A New Measure of Shape Difference

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ABSTRACT

The Modal Assurance Criterion (MAC) is currently the most popular method for measuring whether or not two mode shapes are strongly correlated. In fact, MAC can be applied to any two sets of shape data, e.g. mode shapes, Operating Deflection Shapes (ODS's), or two time or frequency domain waveforms. When used to compare two Frequency Response Functions (FRFs), MAC has been renamed FRAC [3].

MAC values range between **0 & 1**. If **MAC = 1** , the two shapes are ***identical***. A ***"rule of thumb"*** is that two shapes are ***similar*** or ***strongly correlated*** if **MAC > 0.9,** and they are ***different*** or ***weakly correlated*** if **MAC < 0.9**.

MAC is a measure of the ***co-linearity*** of two shapes. That is, it measures whether or not two shapes lie together on the same straight line. MAC has two limitations however;

1) MAC does not measure the ***difference in value*** of two shapes.

2) MAC requires ***at least two*** shape components. MAC = 1 ***always*** for two shapes with ***one matching component*** (two scalars).

In this paper, a new measure, called the **Shape Difference Indicator (SDI)**, is introduced which overcomes the two limitations of MAC. This new measure is more useful for machinery and structural health monitoring applications where, for instance, changes in vibration levels or temperatures are typically used to detect a fault.

An example is given showing how SDI indicates that shape pairs are different even when their MAC values indicate that they are the same, i.e. they are co-linear. A second example shows how SDI can be used not only to detect a fault, but also to correctly identify the fault by comparing its shape values with those in a database of known fault conditions.

KEY WORDS

Modal Assurance Criterion (MAC)
Shape Difference Indicator (SDI)
Operating Deflection Shape (ODS)
Experimental Modal Analysis (EMA) Mode Shape
Finite Element Analysis (FEA) Mode Shape

INTRODUCTION

Figure 1 shows a photo model of the Jim Beam test article. This structure is made up of three aluminum plates fastened together with six Allen screws. Three screws attach the top plate to the vertical plate, and three screws attach the bottom plate to the vertical plate.

A set of 99 Frequency Response Functions (FRFs) was acquired during an impact test of the Jim Beam structure. An instrumented impact hammer was used to measure the impact force, and a tri-axial accelerometer was used to measure the structural responses to the impact force. The structure was impacted at a fixed DOF (15Z), and the accelerometer was attached to each of 33 different Points for each impact of the structure. During each impact, four signals (the force and three acceleration responses) were fed into a 4-channel FFT-based spectrum analyzer where three FRFs were calculated.

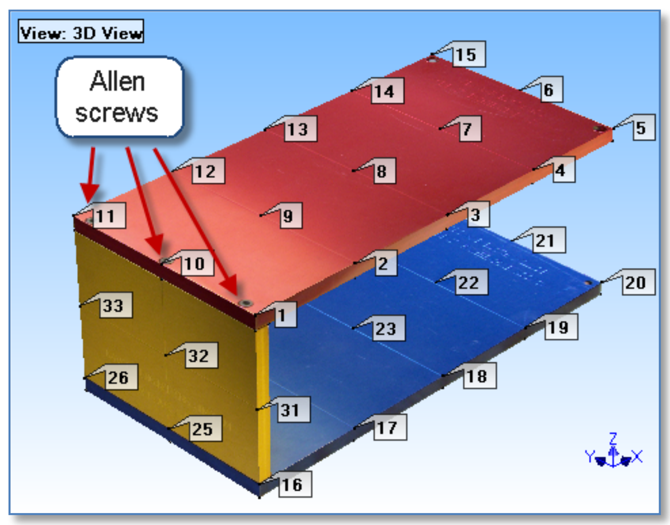


Figure 1 Jim Beam Structure.

The imaginary parts of the 99 FRFs are overlaid in Figure 2. They clearly show the presence of 10 resonance peaks which indicate that at least 10 modes were excited.

The FRFs were then curve fit using a least-squared-error curve fitting method to estimate the modal parameters for the 10 modes. These parameters are listed in Figure 3.

