POST-PROCESSING AMBIENT AND FORCED RESPONSE BRIDGE DATA TO OBTAIN MODAL PARAMETERS

Brian Schwarz & Mark Richardson
Vibrant Technology, Inc.
Jamestown, California

ABSTRACT

In this paper, post-processing methods were applied to three different sets of multiple channel time domain vibration response data taken from the Z24 highway bridge in Switzerland. The objective of this exercise was to compare the mode shapes resulting from each of these data sets with one another. Ideally, all of the tests should yield the same mode shapes, although perhaps not all modes would be excited in each case. The three different test cases were;

Case 1: Two shaker response data, including the simultaneously measured excitation forces. The shakers were driven by uncorrelated random signals.

Case 2: Impact response data, including three reference responses but no measured forces. The impact force was provided by a 100 kg. drop weight impactor.

Case 3: Ambient response data, including three reference (fixed) responses. Excitation was provided by automobile traffic on an adjoining bridge.

Excitation forces were measured in Case 1, so multiple reference Frequency Response Functions (FRFs) were calculated. Since no forces were measured in Cases 2 & 3, a different set of measurements called ODS FRFs ([1], [2]) were calculated for these cases.

All sets of FRFs and ODS FRFs were then "curve fit" to obtain estimates of the modal parameters of the bridge. Modal frequencies are listed in tables for comparison, and the Modal Assurance Criterion (MAC) was used to compare mode shapes.

INTRODUCTION

Modes are used to describe resonant vibration in structures. Modal testing (performing a modal survey) is usually done under controlled stationary (non-time varying) conditions, using one or more exciters. Furthermore, the excitation forces and their corresponding responses are *simultaneously measured*.

FRF Measurement

The most common measurement used for identifying modes is the Frequency Response Function (FRF). The FRF is a frequency domain function, formed as the ratio of a measured response divided by the measured excitation force that caused the response. Response can be measured as displacement, velocity, or acceleration. In these three test cases, all responses were measured as accelerations.

FRFs are ideal measurements for identifying modes since they "isolate" the modes as peaks in the FRF measurements. Since the response spectrum is divided by the force spectrum in each FRF, peaks in the FRFs are *only caused by the modes* of the structure.

MIMO FRF Calculation

Case 1 was a conventional modal test where the excitation forces and their corresponding responses were *simultaneously measured*. This simultaneously measured force and response data was processed using spectrum averaging followed by the FRF calculation. Since two shakers were used instead of one, special Multi-Input Multi-Output (MIMO) processing was used to obtain a set of multiple reference FRFs.

The MIMO spectrum averaging loop is shown in Figure 1. Since the shakers were driven with pure random signals, the resulting sampled waveforms had to have Hanning windows applied to them before the FFT was applied, in order to reduce leakage effects in the resulting spectra.

Spectrum averaging of random signals requires no special triggering since the signals are relatively "continuous" throughout each sampling window (samples used per spectrum average). In Case 2, where transient signals were measured, a trigger was required to define the beginning of each sampling window.

MIMO FRF calculations require the inversion of the excitation force Power Spectrum Matrix (PS Matrix), and multiplication of the Cross Power Spectrum Matrix by the inverse of the PS Matrix to obtain the FRFs. This is also indicated in Figure 1.

Multi-Reference Curve Fitting

For Case 1, the multiple reference FRFs were curve fit using multiple reference FRF curve fitting methods to the estimate modal parameters. Normally, only a single reference set of FRFs is required to identify mode shapes. A set of multi-reference FRFs offers the advantage of identifying modes from one reference that may not be present in the other, or of finding *repeated roots* (two modes with the same natural frequency but different mode shapes).

