Using Impulse Response Functions to Evaluate Baseball Bats

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ABSTRACT
In this paper, we demonstrate a new approach to evaluating the dynamic behavior of baseball bats. Using this approach we can compare the ball striking one spot on a bat versus another, and also compare the performance of one bat design versus another. We can quantify a ball striking the "sweet spot" on a bat versus the "sting" felt at the handle when the ball strikes the wrong spot.

This new approach uses IRFs (Impulse Response Functions), which simulate the impact of a ball striking a bat. The IRFs are synthesized using an experimentally derived modal model of the bat. The modal data is obtained by a standard roving impact test of the bat.

Two different quantitative measures are used for comparing IRFs. One measure is called the SCC (Shape Correlation Coefficient). It is a numerical measure of the *co-linearity* of two deflection shapes. It is the same as the FRAC (Frequency Response Assurance Criterion) calculation, but we apply it to the time domain IRFs as well as frequency domain FRFs.

The second numerical measure is called the SPD (Shape Percent Difference). The SPD is a numerical measure of the *difference* between two deflection shapes. It not only indicates when two shapes are different, but quantifies the magnitude of their difference.

The IRFs of several different baseball bats are compared using both the SCC and SPD calculations over all time samples. These measures show graphically how similar or different the impulse responses of different bats are.

INTRODUCTION

This research was conducted to develop new methods for comparing the performance of baseball bats. The approach taken involved the following steps;

1) Perform a roving impact test on each bat to obtain a calibrated set of FRFs.
2) Curve fit the FRFs to obtain experimental mode shapes.
3) Scale the mode shapes to obtain a modal model.
4) Synthesize acceleration, velocity, or displacement FRFs using the modal model.
5) Inverse FFT the FRFs to obtain a set of IRFs.
6) Compare the impulse responses of the bats at the handle due to an impulsive force on the barrel.

After a modal model was obtained for each bat, its synthesized FRFs were compared with the original FRF test data using SCC and SPD calculations. These calculations were done at each frequency sample to compare the experimental and synthesized FRFs. Additionally, both sets of FRFs were Inverse FFT'd and their corresponding IRFs also compared using SCC and SPD calculations at each time sample. These comparisons validated the accuracy of the modal models.

Finally, the IRFs of the different bats were compared using SCC and SPD calculations. These results quantified not only the similarity or difference of the bat IRFs, but they also showed which bats had a higher level of vibration at the handle due to an impulsive force on the barrel.

Deflection Shape

A deflection shape is defined as the ***deflection of two or more points on a structure***. Stated differently, a deflection shape is the deflection of one point relative to all others. Deflection is a vector quantity, meaning that each of its components has both location and direction associated with it. Deflection measured at a point in a specific direction is called a DOF (Degree of Freedom) [2].

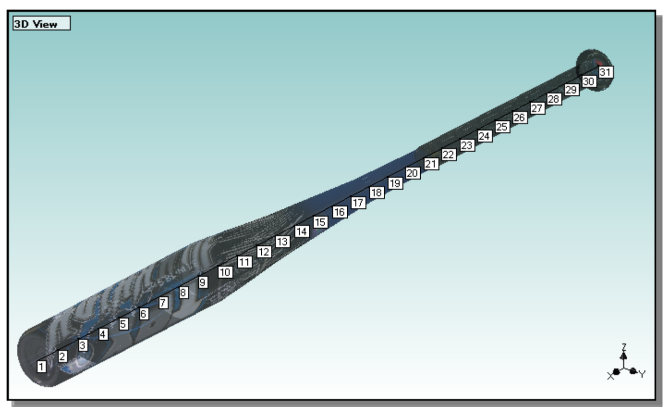


Figure 1. Baseball Bat Showing Test Points

A deflection shape can be defined from any vibration data, either at a moment in time, or at a specific frequency. Different types of time domain data, e.g. random, impulsive, or sinusoidal, or different frequency domain functions [3], e.g. Linear spectra (FFTs), Auto & Cross spectra, FRFs, Transmissibility's, or ODS FRFs can be used to define an ODS.

