Measurements Required for Displaying Operating Deflection Shapes

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ABSTRACT

In order to identify and display the operating deflection shapes or operating mode shapes of a machine or structure, data must be acquired and processed so that all shape DOFs have the correct magnitudes and phases relative to one another. If the data is simultaneously acquired the correct magnitudes and phases are guaranteed. However, in the majority of testing situations, data is acquired in multiple measurement sets, and only the data in each measurement set is simultaneously acquired. When using multiple measurement sets, the data must be processed to correct the magnitudes and phases between sets.

In this paper several processing methods involving the Transmissibility, Auto spectrum, and Cross spectrum are presented for calculating the response spectra required for displaying ODS's. The mathematics behind the processing methods is shown, and the sensitivity to noise and structural changes of the methods is also compared. Various ways of scaling data from multiple measurement sets is also discussed.

In addition, it is shown how operating mode shapes are obtained by curve fitting a set of response spectrum data. Results from the various spectrum estimates are compared using data from several bridge tests.

MEASUREMENTS

ODS's are displayed from response only measurements taken from two or more degrees-of-freedom (DOFs) of a machine or structure. This data is typically a measure of surface motion, and can be in displacement, velocity, or acceleration units.

A variety of measurements can be used to display ODS data. If ODS's are displayed directly from time waveform measurements, they can only be used if the data is ***simultaneously acquired***, or when the measurement is ***repeatable***.

The majority of ODS analysis is done using a multi-channel analyzer or data acquisition system that doesn't have a sufficient number of channels to collect all of the data simultaneously. In this case, data must be collected using multiple measurement sets. This places some limits on the way the data can be collected.

When collecting data using multiple measurement sets, two issues must be addressed. First, the phase relationship between measurement sets must be maintained. Second, if the vibration levels change, possibly due to variation in loads during testing, the effects of variations in amplitude must be addressed.

Cross Spectra

The Cross spectrum is computed by multiplying the Fourier Spectrum of a measured response by the complex conjugate of the Fourier Spectrum of a (fixed) reference response measurement. The result is a complex valued measurement that has the magnitude of the (roving) response times the magnitude of the reference response. The resulting phase is the phase difference between the (roving and reference responses.

Cross Spectrum: $$\mathbf{G_{xy}(\omega) = F_x(\omega)F_y^*(\omega)} \qquad (1)$$

The reference response is a measurement taken at the same DOF for all of the measurement sets. To ensure a good signal, the reference DOF should be chosen at a DOF where the machine or structure has lots of motion. A DOF on a fixed machine base is a poor choice, since $\mathbf{F_y^*(\omega) = 0}$ and therefore the cross spectra will also be zero.

A Cross spectrum is good for ensuring that phases match between measurements in multiple measurement sets. Measurement noise can also be reduced by averaging together several Cross spectra taken from the same pair of DOFs.

However, the Cross spectrum is very sensitive to changes in load levels (and therefore amplitude levels) between measurement sets. Since a load level change affects both the roving and reference responses, any change in load causes a ***squared*** change in the Cross spectrum amplitude. Other methods are less sensitive to load changes.

Transmissibility

A Transmissibility measurement is similar to a Frequency Response Function (FRF) measurement, but uses roving and reference response signals instead of a force and response signal pair. The Transmissibility is ***defined*** as the Fourier spectrum of the roving response divided by the Fourier spectrum of the reference response. It is actually computed by dividing the Cross spectrum between the roving and reference responses by the Auto spectrum of the reference response. The resulting measurement provides the motion of each roving DOF ***normalized*** by the motion of the reference DOF.

Transmissibility:

$$\mathbf{H_{xy}(\omega)} = \frac{\mathbf{F_x(\omega)F_y^*(\omega)}}{\mathbf{F_y(\omega)F_y^*(\omega)}} = \mathbf{G_{xy}(\omega)/G_{yy}(\omega)} \tag{2}$$

Where the Auto spectrum of the reference response is:

$$\mathbf{G_{yy}(\omega) = F_y(\omega)F_y^*(\omega)} \tag{3}$$

Transmissibility's ensure a phase match between measurement sets through the use of the Cross spectrum. Variations in load levels are accounted for as well because the roving response is normalized with respect to the reference motion. Noise in the roving response can also be averaged out of Transmissibility through spectrum averaging, as described before.