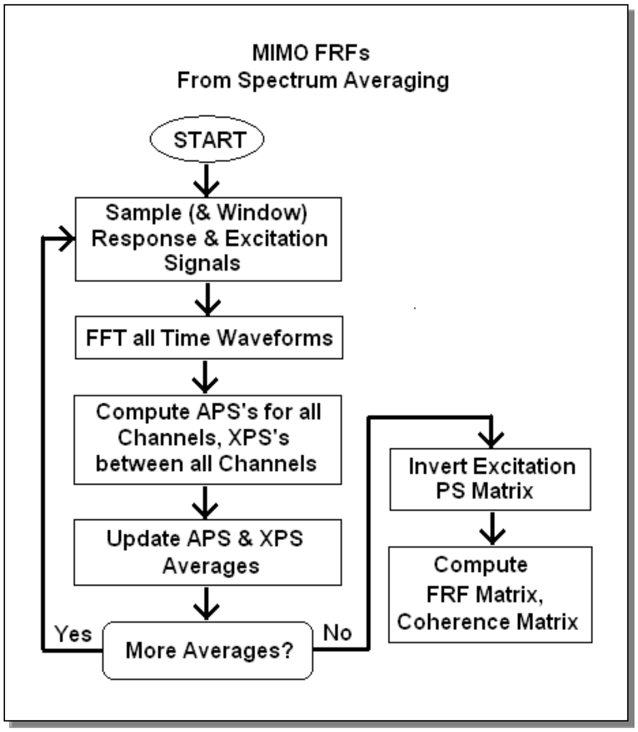


Figure 1. MIMO FRFs.

Transmissibility Measurement

Transmissibility would be the usual measurement to calculate for Cases 2 & 3, where no excitation forces were measured. Transmissibility is calculated in the same way as the FRF, but a ***reference (or fixed)*** response is used instead of the force. Whereas the FRF is the ratio of response divided by force, ***Transmissibility is the ratio of a response divided by a reference response***.

Transmissibility is therefore a ***ratio of two responses***. If the excitation force level varies from one measurement to the next, it is assumed that its effect on both responses is the same, so its effect will be "***canceled out***" in the Transmissibility.

Difficulty with Transmissibility's

The main difficulty with a Transmissibility measurement is that unlike the FRF, peaks in the measurement are not evidence of resonances. Rather, resonant frequencies are located at "***flat spots***" in a Transmissibility. This is shown in Figure 2.

Figure 2 shows a response APS plotted above a Transmissibility. At the frequency of the resonance peak in the APS (inside the cursor band shown with dashed lines), the Transmissibility has a *"flat spot"*, not a peak. Moreover, the peaks in the Transmissibility do not correspond to resonances, but are merely the result of the division of the response spectrum by the reference spectrum at frequencies where the reference spectrum is relatively small.

Therefore, if a set of Transmissibility's is used for identifying mode shapes, at least one response APS measurement must also be used for locating resonance peaks.

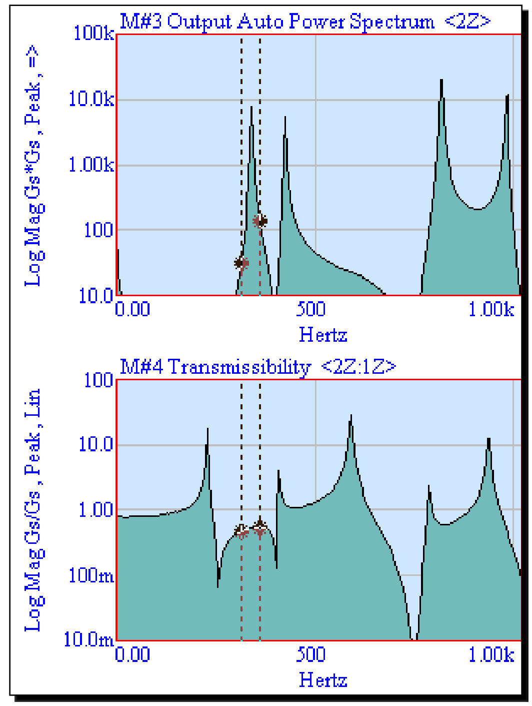


Figure 2. APS and Transmissibility at Resonance Peak.

ODS FRF Measurement

A set of ODS FRF measurements ([1], [2]) can be used to identify mode shapes in a more straightforward manner. The ODS FRF is a complex valued frequency domain function like an FRF, but it is calculated differently.

The ODS FRF calculation is depicted in Figure 3. ***An ODS FRF formed by combining the APS of a roving response with the phase of the XPS between the roving response and a reference response.***

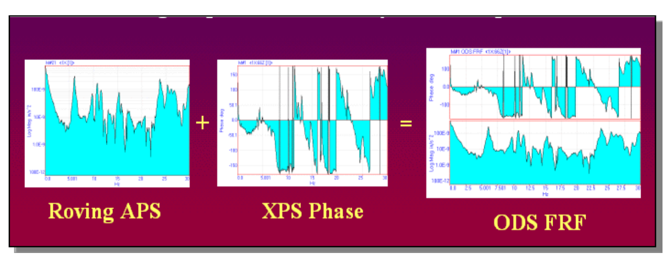


Figure 3. ODS FRF Calculation.

