Simulating Base-Shake Environmental Testing

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ABSTRACT

In most military aircraft and spacecraft applications, each payload structure must be pre-tested on a shake table to ensure that it can withstand the vibration environment that it will experience during flight. Shaker testing is done using a control PSD which is designed to realistically represent the floor motion of the aircraft during takeoff, in flight, or during landing. Qualification testing is typically done by mounting the test article on one or more shakers, and exciting it with a closed loop shaker testing system so that the base of the payload responds with the pre-specified control PSD.

When a test vehicle is too massive to be tested by mounting it on shakers, it is impossible to perform a base-shake test on a shake table. So the question arises; ***"Are there other more convenient driving points from which to excite the structure which will simulate a base-shake test?"***

In this approach, we derive a frequency domain ***Transmissibility*** model which is used to calculate PSDs for convenient driving points as functions of the base-shake PSDs. These calculated PSDs would then be used to control a shaker test that simulates the base-shake test.

The ***Transmissibility*** model is validated by using an ***inverse calculation*** to calculate base-shake PSDs as functions of the new driving point PSDs. Suitable driving points can then be chosen by comparing the calculated base-shake PSDs with the original pre-specified base-shake PSDs.

INTRODUCTION

When a test vehicle is too massive to be tested by mounting it on shakers, our assumption is that it can be tested by shaking it at other driving points which will have the ***same dynamic effects as shaking it from its base***. To test at different driving points, new control PSDs must be calculated which will cause the test article to respond in a manner which is ***similar*** to its response during a base-shake test. In order to accomplish this, the dynamic properties between the base and the other driving points must be correctly modeled.

To calculate new control PSDs, we start with a standard set of time domain equations of motion that model the dynamics of the structure. After partitioning the equations into two sets, (one for moving DOFs (degrees of freedom) and one for the fixed base DOFs), a frequency domain model is derived that relates the responses of the moving DOFs to the fixed base responses. This ***Transmissibility*** matrix model is then used to calculate control PSDs for ***new driving points*** as functions of the base-shake control PSDs.

Modal Test

The base-shake simulation was done using a ***modal model*** of the vehicle sitting on its wheels in its tied down configuration, as shown in Figure 1. The ***modal model*** was used to synthesize elements of a ***Transmissibility model*** that relate base-shake responses to responses at other points on the structure.

To develop a valid modal model, three different model tests were performed on the vehicle. From these three tests, a ***modal model*** was constructed that represented the dynamics of the vehicle vibrating on its wheels. The dominant modes of the model are its ***rigid body*** modes, i.e. the vehicle bouncing on its wheels. In addition, several of the lowest frequency ***elastic modes*** were also excited, and were included in the modal model.

Figure 1 Test Vehicle in Base-Shake Configuration.

Modal testing was done using an electro-dynamic linear stroke shaker, driven by a pure random signal. Twenty triaxial accelerometers were used to measure responses, and a load cell was used to measure the force input. FRFs (Frequency Response Functions) were calculated in the frequency span (0 to 50Hz), using 25 spectrum averages. A total of 60 FRFs were calculated from each of the three modal tests.

The modal model was then used to ***synthesize FRFs*** between each of the wheel hubs and other ***moving DOFs*** on the vehicle. In addition, the modal model was used to calculate ***driving point FRFs*** at the wheel hubs, which were then used to calculate the ***stiffness & damping*** of the tires. Tire stiffness & damping directly influence the response of the vehicle to the base-shake PSDs.

BACKGROUND THEORY

Time Domain Equations

In all FEA (Finite Element Analysis) and modal testing work, it is assumed that Newton's Second Law adequately describes the dynamic behavior of the mechanical structure. Hence, the linear, time-invariant dynamics of the vehicle can be represented with the following set of differential equations;

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \qquad (1)$$

where:

$[M]$ = mass matrix (n by n)
$[C]$ = damping matrix (n by n)
$[K]$ = stiffness matrix (n by n)
$\{\ddot{x}(t)\}$ = acceleration (n vector)
$\{\dot{x}(t)\}$ = velocity (n vector)
$\{x(t)\}$ = displacement (n vector)
$\{f(t)\}$ = force (n vector)
n = DOFs (degrees of freedom)
t = time variable

Frequency Domain Equations

Using Laplace transforms, the time domain equations of motion can be transformed into the frequency domain, and written as;

$$\left([M]s^2+[C]s+[K]\right)\{x(s)\}=\{f(s)\} \qquad (2)$$

where:

s = Laplace variable
$s=\sigma+j\omega$ = complex frequency
$\{x(s)\}$ = Laplace transform of the displacement (n vector)
$\{f(s)\}$ = Laplace transform of the force (n vector)

NOTE: For convenience, the s-variable will be dropped from the displacement and force vectors in the following notation.

