MODAL PARAMETER ESTIMATION FROM AMBIENT RESPONSE DATA

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ABSTRACT

In this paper, a new curve fitting technique is introduced for estimating modal parameters from ambient response data. The curve fitting method is applied to a set of ODS FRFs that were calculated from impact response and ambient response data taken from a concrete bridge. These estimates are then compared with estimates obtained by curve fitting a set of FRFs taken from the same bridge.

This paper uses the results of another IMAC paper [1], where post-processing methods were applied to three different sets of multiple channel time domain vibration response data taken from the Z24 highway bridge in Switzerland. The three different test cases were;

Case 1: Two shaker test, provided simultaneously acquired acceleration response and excitation force time waveforms. The shakers were driven by uncorrelated random signals.

Case 2: Impact test, provided simultaneously acquired acceleration response time waveforms, including three reference (fixed) responses. The impact force was provided by a 100 kg. drop weight impactor, and was not measured.

Case 3: Ambient test, provided simultaneously acquired acceleration response time waveforms, including three reference (fixed) responses. Excitation was provided by traffic on an adjoining bridge.

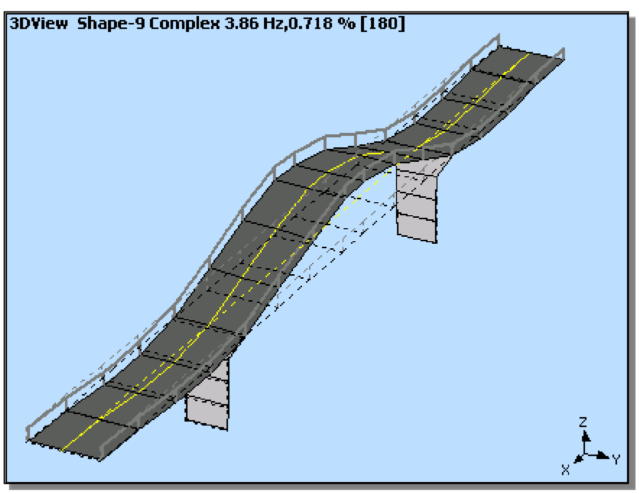

Figure 1. Bending mode of the Z24 Bridge.

Excitation forces were measured in Case 1, so multiple reference Frequency Response Functions (FRFs) were calculated. Since no forces were measured in Cases 2 & 3, a different set of measurements called ODS FRFs ([1], [3]) were calculated for these cases.

All sets of FRFs and ODS FRFs were then curve fit to obtain estimates of the modal parameters of the bridge. Modal frequencies & damping are listed in tables for comparison. Mode shape estimates were compared by calculating the Modal Assurance Criterion (MAC) between pairs of mode shapes.

Each mode shape had 75 DOFs in it. A bending mode of the bridge is shown in Figure 1.

1. INTRODUCTION

The ODS FRF is a complex valued frequency domain function like an FRF, but it is formed differently.

Formation of an ODS FRF is depicted in Figure 2. ***An ODS FRF is formed by combining the Auto Power Spectrum (APS) of a roving response with the phase of the Cross Power Spectrum (XPS) between the roving response and a fixed reference response.***

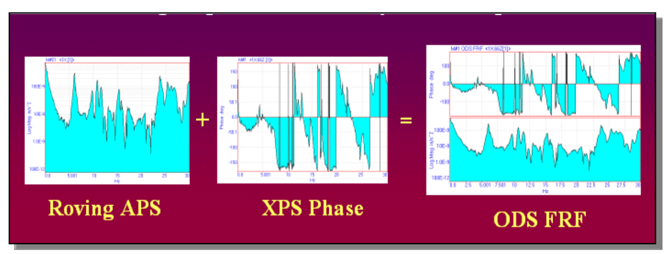

Figure 2. ODS FRF Formation.

The ODS FRF contains the correct magnitude of response (the APS of the roving response), and the correct phase relative to the reference response. Moreover, since the APS has ***peaks at resonances,*** so does the ODS FRF.

MDOF Curve Fitting Method

The new MDOF (multiple degree-of-freedom or multiple mode) curve fitting method using in this paper is a variation of the Global Rational Fraction Polynomial (RFP) method, the details of which are described in references [4], [5], & [6].

Single Versus Multiple Reference Curve Fitting

To obtain mode shapes, a minimum set of measurements for curve fitting would consist of all measurements made between a single fixed reference and multiple roving responses. This is called single reference curve fitting.