Instead of being a ratio of responses, the ODS FRF contains the correct magnitude of response (the APS of the roving response), and the correct Phase relative to the reference response. More importantly, an ODS FRF has ***peaks at resonances***, so it is easier to display ODS's from a set of ODS FRFs and observe mode shapes at resonant frequencies.

Mode Shapes From ODS FRFs

Like FRFs, at least a single reference set of ODS FRFs is required to identify mode shapes. For test Cases 2 & 3, three references of data were acquired. ***Each reference set*** of ODS FRFs was used to obtain mode shape estimates.

CASE 1: MULTI-SHAKER TEST

The data acquired for test Case 1 consisted of **117** time waveforms in **9** different measurement sets. Each measurement set consisted of waveforms that were simultaneously acquired; accelerometer measurements for multiple response DOFs plus force measurements at the **2** shaker locations.

*The shaker locations were designated **1Z** & **2Z** although the shakers were not located at those DOFs.* The shakers were driven by uncorrelated random signals that excited the bridge in the frequency range (**3 to 30 Hz**)

Each time waveform consisted of **65536** samples of uniformly sampled data with a **0.01** second time increment between samples (**100 Hz** sampling rate), giving a total time length of **655.35** seconds.

Each measurement set was processed using MIMO spectrum averaging, with a **2048** sample spectrum size, **20** spectral averages, and a **Hanning** window applied to reduce leakage. The spectrum size and number of averages caused an overlap processing percentage of **21%**, meaning that about 1/5th of the samples from each sampling window were used in the succeeding sampling window.

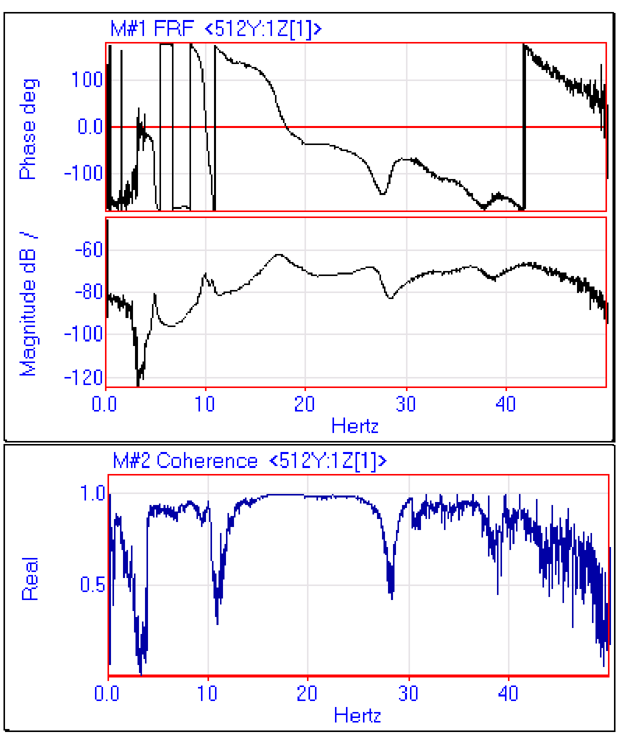


Figure 4. Typical FRF and Coherence.

A typical FRF and Coherence function are shown in Figure 4. Notice that they are both defined of the frequency range (0 to 50Hz), one half of the sampling rate. The MIMO processing yielded a total of **198** FRFs; all DOF pairs involving **99** unique response DOFs and **2** unique reference (shaker) DOFs.

Curve Fitting Results

The FRFs were curve fit using a multiple reference curve fitting technique. **Nine** modes were identified over the frequency range (**0 to 30 Hz**). The modal frequency & damping estimates are shown in Figure 5. In Figure 6, an FRF synthesized from the modal parameters is overlaid on its corresponding FRF measurement. This indicates that the modal parameters provide a good representation of the dynamics of the structure over their frequency range.

Shape	Frequency	Units	Damping	Damping (%)
1	3.872	Hertz	0.029	0.759
2	4.821	Hertz	0.024	0.506
3	9.795	Hertz	0.062	0.632
4	10.489	Hertz	0.114	1.09
5	12.403	Hertz	0.342	2.759
6	13.086	Hertz	0.17	1.3
7	17.259	Hertz	0.713	4.125
8	19.207	Hertz	0.45	2.342
9	26.665	Hertz	0.84	3.149

Figure 5. Shaker Test Modal Frequency & Damping.