Partitioning the matrices into the **fixed base DOFs** (subscript **F**) and the **moving DOFs** (subscript **M**), the equations become;

$$\left(\begin{bmatrix} M_{FF} & M_{FM} \\ M_{MF} & M_{MM} \end{bmatrix}s^2 + \begin{bmatrix} C_{FF} & C_{FM} \\ C_{MF} & C_{MM} \end{bmatrix}s + \begin{bmatrix} K_{FF} & K_{FM} \\ K_{MF} & K_{MM} \end{bmatrix}\right)\begin{Bmatrix} u_F \\ u_M \end{Bmatrix} = \begin{Bmatrix} f_F \\ f_M \end{Bmatrix} \qquad (3)$$

where:

$\{x\}=\begin{Bmatrix} u_F \\ u_M \end{Bmatrix}$

$\{u_F\}$ = displacement of the fixed DOFs
$\{u_M\}$ = displacement of moving DOFs
$\{f_F\}$ = forces applied to fixed DOFs
$\{f_M\}$ = forces applied to moving DOFs

NOTE: The aircraft floor DOFs are referred to as the **fixed base DOFs** because all mode shape components are **zero (or fixed)** at the bottom on the wheels. This is unique to any **base-shake** problem.

Separating the equations into two sets, one for the fixed base DOFs and the other for the moving DOFs;

$$\left(\begin{matrix}[M_{FF} \quad M_{FM}]s^2 + \\ [C_{FF} \quad C_{FM}]s+[K_{FF} \quad K_{FM}]\end{matrix}\right)\begin{Bmatrix} u_F \\ u_M \end{Bmatrix}=\{f_F\} \qquad (4)$$

$$\left(\begin{matrix}[M_{MF} \quad M_{MM}]s^2 + \\ [C_{MF} \quad C_{MM}]s+[K_{MF} \quad K_{MM}]\end{matrix}\right)\begin{Bmatrix} u_F \\ u_M \end{Bmatrix}=\{f_M\} \qquad (5)$$

Assumption: *No forces* are applied at the ***moving DOFs***.

During a base-shake, forces are only applied at the **fixed base DOFs**. Using the moving DOFs equation and assuming that (f_M =0) gives:

$$\begin{aligned}&\left([M_{MF}]s^2+[C_{MF}]s+[K_{MF}]\right)\{u_F\}\\&+\left([M_{MM}]s^2+[C_{MM}]s+[K_{MM}]\right)\{u_M\}=\{0\}\end{aligned} \qquad (6)$$

Rearranging terms gives;

$$\begin{aligned}&\left([M_{MM}]s^2+[C_{MM}]s+[K_{MM}]\right)\{u_M\}\\&=-\left([M_{MF}]s^2+[C_{MF}]s+[K_{MF}]\right)\{u_F\}\end{aligned} \qquad (7)$$

Solving for the **moving DOFs** gives;

$$\begin{aligned}&\{u_M\}=-\left([M_{MM}]s^2+[C_{MM}]s+[K_{MM}]\right)^{-1}\\&\left([M_{MF}]s^2+[C_{MF}]s+[K_{MF}]\right)\{u_F\}\end{aligned} \qquad (8)$$

Assumption: There is ***no inertial coupling*** between the ***fixed base DOFs*** and ***moving DOFs***.

All of the base-shake forces are transmitted through the tires to the wheel hubs, and then to the rest of the structure.

Assuming that ($[M_{MF}] = 0$), equation (7) becomes;

$$\{u_M\} = -\left([M_{MM}]s^2 + [C_{MM}]s + [K_{MM}]\right)^{-1} \left([C_{MF}]s + [K_{MF}]\right)\{u_F\} \tag{9}$$

This equation expresses the ***moving DOFs*** as functions of the ***fixed base DOFs***.