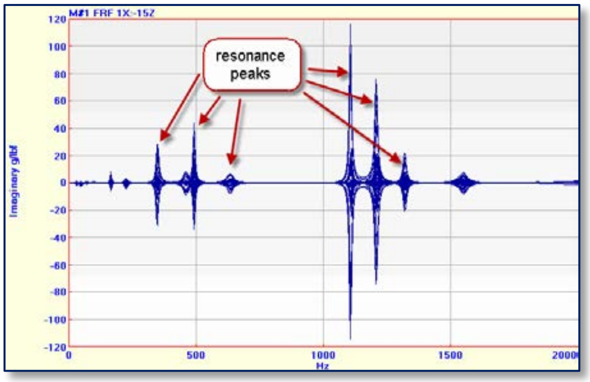
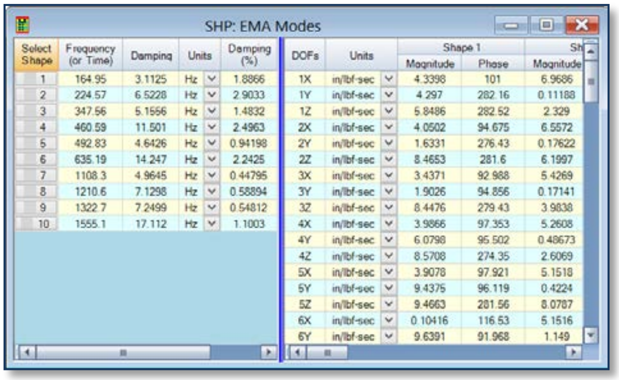


Figure 2. Imaginary Part of 99 FRFs Overlaid.



SHP: EMA Modes

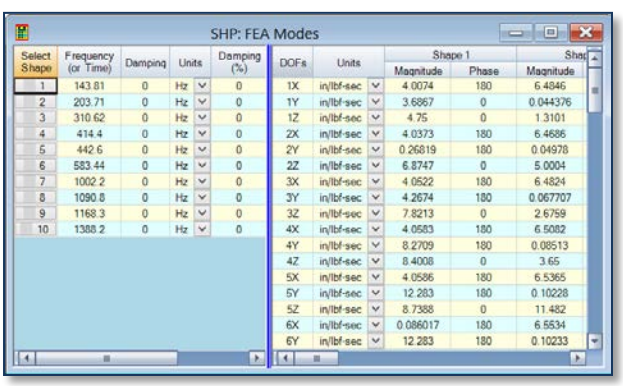
Select Shape	Frequency (or Time)	Damping	Units	Damping (%)
1	164.95	3.1125	Hz	1.8866
2	224.57	6.5228	Hz	2.9033
3	347.56	5.1556	Hz	1.4832
4	460.59	11.501	Hz	2.4963
5	492.83	4.6426	Hz	0.94198
6	635.19	14.247	Hz	2.2425
7	1108.3	4.9645	Hz	0.44795
8	1210.6	7.1298	Hz	0.58894
9	1322.7	7.2499	Hz	0.54812
10	1555.1	17.112	Hz	1.1003

DOFs	Units	Shape 1 Magnitude	Shape 1 Phase	Sh Magnitude
1X	in/lbf-sec	4.3398	101	6.9686
1Y	in/lbf-sec	4.297	282.16	0.11188
1Z	in/lbf-sec	5.8486	282.52	2.329
2X	in/lbf-sec	4.0502	94.675	6.5572
2Y	in/lbf-sec	1.6331	276.43	0.17622
2Z	in/lbf-sec	8.4653	281.6	6.1997
3X	in/lbf-sec	3.4371	92.988	5.4269
3Y	in/lbf-sec	1.9026	94.856	0.17141
3Z	in/lbf-sec	8.4476	279.43	3.9838
4X	in/lbf-sec	3.9866	97.353	5.2608
4Y	in/lbf-sec	6.0798	95.502	0.48673
4Z	in/lbf-sec	8.5708	274.35	2.6069
5X	in/lbf-sec	3.9078	97.921	5.1518
5Y	in/lbf-sec	9.4375	96.119	0.4224
5Z	in/lbf-sec	9.4663	281.56	8.0787
6X	in/lbf-sec	0.10416	116.53	5.1516
6Y	in/lbf-sec	9.6391	91.968	1.149

Figure 3. EMA Mode Shapes.

An FEA model was also created from the Jim Beam photo model [2], and it was solved for its FEA mode shapes, or ***eigenvalues & eigenvectors***. The FEA mode shapes are listed in Figure 4.