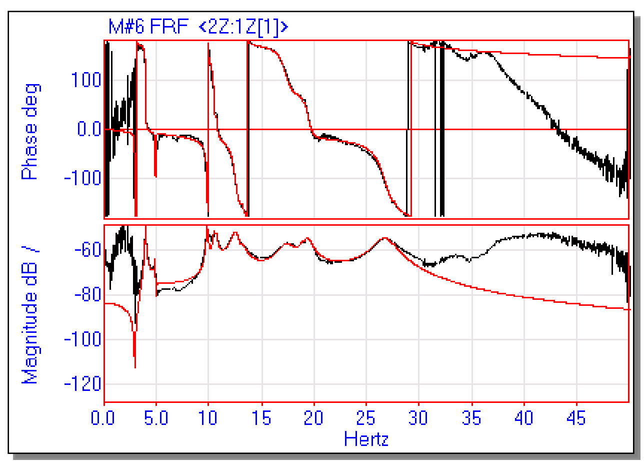


Figure 6. Synthesized FRF Overlaid on Measurement.

Multi-reference curve fitting of the FRFs yielded a separate mode shape estimate for each mode and each reference. Figure 7 contains two different measures of the mode shape estimates from the 2 references. The Modal Assurance Criterion (MAC) values between the mode shapes from the two references are listed in the **MAC** column. The columns with **1Z** and **2Z** as headings contain *modal participation factors*, the *"strength"* of each mode for each reference.

Modes **1**, **2**, **5** and **9** correlate well between the two references. ***Typically, MAC values above 90% indicate that two shapes are nearly alike.*** The other shapes have lower MAC values between the 2 references, indicating that they are substantially different shape estimates.

Mode	Frequency (Hz)	MAC	2Z	1Z
1	3.872	0.99	1.9	0.2
2	4.821	0.96	0.1	0.1
3	9.795	0.84	1.1	1.1
4	10.489	0.80	3.9	0.9
5	12.403	0.90	2.2	7.9
6	13.086	0.30	0.3	1.5
7	17.259	0.28	2.4	10.0
8	19.207	0.14	3.9	5.9
9	26.665	0.94	9.4	3.3

Figure 7. Mode Shape MAC Values and Participation Factors For 2 Shaker References.

The modal participation factors indicate that modes **6**, **7**, and **8** are better defined at reference **1Z**, while modes **3** and **4** are better defined at reference **2Z**

CASE 2: MULTI-REFERENCE IMPACT TEST

The data acquired for test Case 2 consisted of **126** time waveforms in **9** different measurement sets. Each measurement set consisted of waveforms that were simultaneously acquired; accelerometer measurements for multiple response DOFs plus **3** reference accelerometer measurements at locations, **1Z**, **-2Y** & **2Z**.

Each time waveform consisted of **8192** samples of uniformly sampled data with a **0.01** second time increment between samples, giving a total time length of **81.92** seconds.

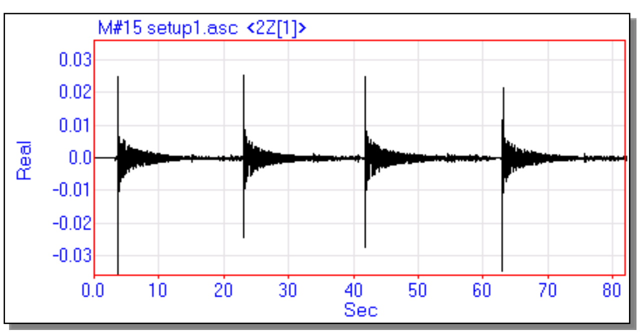


Figure 8. Typical Multiple Impulse Time Waveform.

A typical time waveform is shown in Figure 8. Notice that the bridge was impacted and allowed to "ring down" 4 times over the 81 second acquisition period. Each measurement set was processed using spectrum averaging, with a **900** sample spectrum size and a trigger to begin each sampling window at the start of an impulse. Since each signal contained 4 impulse responses, **4** spectral estimates were averaged together to form APS and XPS estimates, using an averaging loop like the one in Figure 1.

A total of **126** APS's were calculated, one for each of the 99 unique roving response DOFs, and 27 for the 3 reference responses in each of the 9 measurement sets. Also, 287 XPS's were calculated between all pairs of the 99 roving responses and 3 reference responses.

Finally, the APS and XPS estimates were processed to yield **99** ODS FRFs for each of the **3** references, for a total of **287** ODS FRFs. A typical ODS FRF is shown in Figure 9.