FRFs for the Moving DOFs

Using the moving DOFs equation (5) and setting ($\{u_F\} = 0$) gives;

$$\left([M_{MF}]s^2 + [C_{MF}]s + [K_{MF}]\right)\{0\} + \left([M_{MM}]s^2 + [C_{MM}]s + [K_{MM}]\right)\{u_M\} = \{f_M\} \tag{10}$$

or;

$$\{u_M\} = \left([M_{MM}]s^2 + [C_{MM}]s + [K_{MM}]\right)^{-1}\{f_M\}$$

The term $\left([M_{MM}]s^2 + [C_{MM}]s + [K_{MM}]\right)^{-1}$ in the above equation is simply the ***FRF matrix*** between the ***moving DOFs***. Re-writing equation (9) in terms of the FRF matrix $[H_{MM}(s)]$ for the moving DOFs;

$$\{u_M\} = -[H_{MM}(s)]\left([C_{MF}]s + [K_{MF}]\right)\{u_F\} \tag{11}$$

Equation (11) expresses motions of the moving DOFs as functions of the motions of the fixed base DOFs.

Stiffness & Damping Matrix

The term $\left([C_{MF}]s + [K_{MF}]\right)$ is the ***stiffness & damping*** matrix between the ***fixed base DOFs*** (aircraft floor) and the ***moving DOFs***. Meaningful (non-zero) stiffness & damping values only exist between the fixed base and the wheel hub DOFs. Therefore, the motion of all of the moving DOFs other than the wheel hubs depends on two dynamic properties;

1) **FRFs** between the wheel hubs and other moving DOFs.
2) **Stiffness & damping** of the tires, between the fixed base DOFs (aircraft floor) and the wheel hub DOFs.

Transmissibility Matrix

The product of these two matrices (FRFs and Stiffness & Damping) is a ***unit-less Transmissibility matrix***. The FRF matrix has units of (displacement/force) and the Stiffness & Damping matrix has units of (force/displacement), so their product is ***unit-less***. Equation (11) can be re-written as;

$$\{u_M\} = [T(s)]\{u_F\} \tag{12}$$

where:

$$[T(s)] = -[H_{MM}(s)]\left([C_{MF}]s + [K_{MF}]\right)$$

In summary, the FRF matrix $[H_{MM}(s)]$ contains the dynamic properties between the wheel hubs and other moving DOFs of the vehicle. The Stiffness & Damping matrix $\left([C_{MF}]s + [K_{MF}]\right)$ contains the stiffness & damping of the tires, between the fixed base (aircraft floor) and the wheel hubs.

NOTE: The negative sign in front of the Transmissibility matrix is canceled by the negative signs in the ***off-diagonal terms*** of the damping $[C_{MF}]$ and stiffness $[K_{MF}]$ matrices.

Power Spectral Density

The control spectrum for the aircraft floor is specified as a Power Spectral Density (PSD). Multiplying the Transmissibility equation by the ***transposed conjugate*** of itself, gives a new equation in Power spectrum (or PSD) units;

$$\{u_M\}\{u_M\}^t = [T(s)]\{u_F\}\{u_F\}^t[T(s)]^t \tag{13}$$

where:

$\{u_F\}\{u_F\}^t$ = base-shake PSD matrix

$\{u_M\}\{u_M\}^t$ = new control PSD matrix

t – denotes the transposed conjugate

This equation expresses the ***new control PSD matrix*** as a function of the ***base-shake PSD matrix***.

NOTE: The ***diagonal*** elements of these matrices contain Auto PSDs, and the ***off-diagonal*** elements contain Cross PSDs.

It is assumed that each wheel is subjected to independent random vibration. Therefore, the same base-shake PSD will be used for all ***diagonal*** elements and all ***off-diagonal*** elements are set to zero.

Inverse Calculation

The accuracy of the ***new control PSDs*** can be checked by performing an ***inverse calculation***, i.e. calculating the ***base-shake PSD matrix*** as a function of the ***new control PSD matrix***. Pre-multiplying the previous equation by $[T(s)]^t$ and post-multiplying it by $[T(s)]$ gives;

$$[T(s)]^t\{u_M\}\{u_M\}^t[T(s)] = [T(s)]^t[T(s)]\{u_F\}\{u_F\}^t[T(s)]^t[T(s)]$$