Since multiple reference data sets were acquired in all cases (2 references for Case 1, and 3 for Cases 2 & 3), all references could be used together during curve fitting. This is called multiple reference curve fitting. The new ODS FRF curve fitting method described in this paper was used as a single reference curve fitter, but was applied to the data set for each reference.

Global Curve Fitting

This global RFP method obtains modal parameter estimates by a 2-step process. First, modal frequency & damping estimates are obtained by performing least squared error curve fitting on a set of ODS FRFs.

Once the (global) frequency & damping estimates are obtained, ***modal residue***s are obtained by a second least squared curve fit of the ODS FRFs. Finally, the mode shape is assembled from a set of modal residues obtained from curve fitting a set of ODS FRFs corresponding to a single reference response.

In Cases 2 & 3, three sets of ODS FRFs were obtained, one for each of three reference responses. Modal parameters were obtained for each reference, and they were compared with the curve fitting results of Case 1.

Relationship of Response APS and FRF

A response APS is related to an FRF in the following manner. An FRF is defined as the ratio of the Fourier transform of a response over the Fourier transform of the force that caused the response, or,

$$\mathbf{H}(\omega) = \frac{\mathbf{X}(\omega)}{\mathbf{F}(\omega)} \tag{1}$$

where:

$\mathbf{X}(\omega) =$ Fourier transform of response.

$\mathbf{F}(\omega) =$ Fourier transform of excitation force.

$\omega =$ frequency.

The magnitude squared of the FRF can then be written as,

$$\left|\mathbf{H}(\omega)\right|^2 = \frac{\mathbf{X}(\omega)\,\mathbf{X}(\omega)^*}{\mathbf{F}(\omega)\,\mathbf{F}(\omega)^*} \tag{2}$$

where:

$\mathbf{X}(\omega)\,\mathbf{X}(\omega)^* =$ APS of the response.

$\mathbf{F}(\omega)\,\mathbf{F}(\omega)^* =$ APS of the excitation force.

***** - denotes complex conjugate.

We know that peaks in a FRF are due to modes of the structure. That is, because the force spectrum is divided into the response spectrum, any peaks in the FRF must be due to resonances of the structure. Likewise, the same peaks would appear in the APS of the response. Equation (2) leads to the following result.

Flat Force Spectrum: ***If the APS of the excitation force is assumed to be "relatively flat" over the frequency range of measurement, then any peaks in the APS of the response are due to modes of the structure.***

In test Case 2 (the Impact test), the above assumption is valid since an impact force has a relatively flat spectrum. In Case 3, however, there is no way to validate the above assumption. But, in the absence of any evidence to the contrary, it's still valid to assume that random traffic on the adjacent bridge produces random excitation spread over a broad range of frequencies.

Curve Fitting an ODS FRF

Equation (2) is used as the curve fitting model for estimating modal parameters for a set of ODS FRFs. The curve fitting equations using the RFP method in references [4], [5], & [6] need to be slightly modified to curve fit the ***magnitude squared of the FRF model*** to the magnitude of the ODS FRF. The frequency, damping, and residue magnitude are obtained by curve fitting the ODS FRF magnitude. ***The phase of the ODS FRF was used as the phase of the residue.***

Figure 3 shows the results from curve fitting several modes in an ODS FRF. After the curve fitting is completed, a curve fit function can be synthesized using the curve fitting parameters and overlaid on each measurement for comparison.

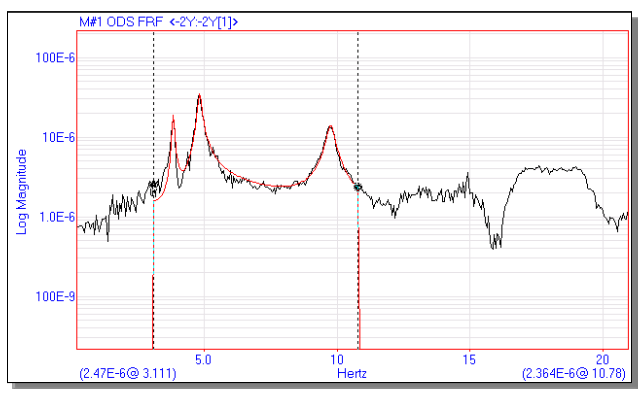

Figure 3. Typical Curve Fit of an ODS FRF

2. COMPARISON OF RESULTS

Case 1 was a "standard" modal test, where two shakers were used to simultaneously excite the bridge. The excitation forces and acceleration responses were simultaneously acquired, and FRFs were calculated from the time waveforms. More details of the post-processing are given in reference [1].