The Jim Beam FEA model was meshed to provide more DOFs before solving for its FEA modes. Hence, the FEA mode shapes had 630 DOFs in them, but only 99 matched with the DOFs of the EMA mode shapes. By comparing the shape component values in Figures 3 & 4, it is clear that the EMA shapes have ***different value***s in them than the FEA shapes.



SHP: FEA Modes

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)
1	143.81	0	Hz	0
2	203.71	0	Hz	0
3	310.62	0	Hz	0
4	414.4	0	Hz	0
5	442.6	0	Hz	0
6	583.44	0	Hz	0
7	1002.2	0	Hz	0
8	1090.8	0	Hz	0
9	1168.3	0	Hz	0
10	1388.2	0	Hz	0

DOFs	Units	Shape 1 Magnitude	Shape 1 Phase	Shap Magnitude
1X	in/lbf-sec	4.0074	180	6.4846
1Y	in/lbf-sec	3.6867	0	0.044376
1Z	in/lbf-sec	4.75	0	1.3101
2X	in/lbf-sec	4.0373	180	6.4686
2Y	in/lbf-sec	0.26819	180	0.04978
2Z	in/lbf-sec	6.8747	0	5.0004
3X	in/lbf-sec	4.0522	180	6.4824
3Y	in/lbf-sec	4.2674	180	0.067707
3Z	in/lbf-sec	7.8213	0	2.6759
4X	in/lbf-sec	4.0583	180	6.5082
4Y	in/lbf-sec	8.2709	180	0.08513
4Z	in/lbf-sec	8.4008	0	3.65
5X	in/lbf-sec	4.0586	180	6.5365
5Y	in/lbf-sec	12.283	180	0.10228
5Z	in/lbf-sec	8.7388	0	11.482
6X	in/lbf-sec	0.086017	180	6.5534
6Y	in/lbf-sec	12.283	180	0.10233

Figure 4. FEA Mode Shapes.

MAC FORMULA

For two shapes ({u},{v}), MAC is calculated with the formula, [1]

$$\mathrm{MAC} = \frac{\left\| \{u\}^h \{v\} \right\|^2}{\{u\}^h \{u\} \{v\}^h \{v\}} \qquad (1)$$

{u}= complex shape (m-vector)
{v}= complex shape (m-vector)
m = number of matching DOFs between the shapes
h - denotes the transposed conjugate vector

MAC is a measure of the ***co-linearity*** of two shapes. That is, if the two shapes lie together on the same straight line, **MAC = 1**. Equation (1) is the Dot Product of the two shapes ***normalized*** by each of their magnitudes squared. Therefore, MAC is not sensitive to the actual values of the shapes themselves, only their ***"shapes"***. If two shapes do not lie on the same line, then **MAC<1**. If **MAC=0**, then the two shapes are ***orthogonal*** to, or ***linearly independent*** of one another.

The MAC values between the EMA & FEA shapes of the Jim Beam are shown in Figure 5. These values clearly indicate that 10 pairs of EMA & FEA mode shapes are ***strongly correlated***. That is, each EMA shape is ***essentially the same*** as its corresponding FEA shape with a MAC value greater than 0.9.

Figure 6 is a bar chart of the ***scale factors*** that should be applied to the EMA shapes to make them equal to the FEA shapes, in a least-squared-error sense [4]. That is, if the EMA shapes are multiplied by the scale factors in Figure 6, the EMA shape components will ***closely approximate*** the FEA shape component values, at least for the 99 matching DOFs.

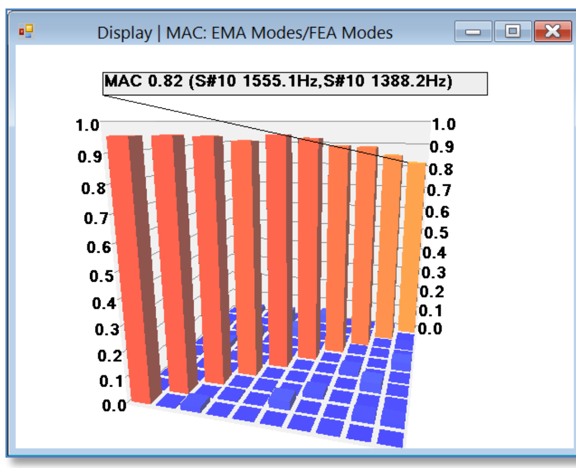


Figure 5. MAC Values Between EMA & FEA Shapes.

