^2}$$

Substituting the residue definition, applying the assumption **4. Light Damping:** $\sigma_k << \omega_k$, and removing insignificant terms gives**,**

$$\left[\frac{\ddot{H}(0)}{2}\right] = \sum_{k=1}^{m} \frac{-[r_k]}{\omega_k^3} = \sum_{k=1}^{m} \frac{-A_k\{u_k\}\{u_k\}^t}{\omega_k^3} \qquad (26)$$

Substituting equations (26), (19), and (23) into equation (25) gives a formula for computing the mass matrix from modal parameters,

$$[M] = ([\phi]^t)^{-1}\left\{\begin{bmatrix}\ddots & & \\ & \frac{4\sigma^2}{A\omega(\sigma^2+\omega^2)} & \\ & & \ddots\end{bmatrix} + \begin{bmatrix}\ddots & & \\ & \frac{(\sigma^2+\omega^2)^2}{A\omega^5} & \\ & & \ddots\end{bmatrix}\right\}[\phi]^{-1}$$

Again, since $\sigma_k << \omega_k$, the first term is negligible compared to the second, so the mass formula can be further simplified,

$$[M[= ([\phi]^t)^{-1}\begin{bmatrix}\ddots & & \\ & \frac{1}{A\omega} & \\ & & \ddots\end{bmatrix}[\phi]^{-1} \qquad (27)$$

Equation (27) is a formula for computing the full mass matrix [**M**] from modal parameters, provided that the inverse of the mode shape matrix $[\phi]^{-1}$ exists.

$$[\phi]^t[M][\phi] = \begin{bmatrix}\ddots & & \\ & m & \\ & & \ddots\end{bmatrix} = \begin{bmatrix}\ddots & & \\ & \frac{1}{A\omega} & \\ & & \ddots\end{bmatrix} \qquad (28)$$

When the mass matrix in equation (27) is pre- and post-multiplied by a mode shape matrix of normal modes, the result is a diagonal matrix, shown in equation (28). ***This is a definition of modal mass.***

Summary of Modal Mass, Stiffness, and Damping

Given the three equations (20), (24), and (28) for diagonalizing the mass, stiffness and damping matrices, it is now a straightforward task to define modal mass, stiffness, and damping as the diagonal matrix elements in each of the respective formulas,

Modal mass: $$m_k = \frac{1}{A_k\omega_k} \qquad k = 1, \ldots m \qquad (29)$$

Modal stiffness: $$k_k = \frac{\sigma_k^2 + \omega_k^2}{A_k\omega_k} \qquad k = 1, \ldots m \qquad (30)$$

Modal damping: $$c_k = \frac{2\sigma_k}{A_k\omega_k} \qquad k = 1, \ldots m \qquad (31)$$

The familiar single degree-of-freedom relationships follow from these definitions ,

$$k_k = m_k(\sigma_k^2 + \omega_k^2) \qquad k = 1, \ldots m$$

$$c_k = m_k(2\sigma_k) \qquad k = 1, \ldots m$$

The dynamics of an SDOF system (a single mass, spring, damper system) is defined by the transfer function,

$$H(s) = \frac{1}{m\,s^2 + c\,s + k} = \frac{1}{s^2 + 2\sigma\,s + (\sigma^2 + \omega^2)}$$

Comparison of these two forms of the transfer function yields the same relationships as the modal formulas above, namely,

$$\frac{c}{m} = 2\sigma \qquad \frac{k}{m} = (\sigma^2 + \omega^2)$$

MASS, STIFFNESS AND DAMPING MATRICES FROM ORTHOGONAL MODES

Equations (19), (23), and (27) are formulas for computing the stiffness, damping, and mass matrices from modal data, but each formula ***still requires that a matrix inverse*** (the inverse of the mode shape matrix $[\phi]^{-1}$), be computed. This is prohibitively difficult, especially when using experimental modal data. However, if a final assumption can be applied to the mode shapes, the mass, stiffness, and damping matrix formulas are greatly simplified.

6. Orthogonal Mode Shapes: If the mode shape vectors are assured to be ***orthogonal with respect to one another*** (and are also normalized to unit magnitudes), then the mode shape matrix has the following properties,

$$[\phi]^t[\phi] = [I] = [\phi]^{-1}[\phi] \quad \Rightarrow \quad [\phi]^t = [\phi]^{-1}$$

In other words, the inverse of the mode shape matrix is equal to its transpose. This is also called a **unitary matrix**.

Substitution of this property into equations (19), (23) and (27) yields the following simplified equations,

Stiffness Matrix: $$[\mathbf{K}] = [\phi]\left[\ddots \mathbf{k} \ddots\right][\phi]^t \quad (32)$$

where,

$\left[\ddots \mathbf{k} \ddots\right]$ = (**m by m**) modal stiffness matrix.

Damping Matrix: $$[\mathbf{C}] = [\phi]\left[\ddots \mathbf{c} \ddots\right][\phi]^t \quad (33)$$

where,

$\left[\ddots \mathbf{c} \ddots\right]$ = (**m by m**) modal damping matrix.

Mass Matrix: $$[\mathbf{M}] = [\phi]\left[\ddots \mathbf{m} \ddots\right][\phi]^t \quad (34)$$

where,

$\left[\ddots \mathbf{m} \ddots\right]$ = (**m by m**) modal mass matrix.

$[\phi]$ = (**n by m**) mode shape matrix.

There are two significant advantages using equations (32), (33) and (34) compared to the complex model calculations (19), (23), and (27). First, there is ***no matrix inversion***. These calculations are much simpler and thus reduce the potential for computational error. Secondly, if **n**-degrees of freedom are measured on a structure but only **m**-modes are identified and $(\mathbf{m} \ll \mathbf{n})$, then (**n by n**) mass, stiffness, and damping matrices can still be computed with the above formulas whereas the complex mode formulas are limited to computing (**m by m**) sized matrices. This second advantage allows a more direct comparison of test results with matrices derived from finite element modeling.

CONCLUSIONS

Formulas for computing the full mass, stiffness, and damping matrices from the transfer function matrix and its derivatives were derived first. Since the transfer function matrix and its derivatives can be synthesized from a set of modal parameters, these formulas provide a means for computing mass, stiffness, and damping from experimental modal parameters.

This approach has a serious computational limitation, however. The first step requires that the flexibility matrix be inverted to obtain the stiffness matrix. Not only is matrix inversion an "error amplifying" process (thus amplifying the errors in experimental modal data), but the number of linearly independent mode shapes required to yield a full rank flexibility matrix in prohibitive for most practical situations.

Next, we made further assumptions regarding the modal parameters of the structure in order to define its modal mass, stiffness, and damping. With these assumptions, the formulas for computing the full mass, stiffness, and damping matrices were greatly simplified. Unfortunately, these new formulas still require a set of linearly independent mode shape vectors, and the number of modes must equal the number of DOFs in the desired mass, stiffness, and damping matrices.

Finally, we made a further restrictive assumption regarding the mode shapes, namely that they are orthogonal to one another. When this assumption can be (approximately) satisfied, the formulas for computing mass, stiffness, and damping from experimental modal data are straightforward and computationally tractable.

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