Data Acquisition

Five different bats were tested using the roving impact hammer method. A (fixed) reference accelerometer was attached to each bat in approximately the same position on the barrel. Then each bat was impacted with an instrumented hammer at 1-inch intervals along the length of the bat from one end to the other.

FRFs were then calculated from the impulse force and accelerometer response signals. Each FRF had 2100 uniform frequency samples, over a span from DC (0Hz) to 2998.6Hz. A typical FRF measurement is shown in Figure 2.

Figure 2. Typical FRF Measurement

Two of the bats were 31 inches long, and a total of 31 FRFs were measured on them. Three of the bats were 29 inches long, and a total of 29 FRFs were measured on them.

Each set of FRFs was curve fit to obtain the experimental modes of the bat. The modal frequency & damping of the bats are shown in Figure 3. These results clearly show that the resonances of baseball bats can be quite different from one another. Modal frequencies range from a **low of 75.6 Hz to a high of 2958 Hz**. Likewise, modal damping decay coefficients range from a low **of 0.47 Hz to a high of 39 Hz.** Typical mode shapes from one of the bats are shown in Figure 4.

Two different numerical methods were used to compare two sets of IRFs. (These same calculations can also be done on two sets of FRFs.) One set of IRFs is called the Baseline IRFs and the other is called the Comparison IRFs. Each set of IRFs contains a deflection shape at each sampled time value. To compare two sets of IRFs, their deflection shapes are compared at each time sample, using two different methods.

	Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	MPC
1	No	75.601	23.804	Hz	30.033	0.94435
2	No	381.21	33.203	Hz	8.6771	0.46565
3	No	1125.1	39.698	Hz	3.5261	0.43255
4	No	1550.4	16.706	Hz	1.0775	0.97044
5	No	1933.2	71.763	Hz	3.7096	0.77813
6	No	2163.2	33.667	Hz	1.5562	0.92827

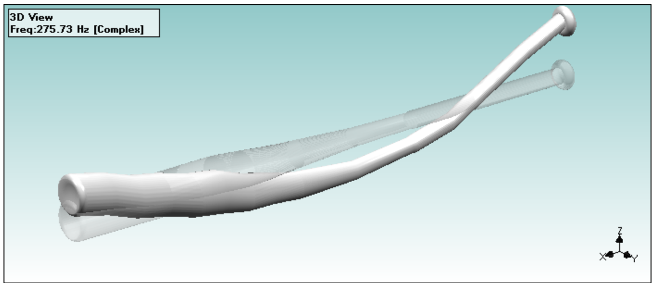
	Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	MPC
1	No	222.59	1.1872	Hz	0.53335	0.073325
2	No	724.71	4.7861	Hz	0.6604	0.80425
3	No	1293.8	9.493	Hz	0.73372	0.49711
4	No	1725	18.082	Hz	1.0481	0.95829
5	No	2111.9	35.935	Hz	1.7013	0.92654
6	No	2958.4	28.862	Hz	0.97555	0.83938

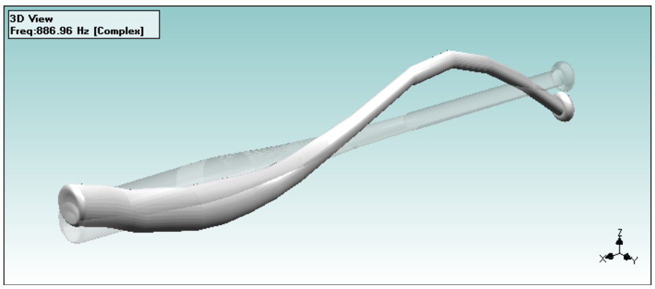
	Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	MPC
1	No	275.73	0.49522	Hz	0.1796	0.7969
2	No	886.96	2.7507	Hz	0.31013	0.99917
3	No	1616.1	7.0195	Hz	0.43434	0.9974
4	No	1969.5	25.245	Hz	1.2817	0.88176
5	No	2712.1	12.312	Hz	0.45397	0.98547
6	No	2896.4	12.173	Hz	0.42027	0.90142