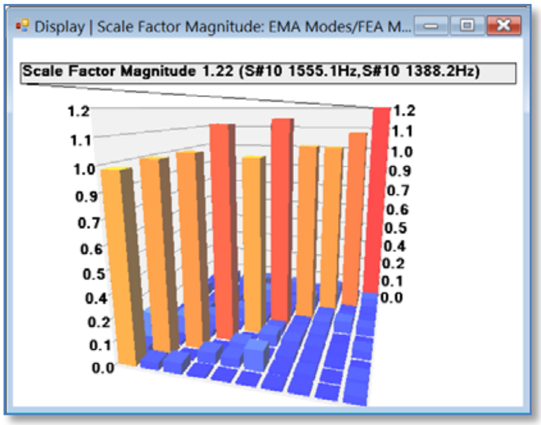


Figure 6. EMA Shape Scale Factors.

The scaling matrix in Figure 6 also contains some ***non-zero off-diagonal*** values, indicating that linear combinations of several EMA shapes are needed in order to more closely match the FEA shape values [4].

After the scale factors in Figure 6 were applied to the EMA shapes, Figure 7 shows the MAC values between the re-scaled EMA shapes and the FEA shapes. Clearly, there is an improvement in the correlation between shape pairs.

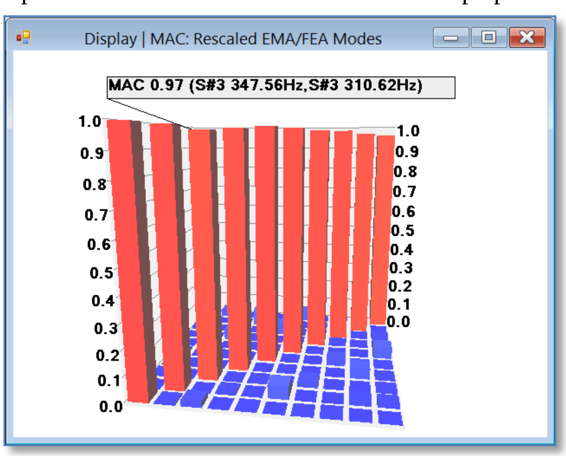


Figure 7. MAC Values Between Rescaled EMA & FEA Shapes

Figure 7 indicates that the rescaled EMA shapes are closer to being co-linear with the FEA shapes. But the question remains, ***"How different are the EMA shape component values from the FEA shape component values?"*** To answer that question, a new measure of the difference between two shapes will be introduced.

SHAPE DIFFERENCE INDICATOR (SDI)

For two shapes ({u},{v}), the Shape Difference Indicator is defined with the formula,

$$\mathrm{SDI}=\left(1-\frac{\left\|\{v\}-\{u\}\right\|^2}{\{v\}^h\{v\}+\{u\}^h\{u\}}\right)^2 \qquad (2)$$

or

$$\mathrm{SDI}=\left(\frac{2\,\mathrm{real}(\{v\}^h\{u\})}{\{v\}^h\{v\}+\{u\}^h\{u\}}\right)^2 \qquad (3)$$

$\mathrm{real}(\{v\}^h\{u\})$ = the real part of the vector dot product
{u}= complex shape (m-vector)
{v}= complex shape (m-vector)
m = number of matching DOFs between the shapes
h - denotes the transposed conjugate vector

SDI values are like MAC values. That is, their values range between **0 & 1**. If **SDI = 1**, the two shapes have ***identical values***. If **SDI < 1**, the two shapes have ***different values*** between their ***matching*** DOFs. Several examples illustrate the range of SDI values.

- If {v} = {u}, SDI = 1
- If {v} = 0 *or* {u} = 0, SDI = 0
- If {v} = 2{u}, SDI = 0.64
- If {v} = 10{u}, SDI = 0.04

The SDI values between the EMA & FEA shapes of the Jim Beam are shown in Figure 8. The SDI values are ***nearly all zero***, indicating that the two sets of shapes have ***different component values***. Closer examination of the shape components in Figures 3 & 4 reveals that the EMA shapes are ***mostly imaginary*** valued (with phases close to 90 & 270 degrees) while the FEA shapes are ***real valued*** (with phases of 0 & 180 degrees). Real valued shapes are also called ***normal*** shapes.

The SDI values between the rescaled EMA shapes and the FEA shapes are shown in Figure 9. Clearly, the rescaled EMA shapes are ***nearly equal*** in value to the FEA shapes.