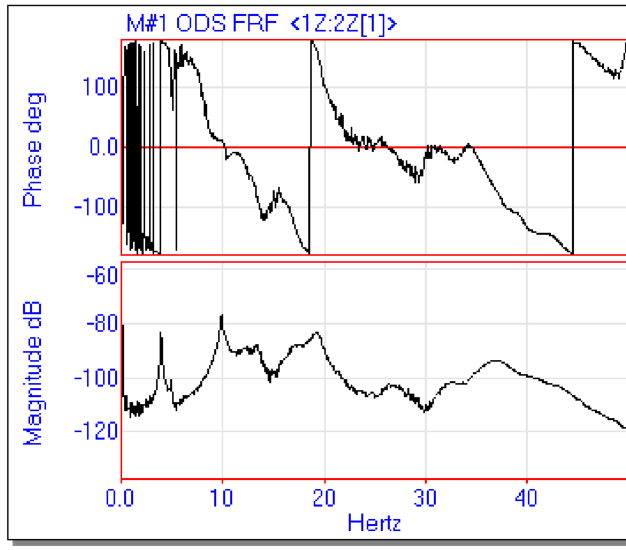


Figure 9. Typical Impact ODS FRF Measurement.

Mode Shapes & Frequencies

Mode shapes and frequencies were obtained by using a "peak cursor" in the vicinity of each resonance peak in each set of ODS FRFs. Mode shapes were assembled from the peak cursor values of the ODS FRFs. The modal frequencies (listed in Figure 9), were obtained by averaging the peak frequencies corresponding to the peak cursor values found in each ODS FRF.

Impact Modal Frequencies (Hz)

	1Z	-2Y	2Z
1	3.834	3.833	3.834
2	4.574	4.739	4.826
3	9.741	9.741	9.742
4	13.235	13.186	13.255
5	16.748	16.802	16.696
6	19.259	19.257	19.235

Figure 9. Modal Frequencies From Impact Data.

Figure 10 lists the MAC values of the mode shapes from the 3 sets of ODS FRF data corresponding to the 3 references.

		1Z	-2Y	2Z
	Frequency	3.834	3.833	3.834
Shape		MAC	MAC	MAC
1	3.834	1.00000	0.98594	0.98937
7	3.833	0.98594	1.00000	0.99246
13	3.834	0.98937	0.99246	1.00000
	Frequency	4.574	4.739	4.826
Shape		MAC	MAC	MAC
2	4.574	1.00000	0.89009	0.48714
8	4.739	0.89009	1.00000	0.53158
14	4.826	0.48714	0.53158	1.00000
	Frequency	9.741	9.741	9.742
Shape		MAC	MAC	MAC
3	9.741	1.00000	0.98492	0.98619
9	9.741	0.98492	1.00000	0.98898
15	9.742	0.98619	0.98898	1.00000
	Frequency	13.235	13.186	13.255
Shape		MAC	MAC	MAC
4	13.235	1.00000	0.94485	0.88673
10	13.186	0.94485	1.00000	0.89863
16	13.255	0.88673	0.89863	1.00000
	Frequency	16.748	16.802	16.696
Shape		MAC	MAC	MAC
5	16.748	1.00000	0.77685	0.85938
11	16.802	0.77685	1.00000	0.49074
17	16.696	0.85938	0.49074	1.00000
	Frequency	19.259	19.257	19.235
Shape		MAC	MAC	MAC
6	19.259	1.00000	0.96786	0.96822
12	19.257	0.96786	1.00000	0.97579
18	19.235	0.96822	0.97579	1.00000

Figure 10. Mode Shape MAC Values For 3 Impact References.

The **3.8**, **9.7**, **13.2** & **19.2 Hz** shapes are strongly correlated from all references. The other shapes are not the same from all references, although some reference pairs do yield the same shapes.

CASE 3: MULTI-REFERENCE AMBIENT TEST

The data acquired for test Case 3 consisted of **126** time waveforms in **9** different measurement sets. Each measurement set consisted of waveforms that were simultaneously acquired; accelerometer measurements for multiple response DOFs, and 3 reference response measurements at locations, **1Z**, **-2Y** & **2Z**.

Each time waveform consisted of **65536** samples of uniformly sampled data with a **0.01** second time increment between samples, giving a total time length of **655.35** seconds. A typical time waveform is shown in Figure 11.