The 2-reference set of FRFs was curve fit using the RFP method described in references [4], [5], & [6]. These Shaker test modal parameters were then used for comparison with the results obtained by curve fitting the ODS FRFs from Cases 2 & 3. The Shaker test frequencies & damping are shown in Figure 4 (a). A typical FRF curve fit over the frequency range (**3 to 30 Hz)** is shown in Figure 4 (b).

Shape	Frequency	Units	Damping	Damping (%)
1	3.888	Hz	0.047	1.209
2	4.803	Hz	0.074	1.548
3	9.777	Hz	0.153	1.563
4	10.487	Hz	0.115	1.096
5	12.401	Hz	0.347	2.793
6	17.332	Hz	0.815	4.699
7	19.244	Hz	0.508	2.638
8	26.668	Hz	0.838	3.142

Figure 4(a). Case 1 Frequencies & Damping.

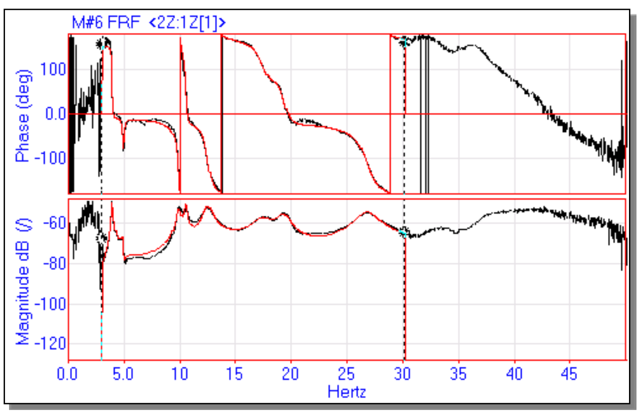

Figure 4(b). Typical FRF Curve Fit.

Comparison of Frequency & Damping

Figure 5 shows a comparison of the Shaker test modal frequency estimates with those obtained from the three references of the Impact test. The three reference sets of ODS FRF data are denoted as **1Z**, **-2Y**, & **2Z**. Notice that the first three modes (**3.8, 4.8, & 9.7 Hz**) were identified from all three references, but only some of the higher frequency modes were identified from individual references.

Shape	Shaker	1Z	-2Y	2Z
1	3.888	3.848	3.851	3.851
2	4.803	4.823	4.839	4.787
3	9.777	9.752	9.75	9.74
4	10.487			
5	12.401	13.006	13.343	13.348
6	17.332		17.284	
7	19.244	19.255		19.228

Figure 5. Cases 1 & 2 Modal Frequencies (Hz).

Figure 6 shows a comparison of the Shaker test modal damping estimates with those obtained from the three references of the Impact test. Notice that the damping values are ***listed in Hz*** instead of percentage of critical damping. Damping in Hz is independent of the modal frequency, and is therefore a more accurate measure of damping for comparison.

Shape	Shaker	1Z	-2Y	2Z
1	0.047	0.037	0.037	0.037
2	0.074	0.061	0.054	0.06
3	0.153	0.152	0.155	0.148
4	0.115			
5	0.347	0.173	0.377	0.383
6	0.815		0.683	
7	0.508	0.417		0.448

Figure 6. Cases 1 & 2 Modal Damping (Hz).

Figure 7 shows a comparison of the Shaker test modal frequency estimates with those obtained from the Ambient test.

Shape	Shaker	1Z	-2Y	2Z
1	3.888	3.86	3.859	3.853
2	4.803	4.898	4.899	4.901
3	9.777	9.747	9.743	9.734
4	10.487			
5	12.401			
6	17.332			
7	19.244			

Figure 7. Cases 1 & 3 Modal Frequencies (Hz).

Figure 8 shows a comparison of the Shaker test modal damping estimates with those obtained from the Ambient test.

Shape	Shaker	1Z	-2Y	2Z
1	0.047	0.028	0.026	0.027
2	0.074	0.055	0.057	0.057
3	0.153	0.098	0.077	0.088
4	0.115			
5	0.347			
6	0.815			
7	0.508			

Figure 8. Cases 1 & 3 Modal Damping (Hz).