Problem with Transmissibility's

One difficulty with a Transmissibility measurement is that the peaks in Transmissibility are not evidence of resonances. Rather, resonant frequencies are located at "flat spots" in the Transmissibility.

The figure below shows a response Auto spectrum plotted above Transmissibility. The response Auto spectrum contains resonance peaks. At the frequency of the resonance peak in the Auto spectrum (inside the Band cursor), the Transmissibility has a "flat spot", not a peak. Moreover, the peaks in the Transmissibility do not correspond to resonances but are merely the result of the division of the roving response spectrum by the reference response spectrum at frequencies where the reference response is relatively small.

Responses from Transmissibility's & Auto Spectra

Roving responses can be computed by multiplying a set of Transmissibility's by a reference Auto spectrum.

$$\overline{\mathbf{F_x(\omega)}} = \mathbf{H_{xy}(\omega)}\sqrt{\mathbf{G_{yy}(\omega)}} \tag{4}$$

This method gives the true amount of motion at each point, along with the relative phase information. Noise can be an issue here because the reference Auto spectrum can contain noise.

An alternative method is to use a reference Fourier Spectrum instead of the Auto spectrum,

$$\mathbf{F_x(\omega) = H_{xy}(\omega)F_y(\omega)} \tag{5}$$

This yields a consistent set of Fourier Spectra for all of roving response DOFs. This data can also be Inverse Fourier Transformed to provide a consistent set of response time waveforms.

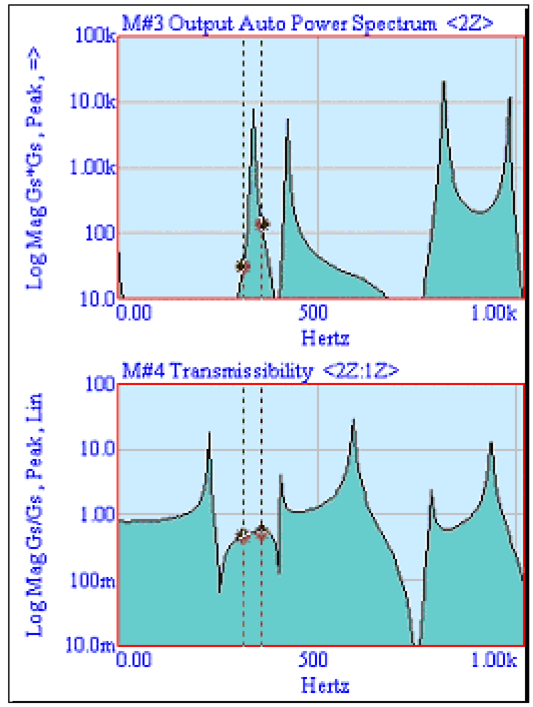

Figure 1. Roving Auto Spectrum & Transmissibility.

ODS FRF

An ODS FRF is another frequency domain function that can be calculated from response only or operating, data. An ODS FRF is computed by replacing the magnitude of the Cross spectrum between a roving and reference responses pair, with the square root of the magnitude of the roving response Auto spectrum. Mathematically, this is equivalent to:

$$\mathbf{ODS\ FRF(\omega)} = \sqrt{\mathbf{G_{xx}(\omega)}}\frac{\mathbf{G_{xy}(\omega)}}{|\mathbf{G_{xy}(\omega)}|} = |\mathbf{F_x(\omega)}|\frac{\mathbf{G_{xy}(\omega)}}{|\mathbf{G_{xy}(\omega)}|} = \overline{\mathbf{F_x(\omega)}} \tag{6}$$

The ODS FRF measures the true amount of motion at each DOF, along with the correct relative phase information between multiple roving responses. Furthermore, the ODS FRF has ***peaks at resonant frequencies***, which makes it easier to identify mode shapes from the response only data.

Also, the ODS FRF has better noise characteristics than a roving response spectrum that is calculated by multiplying a reference Auto spectrum by the Transmissibility. The noise in the roving and reference Auto spectra will generally be about the same. The difference lies in the Cross spectra which will have less noise than a Transmissibility.