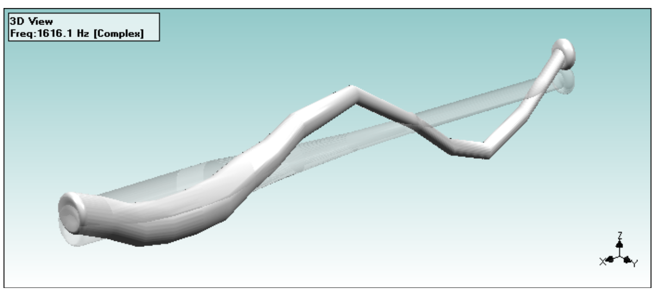
	Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	MPC
1	No	286.87	0.60885	Hz	0.21224	0.83297
2	No	999.06	0.89427	Hz	0.089511	0.95933
3	No	1002.3	1.1487	Hz	0.11461	0.98856
4	No	1933	2.5333	Hz	0.13106	0.99398
5	No	1965.5	2.7897	Hz	0.14193	0.99347
6	No	2564.6	26.294	Hz	1.0252	0.98555

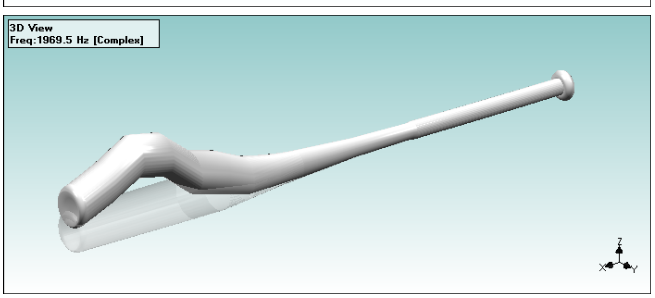
	Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	MPC
1	No	201.17	0.74071	Hz	0.36819	0.95299
2	No	738.78	0.47433	Hz	0.064205	0.77932
3	No	1552.6	0.98221	Hz	0.063262	0.98465
4	No	2026.7	5.1968	Hz	0.25641	0.99921
5	No	2423.3	2.1464	Hz	0.088571	0.99382
6	No	2863.7	34.763	Hz	1.2139	0.97264

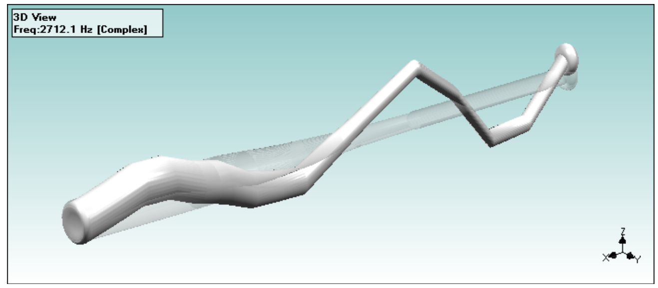
Figure 3. Modal Frequencies & Damping











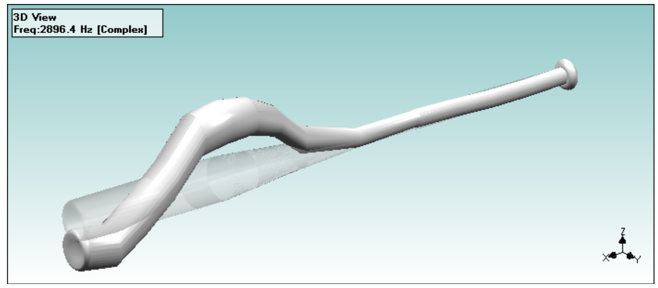


Figure 4. Typical Bat Mode Shapes

One method is called the SCC (Shape Correlation Coefficient), and the other is the SPD (Shape Percent Difference). Both of these calculations yield a percentage value. The SCC measures the *co-linearity* of the two deflection shapes at each time sample. The SPD measures the percent *difference* between the deflection shape of the Baseline IRFs and the deflection shape of the Comparison IRFs at each time sample.

SCC (Shape Correlation Coefficient)

A deflection shape is in general, a *complex* vector with two or more components, each component having a *magnitude & phase*. In this application, each component of the deflection shape is obtained from an IRF at a specific time sample.