Comparison of Mode Shapes

Mode shape estimates from the Impact and Ambient tests where compared with those from the Shaker test by using the Modal Assurance Criterion (MAC). MAC values range between ***0 and 1***. If a MAC values between two shapes is 1, they are identical shapes. As a matter of practice, any MAC value ***above 0.9 indicates that the two shapes are similar***. Any value ***less than 0.9 indicates that they are different shapes***.

Figure 9 shows the MAC values between the shape estimates from the Shaker test and those from the Impact test. These values indicate that the **3.8 & 9.7 Hz** mode shapes are substantially the same for all cases. The MAC values also indicate that the shapes from the three Impact references are substantially the same for all modes.

Shape		Shaker	1Z	-2Y	2Z
Frequency		3.888	3.848	3.851	3.851
Shape	Frequency	MAC	MAC	MAC	MAC
1	3.888	1.000	0.978	0.989	0.994
8	3.848	0.978	1.000	0.987	0.989
13	3.851	0.989	0.987	1.000	0.994
19	3.851	0.994	0.989	0.994	1.000
Frequency		4.803	4.823	4.839	4.787
Shape	Frequency	MAC	MAC	MAC	MAC
2	4.803	1.000	0.507	0.680	0.551
9	4.823	0.507	1.000	0.943	0.950
14	4.839	0.680	0.943	1.000	0.947
20	4.787	0.551	0.950	0.947	1.000
Frequency		9.777	9.752	9.75	9.74
Shape	Frequency	MAC	MAC	MAC	MAC
3	9.777	1.000	0.909	0.927	0.916
10	9.752	0.909	1.000	0.984	0.984
15	9.75	0.927	0.984	1.000	0.988
21	9.74	0.916	0.984	0.988	1.000
Frequency		12.401	13.006	13.343	13.348
Shape	Frequency	MAC	MAC	MAC	MAC
5	12.401	1.000	0.134	0.117	0.132
11	13.006	0.134	1.000	0.953	0.934
16	13.343	0.117	0.953	1.000	0.966
22	13.348	0.132	0.934	0.966	1.000
Frequency		19.244	19.255	19.179	19.228
Shape	Frequency	MAC	MAC	MAC	MAC
7	19.244	1.000	0.830	0.834	0.825
12	19.255	0.830	1.000	0.978	0.989
18	19.179	0.834	0.978	1.000	0.977
23	19.228	0.825	0.989	0.977	1.000

Figure 9. Cases 1 & 2 MAC Values.

Figure 10 shows the MAC values between the shape estimates from the Shaker test and those from the Ambient test. These values also indicate that the **3.8 & 9.7 Hz** mode shapes are substantially the same for all cases, the same as with the Impact results. The MAC values also indicate that the shapes from the three Ambient references are substantially the same for all modes.

Shape		Shaker	1Z	-2Y	2Z
Frequency		3.888	3.86	3.859	3.853
Shape	Frequency	MAC	MAC	MAC	MAC
1	3.888	1.000	0.996	0.985	0.935
8	3.86	0.996	1.000	0.987	0.934
11	3.859	0.985	0.987	1.000	0.967
14	3.853	0.935	0.934	0.967	1.000
Frequency		4.803	4.898	4.899	4.901
Shape	Frequency	MAC	MAC	MAC	MAC
2	4.803	1.000	0.875	0.880	0.829
9	4.898	0.875	1.000	0.984	0.873
12	4.899	0.880	0.984	1.000	0.918
15	4.901	0.829	0.873	0.918	1.000
Frequency		9.777	9.747	9.743	9.734
Shape	Frequency	MAC	MAC	MAC	MAC
3	9.777	1.000	0.765	0.617	0.718
10	9.747	0.765	1.000	0.869	0.914
13	9.743	0.617	0.869	1.000	0.877
16	9.734	0.718	0.914	0.877	1.000

Figure 10. Cases 1 & 3 MAC Values.

3. CONCLUSIONS

A new curve fitting algorithm that can extract modal parameters from both APS and ODS FRF measurements was applied to several sets of impact and ambient response data taken from a concrete road bridge. These results were compared with modal parameter estimates obtained by curve fitting FRF measurements, from a 2-shaker test on the same bridge.