Scaling ODS FRFs

The ODS FRF does not account for variations in load level as the Transmissibility does. However, each measurement set of ODS FRFs can be re-scaled to account for variations in load between measurement sets. An effective way of doing this is to create a scale factor (SF_i) for the i^{th} measurement set as the ratio of the average reference Auto spectrum for ***all*** measurement sets divided by the reference Auto spectrum for the i^{th} measurement set.

$$\mathbf{SF_i} = \sqrt{\frac{\sum_{k=1}^{\#\text{Meas Sets}} \left(\sum_{j=\omega_1}^{\omega_2} \mathbf{G_{yy}(\omega_j)_k} \right)}{\sum_{j=\omega_1}^{\omega_2} \mathbf{G_{yy}(\omega_j)_i}}} \qquad (7)$$

This scale factor can be calculated for any frequency range (ω_1, ω_2) of interest, or just at a specific frequency $\omega_1 = \omega_2$.

Coherence

The Coherence function can also be used with operating data to ensure that phases in the measurements are valid. A valid Coherence measurement requires that at least two averages of spectrum data are taken. A Coherence value ***close*** to one (1) at all frequencies is an indication that;

1. The roving and reference responses are ***linearly related***.
2. The roving and reference spectra have ***minimum leakage***.
3. Measurement ***noise is low***, meaning that the signal to noise ratio is high.

Wherever the Coherence is less than one (1), one (or more) of the above conditions is not being met.

Modal Parameter Estimation using Operating Data

A set of ODS FRFS can be curve fit to estimate modal parameters, provided that the following assumption is made.

Flat Force Spectrum: ***If the excitation force spectrum matrix can be assumed to be "relatively flat" over in the frequency range of the modes of interest, then ODS FRFs can be curve fit using an FRF curve fitting model.***

Modal parameter estimation is based upon the use of an analytical model for an FRF. During curve fitting, the modal frequency, damping and mode shape component (residue) are estimated for each mode by matching the FRF model to experimental data, usually in a least squared error sense. (Equivalently, and analytical expression for the Impulse Response Function, or IRF, can be curve fit to experimental impulse response data.)

In order to curve fit operating data using an FRF model, the following assumptions must be met;

1. The dynamic behavior of the machine or structure adequately satisfies a set of ***linear***, ***stationary*** (non-time varying), ***2nd order*** differential equations.
2. Maxwell's ***reciprocity***, or symmetry is valid.

In the frequency domain, meeting the above assumptions is equivalent to satisfying an FRF matrix model. The FRF matrix model contains FRFs, Fourier spectra of applied forces, and Fourier spectra of resulting displacement responses. Each element of the FRF matrix model can be expressed as

$$\mathbf{F_x(j\omega) = H_{x,f}(j\omega)F_f(j\omega)} \qquad (8)$$

where:

$\mathbf{F_x(j\omega)} =$ Fourier spectrum of the response.

$\mathbf{F_f(j\omega)} =$ Fourier spectrum of the force.

$\mathbf{H_{x,f}(j\omega)} =$ FRF between the force and response DOFs.

If it is assumed that the Auto spectrum of the excitation force (or forces) is flat in the frequency range surrounding a mode, then the magnitude of the Fourier spectrum of the force can be represented by a constant value.

$$\mathbf{|F_f(j\omega)| = C} \qquad (9)$$

The structural response then becomes proportional to the FRF,

$$\mathbf{F_x(j\omega) \propto CH_{x,f}(j\omega)} \qquad (10)$$

If the Spectrum of the applied forces is flat in the region of a mode, then the frequency, damping and magnitude of the mode is preserved between the FRF and the ODS.

The simplest techniques used to estimate modal parameters are SDOF methods. These methods assume only one mode dominates at a frequency and there are a variety of SDOF methods that can be used to estimate the modal parameters that work with both FRFs and ODS FRFs. Reference [1] compares curve fitting results using two popular MDOF methods, the Complex Exponential method and the Polynomial method.

Example 1: Bridge Data

In this example, some experimental data taken from a highway Bridge, that has been documented in previous references [1]-[3], will be post processed to calculate the force spectra of the unmeasured forces, and to compare examine the noise content the ODS FRF versus Transmissibility.