The SCC measures the similarity between two complex vectors. When this coefficient is used to compare two mode shapes, it is called a MAC (Modal Assurance Criterion) [1]. The SCC is defined as;

$$\mathrm{SCC} = \frac{\left\| \mathrm{DS_C} \circ \mathrm{DS_B^*} \right\|}{\left\| \mathrm{DS_C} \right\| \left\| \mathrm{DS_B} \right\|}$$

where: $\mathrm{DS_B}$ = Baseline deflection shape

$\mathrm{DS_C}$ = Comparison deflection shape

$\mathrm{DS_B^*}$ = complex conjugate of DS_B

$\| \; \|$ indicates the magnitude squared

$\circ$ indicates the DOT product between two vectors

The SCC is a *normalized DOT product* between two complex vectors. It has values between 0 and 1. A value of 1 indicates that the two deflection shapes are the same. As a "*rule of thumb*", an SCC value greater than 0.90 indicates that two shapes are similar. *A value less than 0.90 indicates that two shapes are different.*

The SCC provides a single numerical measure of the similarity of two deflection shapes. It measures whether or not two vectors are *co-linear*, or lie along the same line. If two deflection shapes are co-linear but have different magnitudes, the SCC will still have a value of 1. Therefore, a measure of the *difference* in the magnitudes between two deflection shapes is required.

SPD (Shape Percent Difference)
A direct measure of the difference between two deflection shapes is the SPD (Shape Percent Difference).

$$SPD = \frac{|DS_C - DS_B|}{|DS_B|}$$

where: DS_B = Baseline deflection shape

DS_C = Comparison deflection shape

$|\ |$ indicates the magnitude of the vector

If $|DS_C| < |DS_B|$ then the SPD is negative

The SPD measures the percentage difference between the two shapes relative to the Baseline deflection shape. A value of 0 indicates no difference, and a value of 1 is a 100% difference between the two shapes.

To summarize, if two deflection shapes are the same, their *SCC will be at or near 1*, and their *SPD will be at or near 0*. As the two shapes become different from one another, the *SCC will decrease toward 0*, and the *SPD* will *increase* or *decrease* depending on which shape, Comparison or the Baseline shape, has a greater magnitude.

Mode Shape Interpolation
In order to compare deflection shapes between all sets of IRFs, they all have to have a common set of DOFs. Two of the bats were tested at 31 points spaced 1 inch apart, and the other three bats were tested at 29 points spaced 1 inch apart.

To obtain five sets of mode shapes with common DOFs, the mode shapes with 29 DOFs were *interpolated* so that they contained 31 evenly spaced DOFs. With each modal model having 31 DOFs, they could then be used to synthesize FRFs (and obtain IRFs) with the same number of DOFs.

The mode shapes with 29 DOFs were interpolated into 31 DOFs by using geometric interpolation. Geometric interpolation uses a weighted summation of the mode shape components at 29 evenly spaced DOFs to calculate new mode shape components at 31 evenly spaced DOFs. A typical 29 DOF mode shape and its interpolated 31 DOF mode shape are shown in Figure 5.

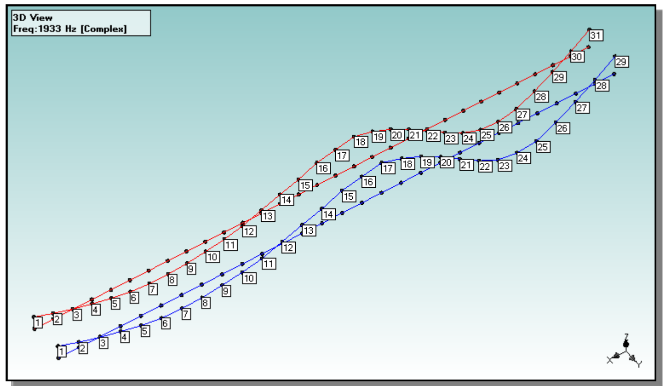
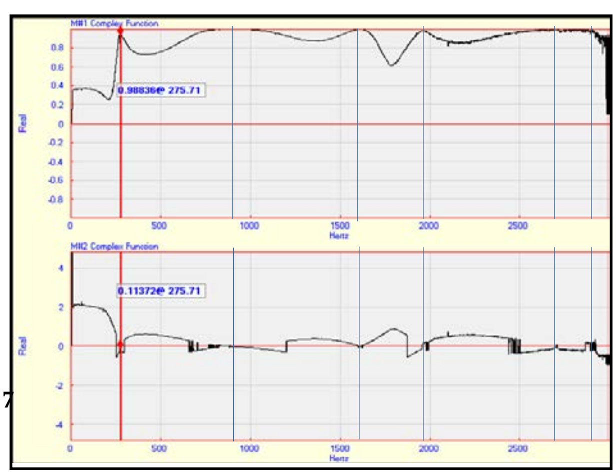