Solving for the ***base-shake PSD matrix*** $\{u_F\}\{u_F\}^t$ gives;

$$\{u_F\}\{u_F\}^t = [A(s)]\{u_M\}\{u_M\}^t[A(s)]^t \tag{14}$$

where:

$$[A(s)] = \left[[T(s)]^t[T(s)]\right]^{-1}[T(s)]^t$$

This equation expresses the ***base-shake PSD matrix*** as a function of the ***new control PSD matrix.***

Clearly, there are cases where testing a vehicle using certain new control PSDs ***will not simulate*** the intended base-shake excitation**.** It depends on the dynamic properties represented by the modal model at the chosen ***moving DOFs*** of the structure. For example, shaking at any moving DOF where one or more mode shapes are at or near a ***nodal point (zero motion)*** will not excite all of the modes, and therefore may not accurately simulate a base-shake test. The ***inverse calculation*** should help locate suitable driving points which will ***closely simulate*** a base-shake test.

BASE-SHAKE PSDs

For testing payloads in military aircraft, a base-shake PSD is pre-specified for the aircraft floor. It is assumed that the PSD is applied at the base of each wheel in three directions (X, Y, & Z). Therefore, the ***base-shake PSD matrix*** is a **(12 by 12)** ***diagonal matrix.*** Base-shake PSDs for the C-5 and C-17 aircraft are defined in Figure 2.

TIRE STIFFNESS & DAMPNG

The Stiffness & Damping matrix $([C_{MF}]s+[K_{MF}])$ contains stiffness and damping values between the floor (bottom of each tire) and each wheel hub. This ***(12 by 12) off-diagonal matrix*** contains different values for each wheel and each direction. Each stiffness and damping element is obtained from the **driving point FRF** at each wheel hub, which is synthesized from the modal model.

- Each element of the Stiffness matrix $[K_{MF}]$ is obtained as the ***inverse of the flexibility line*** near DC of the driving point FRF. Examples are shown in Figure 3.
- The elements of the Damping matrix $[C_{MF}]$ are calculated with the following formula;

C = Mass x (damping decay constant from the IRF)

C-5 Aircraft	
Hz	**g²/Hz**
15	0.003
1000	0.003
2000	0.00075
rms = 2.11g	

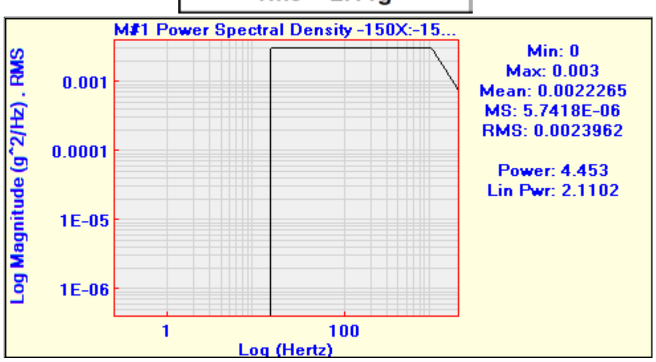

Figure 2A C-5 Aircraft PSD

C-17 Aircraft	
Hz	**g²/Hz**
5	0.005
66.897	0.005
150	0.025
500	0.025
2000	0.0016
rms = 4.43g	