Each measurement set was processed using spectrum averaging, with a **2048** sample spectrum size, **50** spectral averages, and a **Hanning** window to reduce leakage. The

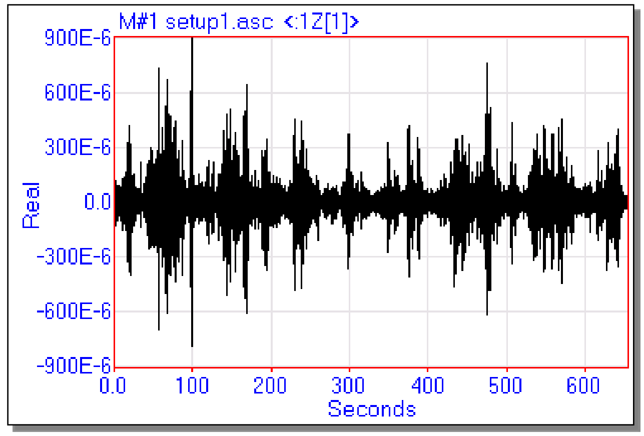


Figure 11. Typical Ambient Time Waveform.

spectrum size and number of averages caused an overlap processing percentage of **69%**. A total of **287** ODS FRFs were calculated; **99** ODS FRFs for each of the **3** references. A typical Ambient ODS FRF is shown in Figure 12.

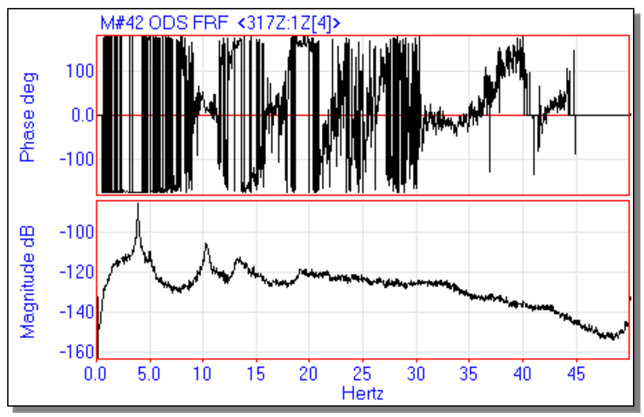


Figure 12. Typical Ambient ODS FRF Measurement.

Mode Shapes & Frequencies

Mode shapes and frequencies were obtained by using a "peak cursor" on each set of ODS FRFs, the same as with the Impact test results. The modal frequencies are listed in Figure 13, and the mode shapes MAC values among the shapes from the 3 references are listed in Figure 14.

Ambient Modal Frequencies (Hz)

	1Z	-2Y	2Z
1	3.855	3.856	3.856
2	4.902	4.902	4.903
3	9.777	9.773	9.766
4	10.257	10.275	10.238
5	12.703	12.329	12.689
6	13.819	13.062	

Figure 13. Modal Frequencies From Ambient Data.

		1Z	-2Y	2Z
	Frequency	3.855	3.856	3.856
Shape		MAC	MAC	MAC
1	3.855	1.00000	0.99203	0.94582
7	3.856	0.99203	1.00000	0.96936
13	3.856	0.94582	0.96936	1.00000
	Frequency	4.902	4.902	4.903
Shape		MAC	MAC	MAC
2	4.902	1.00000	0.98520	0.85204
8	4.902	0.98520	1.00000	0.90357
14	4.903	0.85204	0.90357	1.00000
	Frequency	9.777	9.773	9.766
Shape		MAC	MAC	MAC
3	9.777	1.00000	0.88813	0.90259
9	9.773	0.88813	1.00000	0.89641
15	9.766	0.90259	0.89641	1.00000
	Frequency	10.257	10.275	10.238
Shape		MAC	MAC	MAC
4	10.257	1.00000	0.43877	0.38226
10	10.275	0.43877	1.00000	0.81484
16	10.238	0.38226	0.81484	1.00000
	Frequency	12.703	12.329	12.689
Shape		MAC	MAC	MAC
5	12.703	1.00000	0.56002	0.91335
11	12.329	0.56002	1.00000	0.59083
17	12.689	0.91335	0.59083	1.00000
	Frequency	13.819	13.062	
Shape		MAC	MAC	
6	13.819	1.00000	0.83584	
12	13.062	0.83584	1.00000	

Figure 14. Mode Shape MAC Values
For 3Ambient References.

The **3.8** and **4.9** Hz mode shapes have acceptable MAC values, while the **9.7** and **12.7** Hz shapes are acceptable for references **1Z** and **2Z**. All of the other MAC values indicate questionable results.