Figure 5. Mode Shape Interpolated From 29 to 31 DOFs

Synthesized Vs. Experimental FRFs
To compare the IRFs of the five bats, FRFs were first synthesized for each bat using its modal model (scaled mode shapes). Then the FRFs were Inverse FFT'd to obtain the IRFs. However, before calculating the IRFs, the synthesized FRFs were compared with the experimental FRFs, both visually and using SCC and SPD. A typical result is shown in Figure 6. Two FRFs are overlaid in Figure 6A. SCC and SPD values comparing all 31 FRFs are displayed in Figure 6B.

Figure 6B shows that when the SCC is close to "1", the SPD is also close to "0", indicating that *all 31 synthesized and experimental FRFs* are closely matched.

At the first cursor position (275.71Hz) the SCC value is **"0.99"** and the SPD value is "0.11". The additional cursor positions show the frequencies of the other 5 modes in the model. It is evident that the SCC is near 1.0, and the SPD is near 0.0 at all of these frequencies.



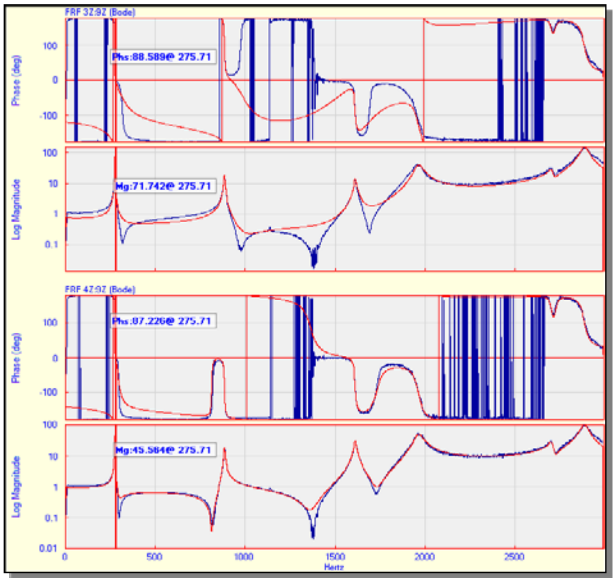


Figure 6A. Two Synthesized & Experimental FRFs Overlaid

Figure 6B. SCC (top) & SPD (bottom) from 31 FRFs

Synthesized Vs. Experimental IRFs

Figure 7 shows the IRF comparisons for the same bat as shown in Figure 6. (The IRFs are compared at each time sample, whereas the FRFs were compared at each frequency.) The IRFs match differently than the FRFs, but the result is the same.

In the cursor band **(0 to 0.1 sec)** shown in Figure 7B, the maximum of the SCC **is 0.99**, the minimum is **0.19**, and the mean is **"0.90".** The maximum of the SPD is **"-0.14"** the minimum is **0.05** and the mean is **"-0.0012"** indicating that *all 31 synthesized and experimental IRFs* are closely matched. (For the SPD calculations, the deflection shape from the *peak response* of the IRFs was used to normalize the shape difference.)