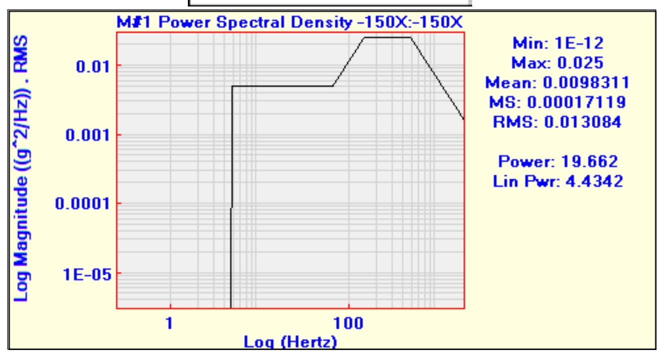

Figure 2B C-17 Aircraft PSD

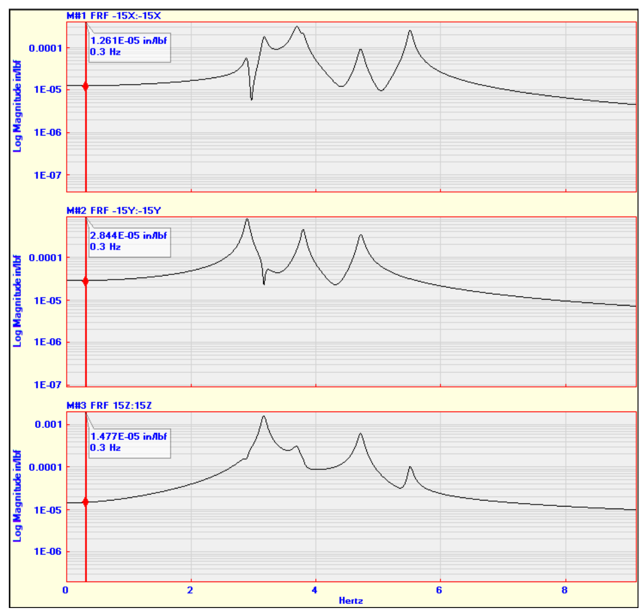

Figure 3 Flexibility Line at Low Frequency (in/lbf = 1/Stiffness)

The damping decay constant is the ***slope of the envelope of the log magnitude of the driving point IRF*** (Impulse Response Function), the inverse FFT of each driving point FRF. The damping decay constant is obtained by curve fitting a straight line to the **driving point IRF.** Examples of logarithmic decay envelopes are shown in Figure 4.

Mass is obtained from the **mass line** (at high frequency) of the driving point inertance (acceleration/force) FRF in each direction at each wheel hub. The driving point inertance is obtained by ***differentiating the synthesized (displacement/force) FRF twice***. Examples of mass lines are shown in Figure 5.

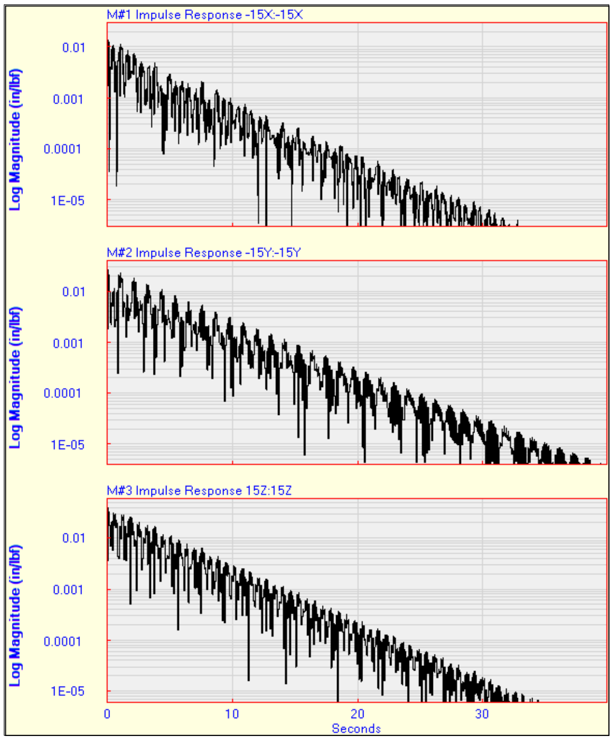

Figure 4 Logarithmic Decrement of Impulse Responses.

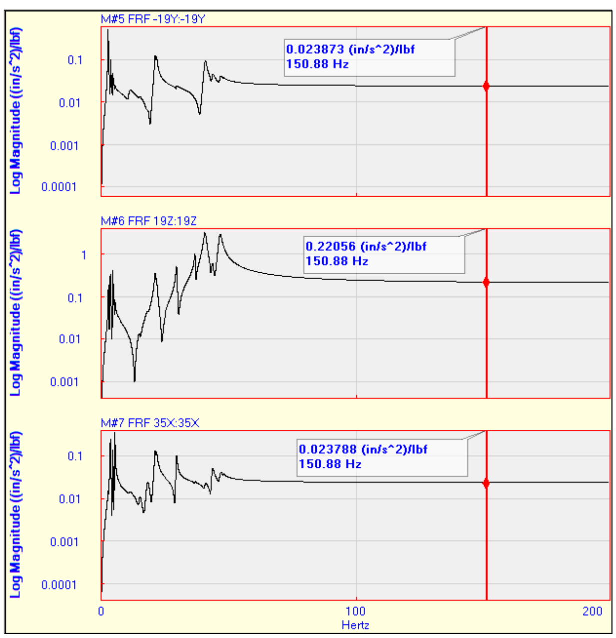

Figure 5 Inertance Mass Line (in/sec^2-lbf = 1/mass).