Figure 4 shows the frequency & damping estimates, plus a typical FRF curve fit from the Shaker test. All of the modes in the frequency range (**3 to 30 Hz**) were identified. This was the frequency band of excitation.

Modal frequency & damping estimates from the three test Cases are listed in Figures 5 to 8. From the comparisons in Figures 5 & 7, it is clear that modal frequencies for the first three modes (**3.8, 4.8, & 9.7 Hz**) matched very well among all Cases and all references.

The Case 2 results (Figure 5) also show a match of the **17 & 19** Hz modal frequencies with those of Case 1, but not for all references. The most probable explanation for these results is that ***only the first three modes were adequately excited in Cases 2 & 3***.

Modal damping estimates from Cases 1 & 2 (shown in Figure 6), were within 10% of one another for the first three modes. However, the Ambient test results (shown in Figure 8), yielded consistently lighter damping estimates for the first three modes.

Mode shape comparisons are shown in Figures 9 & 10. Both Figures indicate that the modes shapes from all three references in Cases 2 & 3 matched well with each other. That is, their MAC values were close to ***0.9 or above***. This demonstrates the ODS FRF curve fitting results are consistent and independent of the reference used for curve fitting.

Nevertheless, Figure 9 indicates that only the **3.8 & 9.7 Hz** mode shapes from the Impact test matched those from the Shaker test. Likewise, Figure 10 indicates that only the **3.8 Hz** mode shape from the Ambient test matched the Shaker test mode well.

The mismatch between the **4.8 Hz** mode shapes can be explained by the Complexity plots in Figure 11. Notice that the mode shape components in Figure 11 (a) lie approximately along the ***0-180 degree line***. This indicates that the Shaker test mode shape is a ***normal mode***, typical of a lightly damped structure.

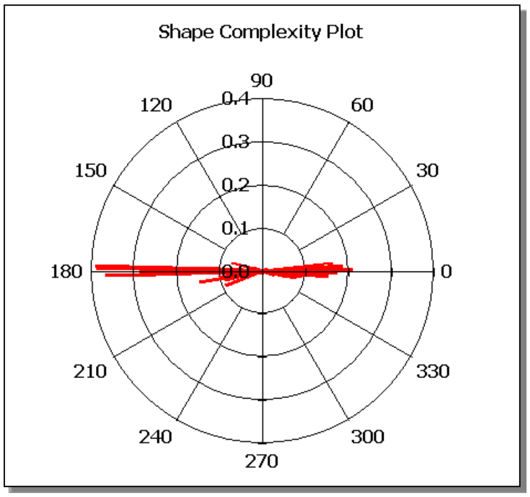

Figure 11 (a). Complexity of 4.8 Hz Shaker Mode.

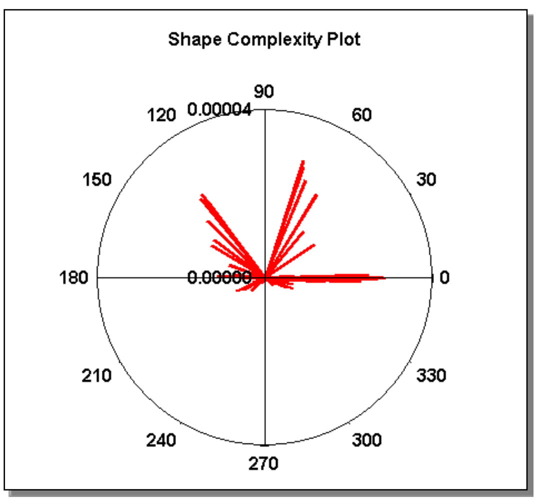

Figure 11 (b). Complexity of 4.8 Hz Impact Mode.

On the other hand, Figure 11 (b) indicates that the mode shape estimate from the Impact test is a ***complex mode***, with phases that don't lie along a straight line. This is a result of the way the mode shape phase was taken from the ODS FRFs.

Only the ***magnitude*** of each mode shape component was obtained by curve fitting the magnitude of each ODS FRF, and this is believed to be accurate. However, the phase of each mode shape component was taken from the phase of each ODS FRF. This phase estimate is in error because the ODS FRF phase contains the influence of all modes at any frequency.

Clearly, a better estimate of mode shape phase is needed to obtain mode shapes that match those from the Shaker test.