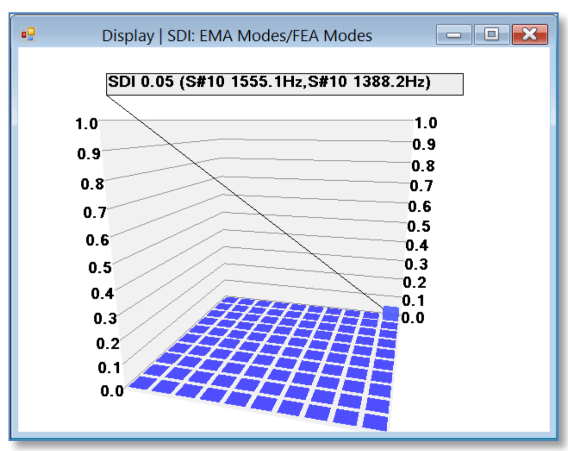


Figure 8. SDI Values For EMA & FEA Shapes

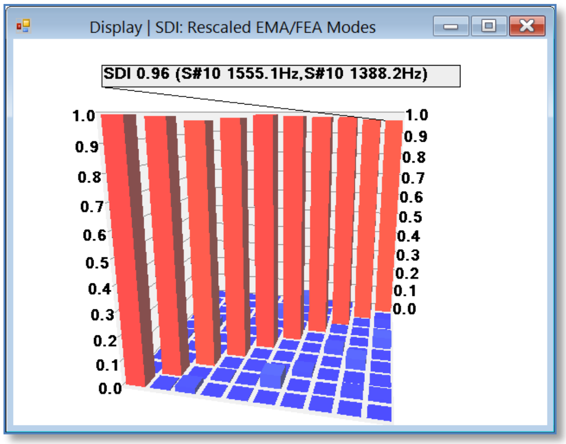


Figure 9. SDI Values For Rescaled EMA & FEA Shapes

SDI AND CAP SCREW TORQUE

SDI can be used to detect differences between two shapes, no matter what type of data they contain. To illustrate this, different amounts of torque where applied to one of the Allen screws that attach the top plate to the back plate of the Jim Beam, as shown in Figure 10.

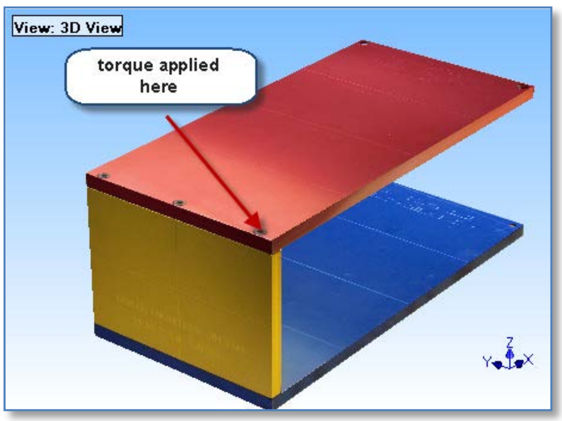
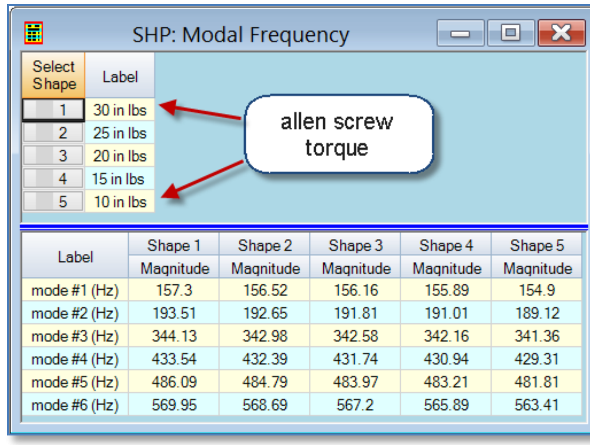


Figure 10. Torque Applied To Allen Screw

For each of six different torque values; **30 in-lbs, 25 in-lbs, 20 in-lbs, 15 in-lbs, and 10 in-lbs**, the modal frequency and damping of six modes of the Jim Beam were stored as shape components. The modal frequency shapes are listed in Figure 11, and the modal damping shapes in Figure 12.