Figure 6 shows all 12 elements of the Stiffness and Damping matrix in overlaid format. These 12 ***off diagonal*** terms (indicated by the DOFs in the spreadsheet), contain both a Stiffness (real part) and the Damping multiplied by frequency (imaginary part). This matrix is multiplied by the **moving DOFs FRF matrix** to obtain the **Transmissibility** matrix.

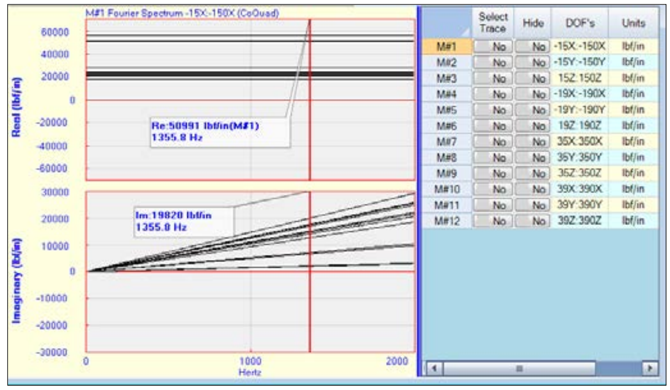

Figure 6 Stiffness & Damping matrix.

TRANSMISSIBILITY MATRIX

The Transmissibility matrix is the product of the moving DOFs FRF matrix and the Stiffness & Damping matrix. It is clear from this product that the Stiffness & Damping matrix ***scales*** each element of the **moving DOFs FRF matrix** to reflect the stiffness and damping of each tire in a direction.

If the tires had no stiffness or damping in them (a hypothetical case), then base-shaking the vehicle would cause no motion of the **moving DOFs.** On the other hand, if the tires were very stiff or had lots of damping, then base-shaking the vehicle would result in large motions of the **moving DOFs.**

The moving DOFs FRF matrix need only be synthesized between the moving DOFs which might be used as the new driving points, and the DOFs of the wheel hubs. Rather than use all 60 moving DOFs (the number of components in the experimental mode shapes), a smaller set of 9 DOFs was considered. Figure 7 contains red arrows indicating the 9 candidate driving point DOFs.

Figure 8 shows the Transmissibility's for one moving DOF (-10Y) due to the 4 base-shake PSDs at each wheel in the Z-direction. The entire Transmissibility matrix contains 108 terms, **nine rows** for the moving DOFs and **twelve columns** for the fixed base DOFs (three directions at each wheel).

NEW CONTROL PSD MATRIX

A new control PSD matrix is calculated using the base-shake PSD matrix and the Transmissibility matrix in equation (13). The ***base-shake PSD matrix*** $\{u_F\}\{u_F\}^t$ is a **(12 by 12) *diagonal matrix***. The ***Transmissibility matrix*** $[T(s)]$ is a **(9 by 12) *full matrix***. The resulting matrix of new control PSDs is a **(9 by 9) *matrix***. Elements of this matrix for only four moving DOFs due to base-shake PSDs in the Z direction are shown in Figure 9. The new driving Point DOFs are -17Z, 41Z, -45Z, and 55Z. The base-shake PSDs for

DOFs 150Z, 190Z, 350Z, and 390Z were used to calculate the new control PSDs.

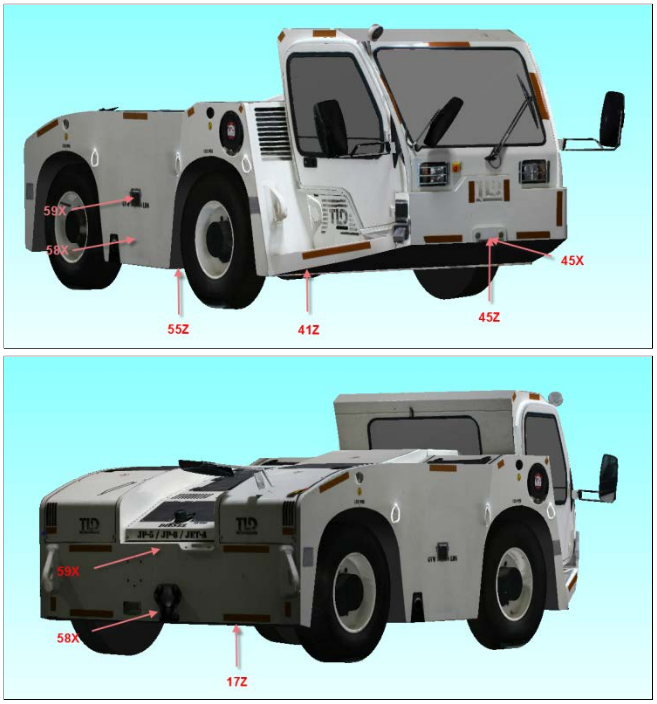

Figure 7 Convenient Driving Points.