Improved Phase Estimates

In an attempt to reduce noise in the ODS FRF measurements, we applied an ***exponential window*** to their Inverse Fourier transforms. Normally, when an exponential window is applied to the Inverse Fourier transform of an FRF (yielding an Impulse Response Function), a known amount of damping is added to all modes. This is an effective method for reducing noise in the measurements, with a recoverable effect on the modal parameter estimates.

The Inverse Fourier transform of an ODS FRF is similar to a Cross Correlation function. Applying an exponential window to a Cross Correlation function yields an approximation of an Impulse Response Function. The Fourier Transform of this approximation yields a ***modified ODS FRF that is similar to an FRF***. Therefore, it can be curve fit with an FRF model.

Figure 12 shows the MAC values between the shape estimates from the Shaker test and those from the Impact test after the ODS FRFs were modified with the exponential window. These MAC values indicate improved shape estimates from the Impact data for all of the modes listed, especially the **4.8 Hz** mode.

Shape		Shaker	1Z	-2Y	2Z
Frequency		3.888	3.843	3.887	3.85
Shape	Frequency	MAC	MAC	MAC	MAC
1	3.888	1.000	0.977	0.978	0.994
8	3.843	0.977	1.000	0.987	0.991
13	3.887	0.978	0.987	1.000	0.989
17	3.85	0.994	0.991	0.989	1.000
Frequency		4.803	4.883	4.801	4.797
Shape	Frequency	MAC	MAC	MAC	MAC
2	4.803	1.000	0.910	0.749	0.904
9	4.883	0.910	1.000	0.802	0.813
14	4.801	0.749	0.802	1.000	0.709
18	4.797	0.904	0.813	0.709	1.000
Frequency		9.777	9.777	9.774	9.742
Shape	Frequency	MAC	MAC	MAC	MAC
3	9.777	1.000	0.911	0.946	0.912
10	9.777	0.911	1.000	0.980	0.982
15	9.774	0.946	0.980	1.000	0.975
19	9.742	0.912	0.982	0.975	1.000
Frequency		19.244	19.243	19.637	19.417
Shape	Frequency	MAC	MAC	MAC	MAC
7	19.244	1.000	0.899	0.839	0.904
12	19.243	0.899	1.000	0.856	0.935
16	19.637	0.839	0.856	1.000	0.939
20	19.417	0.904	0.935	0.939	1.000

Figure 12. Cases 1 & 2 MAC Values After Windowing.

Shape		Shaker	1Z	-2Y	2Z
Frequency		3.888	3.86	3.854	3.856
Shape	Frequency	MAC	MAC	MAC	MAC
1	3.888	1.000	0.992	0.987	0.918
8	3.86	0.992	1.000	0.993	0.942
11	3.854	0.987	0.993	1.000	0.934
14	3.856	0.918	0.942	0.934	1.000
Frequency		4.803	4.898	4.893	4.883
Shape	Frequency	MAC	MAC	MAC	MAC
2	4.803	1.000	0.848	0.850	0.580
9	4.898	0.848	1.000	0.833	0.575
12	4.893	0.850	0.833	1.000	0.753
15	4.883	0.580	0.575	0.753	1.000
Frequency		9.777	9.704	9.814	9.807
Shape	Frequency	MAC	MAC	MAC	MAC
3	9.777	1.000	0.439	0.299	0.662
10	9.704	0.439	1.000	0.313	0.588
13	9.814	0.299	0.313	1.000	0.569
16	9.807	0.662	0.588	0.569	1.000

Figure 13. Cases 1 & 3 MAC Values After Windowing.

Figure 13 shows the MAC values between the shape estimates from the Shaker test and those from the Ambient test after the ODS FRFs were modified with the exponential window. These MAC values indicate that exponential windowing of the ODS FRFs didn't improve the mode shape estimates.

More study of the effect of exponential windowing on ODS FRFs is required. The windowed ODS FRFs appear to provide a more accurate estimate of mode shape phase. Our original method (simply using the ODS FRF phase at the frequency of the mode) is not valid because of the influence of surrounding modes on the ODS FRF phase.

Frequency & damping estimates from the windowed ODS FRFs also appear to be accurate, although we don't show any comparisons with the Shaker modes here.

Beyond the first three modes (**3.8, 4.8, & 9.7 Hz**) the best explanation for the failure of both the Impact and Ambient tests to match the Shaker results is that the higher frequency modes simply were not excited during those tests.

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