Three tests were performed on the bridge, as documented in [3]. The first test was a traditional modal test using two shakers driven by random signals for excitation. FRFs were calculated and curve fit to obtain experimental modal parameters. In the second test, excitation was provided by impacting the bridge (separately at three different locations), but only bridge responses were measured. In the third test, only the ambient response to bridge traffic was measured.

Figure 1 shows a 3 by 2 matrix of FRF measurements taken from the bridge during the multi-shaker test.

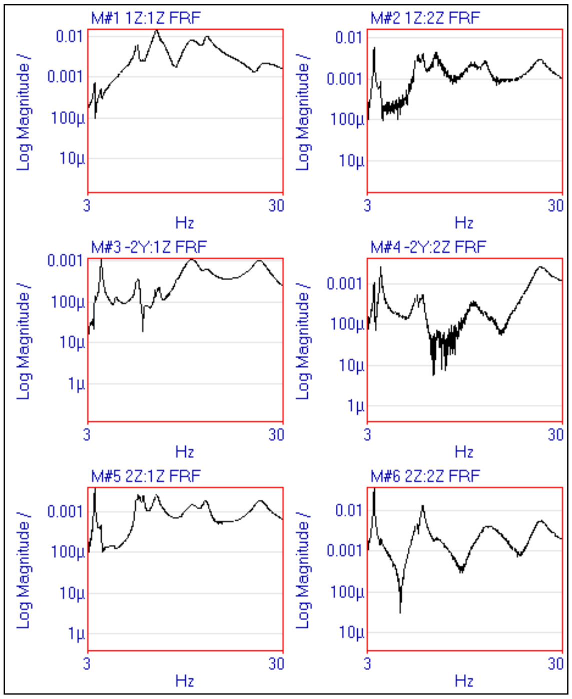

Figure 1. 3 by 2 FRF Matrix from Z24 Bridge.

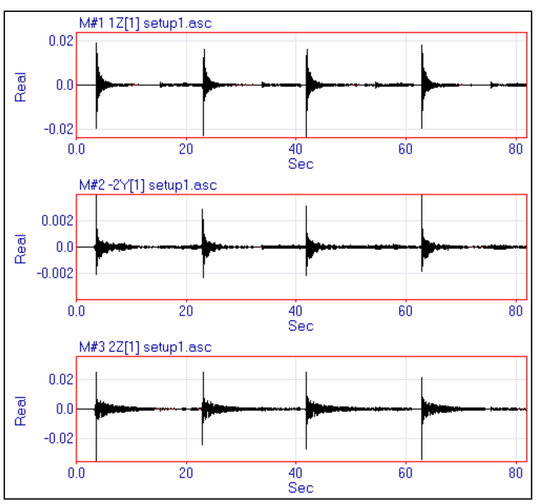

Figure 2. Impact Responses.

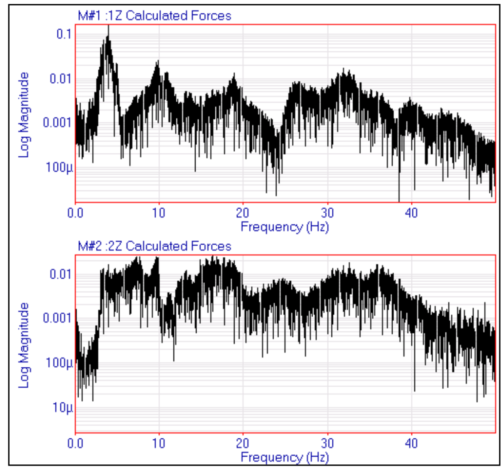

Figure 3. Calculated Impact Force Spectra.

Figure 2 shows typical impact responses taken during the one of the impact tests. Figure 3 shows the (unmeasured) force spectra at DOFs 1Z and 2Z calculated from the data n Figures 1 & 2.

Clearly, the force spectra are not "flat" over the entire frequency span, but they do have sufficient levels of the band (3 to 30 Hz) where the modes where excited during the shaker test.