The SDI values between all frequency shape pairs are displayed in the bar chart in Figure 13. The SDI values for all damping shape pairs are displayed in the bar chart in Figure 13.

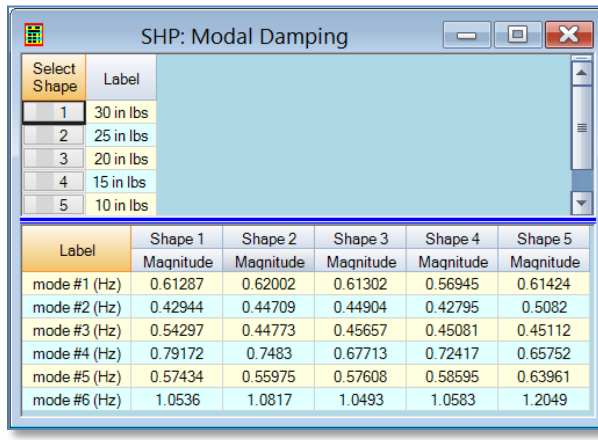
All bars in Figure 13 not only indicate a clear change in the modal frequencies for each torque value, but they also indicate that the SDI value ***dropped monotonically*** as the Allen screw was loosened from 30 in-lbs to 10 in-lbs,. In other words, the modal frequencies shifted more between cases, as the torque was reduced. On the other hand, the SDI bars in Figure 14 indicate that there was ***no significant change*** in the modal damping due to loosening the Allen screw.



Select Shape	Label
1	30 in lbs
2	25 in lbs
3	20 in lbs
4	15 in lbs
5	10 in lbs

Label	Shape 1 Magnitude	Shape 2 Magnitude	Shape 3 Magnitude	Shape 4 Magnitude	Shape 5 Magnitude
mode #1 (Hz)	157.3	156.52	156.16	155.89	154.9
mode #2 (Hz)	193.51	192.65	191.81	191.01	189.12
mode #3 (Hz)	344.13	342.98	342.58	342.16	341.36
mode #4 (Hz)	433.54	432.39	431.74	430.94	429.31
mode #5 (Hz)	486.09	484.79	483.97	483.21	481.81
mode #6 (Hz)	569.95	568.69	567.2	565.89	563.41

Figure 11. Modal Frequency Shapes



Select Shape	Label
1	30 in lbs
2	25 in lbs
3	20 in lbs
4	15 in lbs
5	10 in lbs

Label	Shape 1 Magnitude	Shape 2 Magnitude	Shape 3 Magnitude	Shape 4 Magnitude	Shape 5 Magnitude
mode #1 (Hz)	0.61287	0.62002	0.61302	0.56945	0.61424
mode #2 (Hz)	0.42944	0.44709	0.44904	0.42795	0.5082
mode #3 (Hz)	0.54297	0.44773	0.45657	0.45081	0.45112
mode #4 (Hz)	0.79172	0.7483	0.67713	0.72417	0.65752
mode #5 (Hz)	0.57434	0.55975	0.57608	0.58595	0.63961
mode #6 (Hz)	1.0536	1.0817	1.0493	1.0583	1.2049

Figure 12. Modal Damping Shapes

CONCLUSIONS

A new measure of the difference between two shapes, called the **Shape Difference Indicator** or **SDI,** was presented. Like MAC values, the SDI values range **between 0 & 1**. When **SDI = 1**, the two shapes are ***identical***. When **SDI < 1**, the two shapes have ***different*** values.

Unlike MAC though, SDI is sensitive to differences in the shape values, and also gives meaningful results even between ***two scalars***.

Both MAC and SDI were calculated between the EMA & FEA mode shapes of the Jim Beam. The MAC values shown in Figure 5 indicate an acceptable level of correlation between the EMA & FEA mode shapes.

The SDI values between the EMA & FEA shape pairs are shown in Figure 8 and are all ***nearly zero***, indicating that the EMA shapes have different values than the FEA shapes. A cursory comparison of the shape values in Figures 3 & 4 verifies that the phases of the EMA shape components are approximately 90 degrees apart from the phases of the FEA shape components. MAC rightly indicates that the ***shapes are nearly the same***, but SDI indicates that their ***component values are different***.