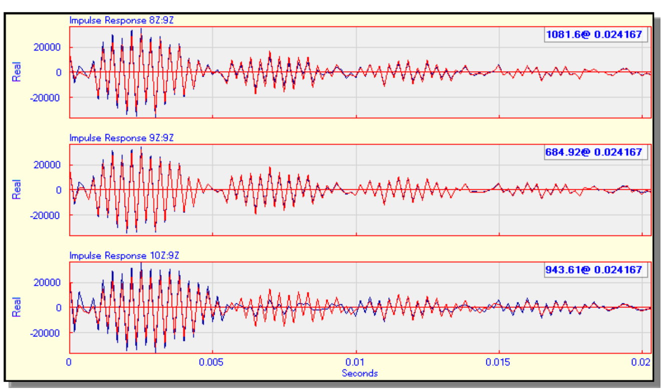


Figure 7A. Three Synthesized & Experimental IRFs Overlaid

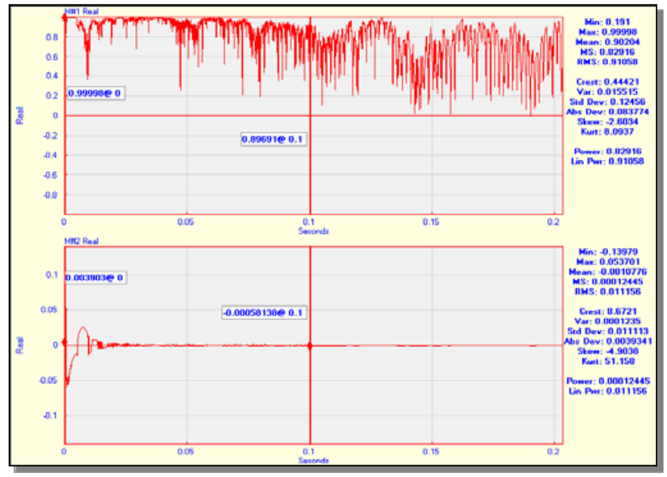


Figure 7B.SCC (top) & SPD (bottom) from 31 IRFs

The FRF and IRF comparisons both confirm that a modal model is sufficiently accurate so that synthesized IRFs can be used for comparing the impulse responses of the different bats.

The advantage of using the modal model is that impulse responses can be calculated between *any pair of DOFs* (impact and response DOFs) where the mode shapes are defined. Hence, an impact force could be simulated at any DOF on the barrel, and the response simulated at any DOF on the handle.

The experimental IRFs themselves could also be used for comparisons, but experimental data only contains a limited number of reference (impact) DOFs, usually only one.

Comparison of IRFs

Figure 8 shows synthesized IRFs of all five bats. Each IRF is the response at 25Z (the bat handle) due to an impulsive force applied at 9Z (on the barrel of the bat). The **initial 0.2 seconds** of each IRF are shown. Comparing the IRFs makes it clear that the bats respond quite differently. The vibration of Bat#1 (on the left) is completely damped out while Bat#5 (on the right) still has substantial vibration after 0.2 seconds. Furthermore, the IRFs make it clear that the peak responses (in g's/lb) of the Bats are different. For example, the peak response of Bat#2 is much less than the response of Bat#4.

Comparison of Deflection Shapes

Figure 9 shows the SCC and SPD of the deflection shapes of Bat#1 compared with all five Bats. When Bat#1 is compared with itself (on the left), its SCC values (upper graph) are 1, and its SPD values (lower graph) are 0.

The IRFs of the other four Bats correlate well with Bat#1 near the beginning of the impulse responses. However, the shapes of the other Bats soon digress from the shapes of Bat#1, indicated by SCC values **less than 0.5**. Moreover, when the SCC values are near 1, the SPD values **are also high (0.5 or 50%)**, indicating that the deflection shapes of

Bat#1 are quite different from the deflection shapes of the other Bats.

CONCLUSIONS

A new method for comparing the impulse responses of baseball bats was introduced in this paper. It is based on comparing the deflection shapes between two sets of IRFs.

IRFs contain the combined response all of modes that are excited, which depends on their mode shapes, frequencies, damping, and the impact and response DOFs. IRFs were synthesized for each bat using experimentally derived modal data. Using a modal model provides the flexibility of synthesizing IRFs between *any pair of DOFs* of the bat where the mode shapes are defined.

Two measures for comparing deflection shapes were introduced. The SCC (shape correlation coefficient) quantifies the *co-linearity* between two shapes, and the SPD (shape percent difference) measures the *difference* between two shapes. Both of these measures provided clear graphic evidence of the differences between the impulse responses of five different baseball bats.