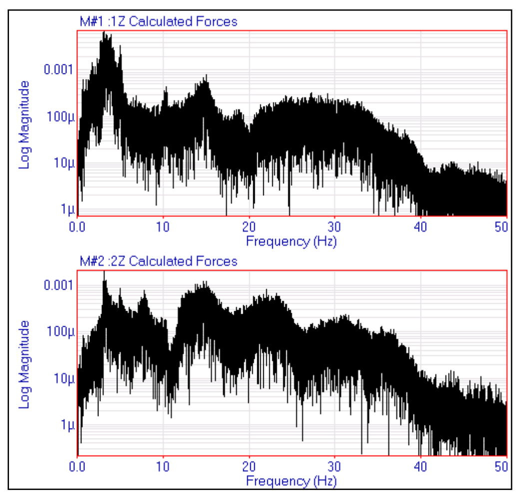

Figure 4. Calculated Ambient Force Spectra.

Figure 4. shows the calculated force spectra of the ambient forces, using ambient response data and the FRF model in Figure 1.

Comparing these to the impact spectra, it is clear that the ambient force spectra have more noise and also violate the "flat spectrum" assumption.

Nevertheless, the ODS FRF and Transmissibility can still be calculated from the response only data. Some results are shown in Figure 5. The ODS FRFs are on the left, and Transmissibility based responses are on the right. Although the results are the same for 1Z:1Z, in the other cases, the Transmissibility based responses have more noise in them.

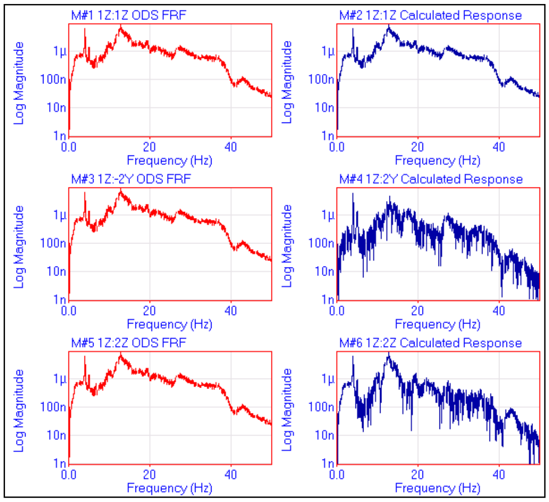

Figure 5. ODS FRFs versus Transmissibility Responses.

Example 2: Modes From Response Only Data

In this example, a set of single reference experimental FRFs is used together with a synthesized random time domain force signal to calculate a set of forced responses for a beam structure. Then, a set of ODS FRFs is calculated from the random response only data. Finally, the set of ODS FRFs is curve fit and the response only modal parameters are compared with the experimental modes obtained from the FRFs,

Figure 6 contains some of the typical FRFs measured on the beam.

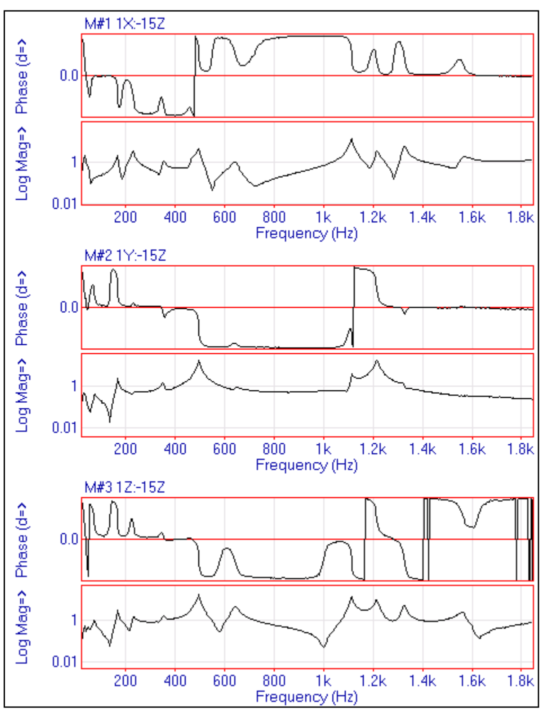

Figure 6. Typical FRFs from the Beam.