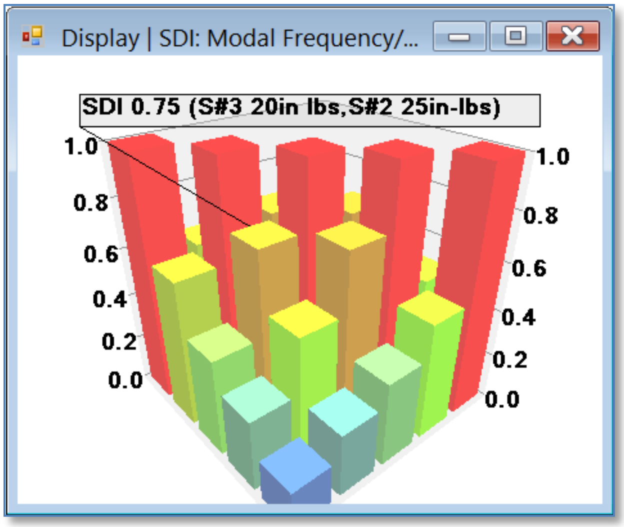


Figure 13. SDI for Modal Frequency Shapes

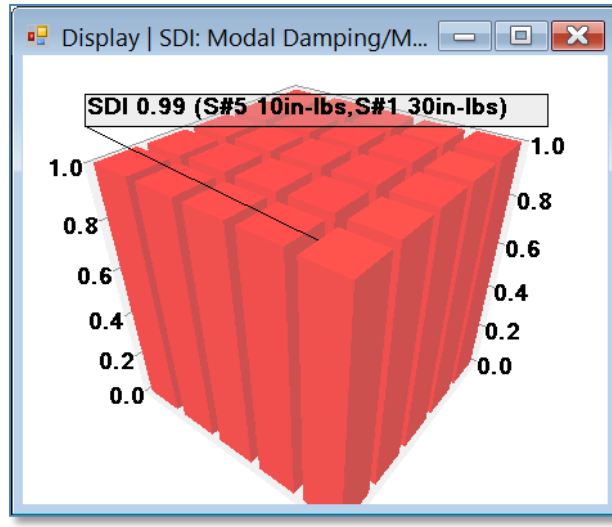


Figure 14. SDI for Modal Damping Shapes

After the EMA shapes were rescaled to more closely match the FEA shapes [4], the MAC values in Figure 7 and the SDI values in Figure 9 both indicate a ***strong correlation*** between the EMA & FEA shape pairs.

Next, the SDI calculation was used to classify the torque applied to one of the Allen screws used to hold the plates together on the Jim Beam. Five different torque values were applied to one Allen screw, as shown in Figure 10. With each torque value applied to the screw, FRFs were acquired and curve fit to obtain the modal frequency & damping of the first six modes of the beam. Those values are listed as in Figures 11 & 12.

Each table contains five shapes, one for each torque value. Each shape contains six components, each component containing either a modal frequency or damping value. The SDI bar chart values shown in Figure 13 were calculated between all frequency shape pairs in Figure 11. Likewise, the SDI bar chart values shown in Figure 14 were calculated between all pairs of shapes in Figure 12.

The SDI values in Figure 13 indicate a ***significant difference*** in modal frequencies caused by the five different torque values. However, the SDI values in Figure 14 indicate ***very little difference*** in modal damping values between the five different torque values.

FAULT CORRELATION TOOL (FaCTs™)

SDI has been implemented in the Vibrant Technology MechaniCom Machine Surveillance System™ and the MechaniCom Qualification Testing System™ as a tool for ***detecting and diagnosing faults*** in machinery and structures. This Fault Correlation Tool, called FaCTs™, uses an ***ordered table & bar chart*** for correlating shapes based on their SDI values.

To illustrate, suppose that multiple Jim Beam structures were being tested for proper assembly, and that 25 in-lbs was the correct torque value for tightening the Allen screw shown in Figure 10. If shapes like those in Figure 11 were stored in the MechaniCom database, then each time a Jim Beam was tested, the SDI values between its frequency shape and the stored shapes would be ordered in a FaCTs™ table, from the highest to lowest SDI value. The stored shape with the highest SDI value would then indicate whether the current torque used on the Allen screw was ***higher*** or ***lower*** than 25 in-lbs.

Any type of engineering data, including vibration, temperatures, pressures, flow rates, voltage, current, etc. can be used with SDI. A FaCTs™ table then, is not only useful for ***detecting*** a machine failure or improper structural assembly, but also for ***correlating*** it with a known machine failure or improper assembly.

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