COMPARISON OF RESULTS

Mode shapes and modal frequencies were obtained from all three test cases, and for multiple references in each test case. Figure 15 is a listing of typical modal frequencies from each test case

Modal Frequency Comparison (Hz)

Shape	Shaker	Impact	Ambient
1	3.872	3.834	3.856
2	4.821	4.574	4.902
3	9.795	9.741	9.773
4	10.489		10.275
5	12.403		12.329
6	13.086	13.235	13.062
7	17.259	16.748	
8	19.207	19.259	
9	26.665		

Figure 15. Modal Frequencies From 3 Cases.

For these results, only one reference was chosen from each test case. Figure 16 is a listing of MAC values for typical mode shapes obtained from the all three test cases. Mode shapes with high MAC values among references were chosen from each test case.

In general, for the shaker test (Case 1), reference **2Z** yields better mode shapes. For both the Impact and Ambient tests (Cases 2 & 3), reference **1Z** yielded better shapes.

			Shape 1	Shape 9	Shape 15
Frequency			3.872	3.834	3.855
Damping (%)			0.759	0.0	0.0
Shape	Frequency	Damping (%)	MAC	MAC	MAC
1	3.872	0.759	1.00000	0.98114	0.99265
9	3.834	0.0	0.98114	1.00000	0.96480
15	3.855	0.0	0.99265	0.96480	1.00000
			Shape 2	Shape 10	Shape 16
Frequency			4.821	4.574	4.902
Damping (%)			0.506	0.0	0.0
Shape	Frequency	Damping (%)	MAC	MAC	MAC
2	4.821	0.506	1.00000	0.43913	0.87576
10	4.574	0.0	0.43913	1.00000	0.28969
16	4.902	0.0	0.87576	0.28969	1.00000
			Shape 3	Shape 11	Shape 17
Frequency			9.795	9.741	9.777
Damping (%)			0.632	0.0	0.0
Shape	Frequency	Damping (%)	MAC	MAC	MAC
3	9.795	0.632	1.00000	0.92281	0.79888
11	9.741	0.0	0.92281	1.00000	0.77927
17	9.777	0.0	0.79888	0.77927	1.00000
			Shape 6	Shape 12	Shape 20
Frequency			13.086	13.235	13.819
Damping (%)			1.3	0.0	0.0
Shape	Frequency	Damping (%)	MAC	MAC	MAC
6	13.086	1.3	1.00000	0.69121	0.70403
12	13.235	0.0	0.69121	1.00000	0.67104
20	13.819	0.0	0.70403	0.67104	1.00000

Figure 16. Typical MAC Values
Between Shapes From Each Test Case.

CONCLUSIONS

Three different tests were conducted on a concrete and steel highway bridge. The results of these tests were three sets of sampled time waveforms that measured the 3D motion of the bridge at 69 points. Mode shapes of the 4 modes common to all tests are shown in Figure 17.

All three test cases consisted of 9 separate measurement sets of simultaneously acquired data. Case 1 included time waveforms of the forces applied to the bridge during the 2-shaker test. Cases 2 & 3 included reference (fixed) responses at three different DOFs.

The Case 1 data was post-processed using spectrum averaging to reduce noise, and a Hanning window was applied to reduce leakage. Twenty spectrum averages were computed by overlap processing the time data, and 2-reference FRFs and Coherences were computed using MIMO processing.

Cases 2 & 3 were also processed using spectrum averaging to reduce noise. Since Case 2 involved impulse responses, a trigger was used to start the sampling window at the beginning of each impulse. The ambient responses in Case 3 were non-periodic in the sampling window, so a Hanning window was used to reduce leakage.

Case 1 yields the best results, as expected. Whenever FRFs can be measured under controlled and measured excitation conditions, the modal parameters are usually the most accurate.

However, Cases 2 & 3 are more representative of tests that can be performed under a wider set of circumstances. It is not always possible to control or measure the forces exciting a structure. Yet, these results show that meaningful modal parameter estimates can still be obtained from ***response only data***.