A total of 99 FRFs were measured, in three directions at 33 points. The synthesized random excitation signal is shown in Figure 7. Its spectrum has unit amplitude and random phase. The time waveform has 82000 samples over a time length of

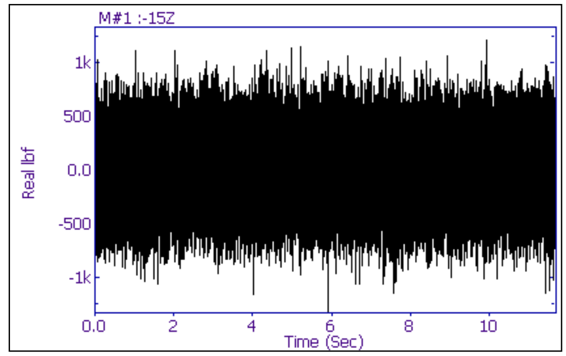

Figure 7. Random Excitation Force Signal.

99 forced response time waveforms were calculated using the data in Figures 6 & 7. These random responses were then used to calculate a set of ODS FRFs using the response at DOF 15Z as the reference response.

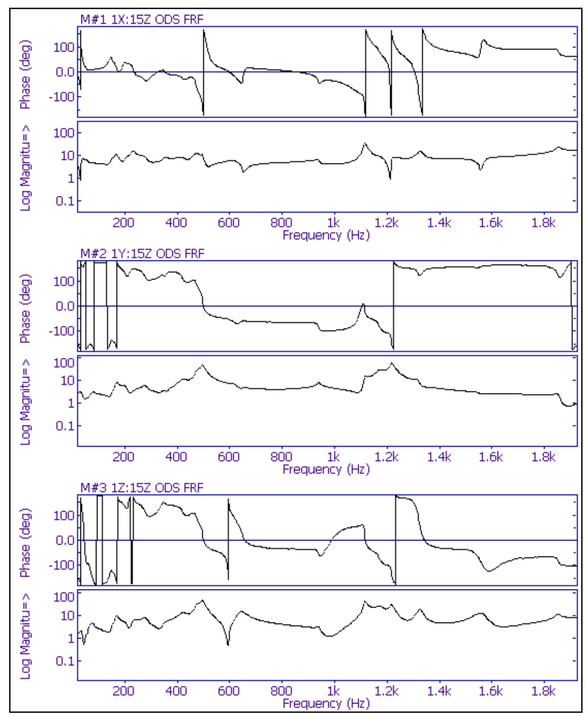

Figure 8. Bode Plots of Typical ODS FRFs.

Some of the 99 ODS FRFs are shown in Figure 8. Before curve fitting, these measurements were inverse FFT'd and multiplied by an exponential window to insure that they took the form of decayed sinusoidal functions, similar to impulse responses. This was followed by an FFT operation to yield a set of measurements that are similar to FRFs, hence can be curve fit using an FRF model. Table 1 contains the modal frequency estimates for the two sets of experimental data. Table 2 contains the MAC values of the mode shape estimates.

Mode	FRF Frequency (Hz)	ODS FRF Frequency (Hz)
1	165.0	165.0
2	224.6	224.5
3	347.9	346.7
4	460.4	473.0
5	493.0	493.6
6	635.5	633.7
7	1109.0	1111.0
8	1211.0	1215.0
9	1323.0	1319.0
10	1557.0	1547.0

Table 1. Modal Frequency Comparison.

Mode	Modal Assurance Criterion
1	0.965
2	0.024
3	0.986
4	0.527
5	0.964
6	0.932
7	0.964
8	0.955
9	0.961
10	0.918

Table 2. Modal Shape Comparison.

CONCLUSIONS

It was first shown that ODS FRF estimates are more noise-free than Transmissibility's multiplied by the reference Auto spectrum. This is because the denominator of the Transmissibility also contains the measurement noise of the reference Auto spectrum, which adds more noise to the final result. The ODS FRF only contains the additive noise of the roving response Auto spectrum.

Next, it was shown that modal parameters can be obtained by using an FRF model to curve fit a set of ODS FRFs. These results were compared to the parameter estimates obtained from a set of FRFs from the same structure.

Obtaining modal parameter estimates from response only data has been dubbed ***Operational Modal***. It was shown that this approach has limitations, and produces noisier measurements and consequently less accurate modal parameters than the more traditional FRF based technique.

Nevertheless, in situations where the excitation forces cannot be measured, the use of response only data can still provide some useful results.

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