Two other innovations were used in this research. First, geometric interpolation was used to create mode shape components for three of the bats to match the same DOFs of the other two bats. Secondly, SCC and SPD were used to verify that the FRFs and IRFs synthesized from the modal models correlated well with the experimental data.

This quantitative approach to comparing the dynamic behavior of structures should be useful in many other applications. Once a modal model is validated, the mode shapes themselves can be integrated or differentiated and then used to synthesize and compare the displacement, velocity, and acceleration responses of structures.

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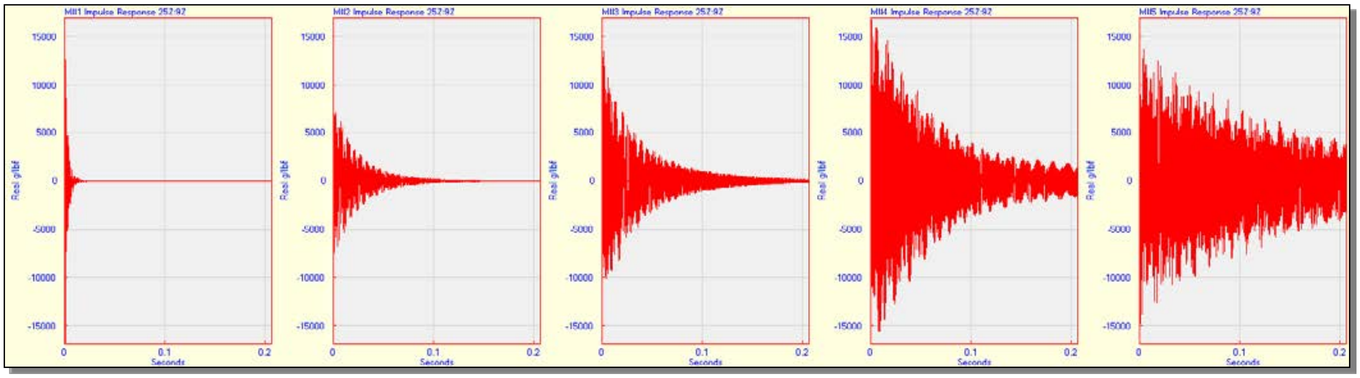

Figure 8. Synthesized IRFs (Response at 25Z due to impulsive force at 9Z)

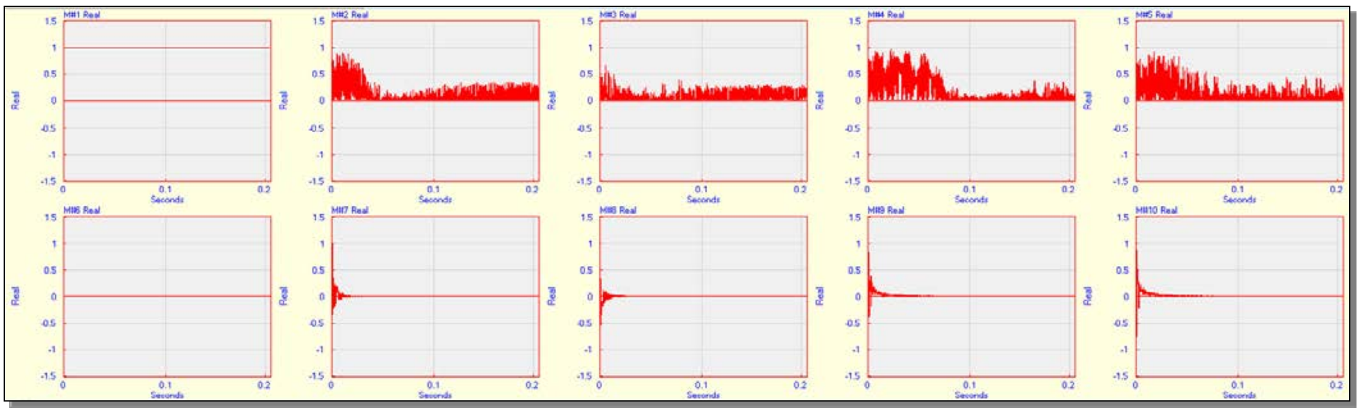

Figure 9. SCC (top) & SPD (bottom) Bat #1 Compared With Five Bats