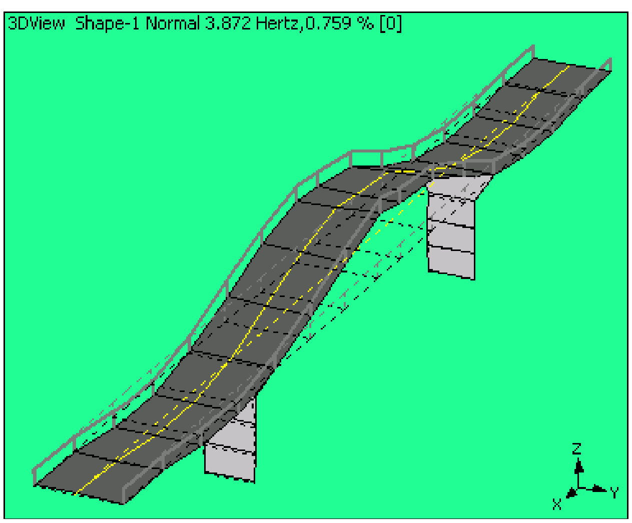
The results show that all three test cases accurately identified the first two modes (**3.8** & **4.8 Hz**). For the higher frequencies, there is still good agreement among some of the results in each case, but not among all three cases.

All of the post-processing and graphics presented in this paper was done with the ME'scopeVES™ software, which is commercially available from Vibrant Technology, Inc.

REFERENCES

[1] Richardson, M. H., Is It A Mode Shape Or An Operating Deflection Shape?, Sound and Vibration Magazine, February, 1997.

[2] Vold, Havard, Schwarz, Brian & Richardson, M. H, Measuring Operating Deflection Shapes Under Non-Stationary Conditions, Proceedings of the 18th International Modal Analysis Conference, Kissimmee, Florida, February, 2000.



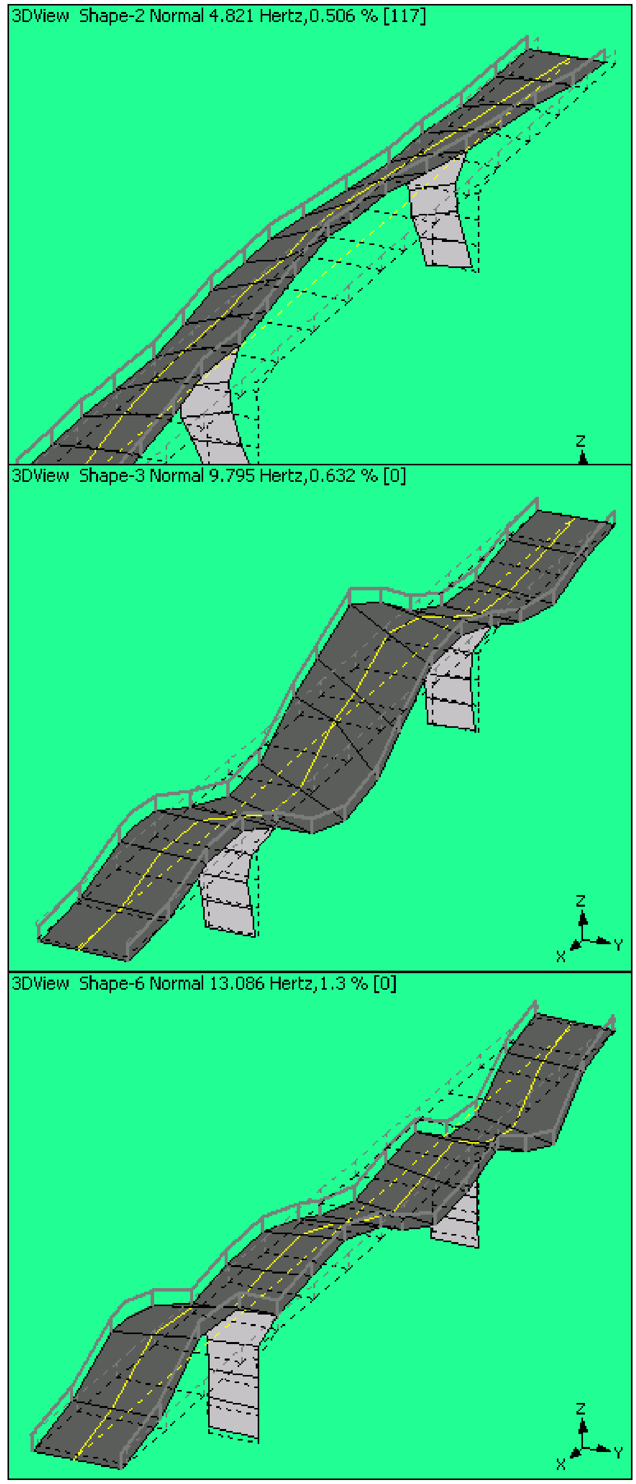


Figure 17. Mode Shapes From All Three Tests.