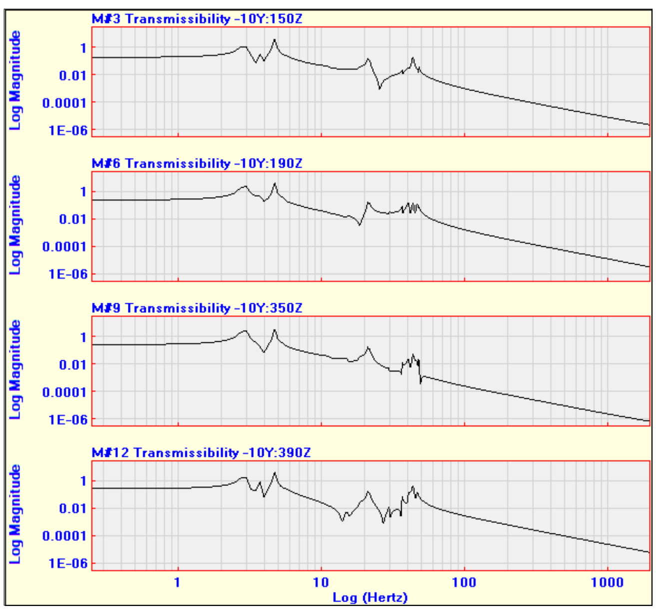

Figure 8 Transmissibility's for moving DOF (-10Y)

SIMULATED BASE SHAKE

The validity of the new control PSDs is confirmed by using the Inverse equation (14) to calculate base-shake PSDs from the new control PSDs. Figure 10 shows the base-shake PSDs which were calculated from the (4 by 4) control PSD matrix for DOFs -17Z, 41Z, 45Z, and 55 Z. For this case, it is clear that the (4 by 4) matrix of base-shake PSDs is reproduced with over three decades of accuracy.

CONCLUSIONS

The question that was addressed in this paper is; ***"Is it possible to shake an aircraft payload from other more convenient driving points using control PSDs that simulate a base shake test?"*** To simulate a base-shake, a unique Transmissibility model was derived to relate the base-shake PSDs to the new control PSDs required to simulate the base-shake.

An experimentally derived modal model of the vehicle on its fixed base was used to ***synthesize*** the Transmissibility's. The modal model was developed from three different shaker tests of the vehicle, using pure random excitation signals and 60 tri-axial accelerometers. The resulting FRFs were curve fit to obtain experimental mode shapes.

The modal model was then used to ***synthesize FRFs*** between the wheel hubs and all other moving DOFs on the vehicle. By measuring motions at each wheel hub, all of the vehicle suspension dynamics were included in the modal model. The modal model was also used to ***synthesize driving point FRFs*** at the wheel hubs, from which the stiffness & damping of the vehicle tires were obtained. Then, ***Transmissibility's*** were calculated as the product of the tire stiffness & damping times the FRFs between the wheel hubs and the other moving DOFs,

Finally, the new control PSDs were calculated using the Transmissibility's and the base-shake PSDs. The new control PSDs were validated using a ***round trip*** calculation as follows;

Base-Shake PSDs > Transmissibility Model > Control PSDs

Control PSDs > Inverse Model > Simulated Base-Shake PSDs

The round trip was then used to show that four convenient driving Points on the vehicle simulated the base-shake quite accurately, for the Z direction.

If a real vehicle behaves in a linear manner, so that its modal model adequately represents its dynamics, then the ***round trip*** calculation can be used to prove that a base-shake simulation is indeed a valid test. Of course, conclusive proof can only be obtained by performing a true base-shake on a vehicle and comparing the base-shake and ***simulated*** base-shake results.

REFERENCES

1. Zhuge, J., Formenti, D., Richardson, M. "A Brief History of Modern Digital Shaker Controllers" Sound & Vibration magazine, September, 2010.

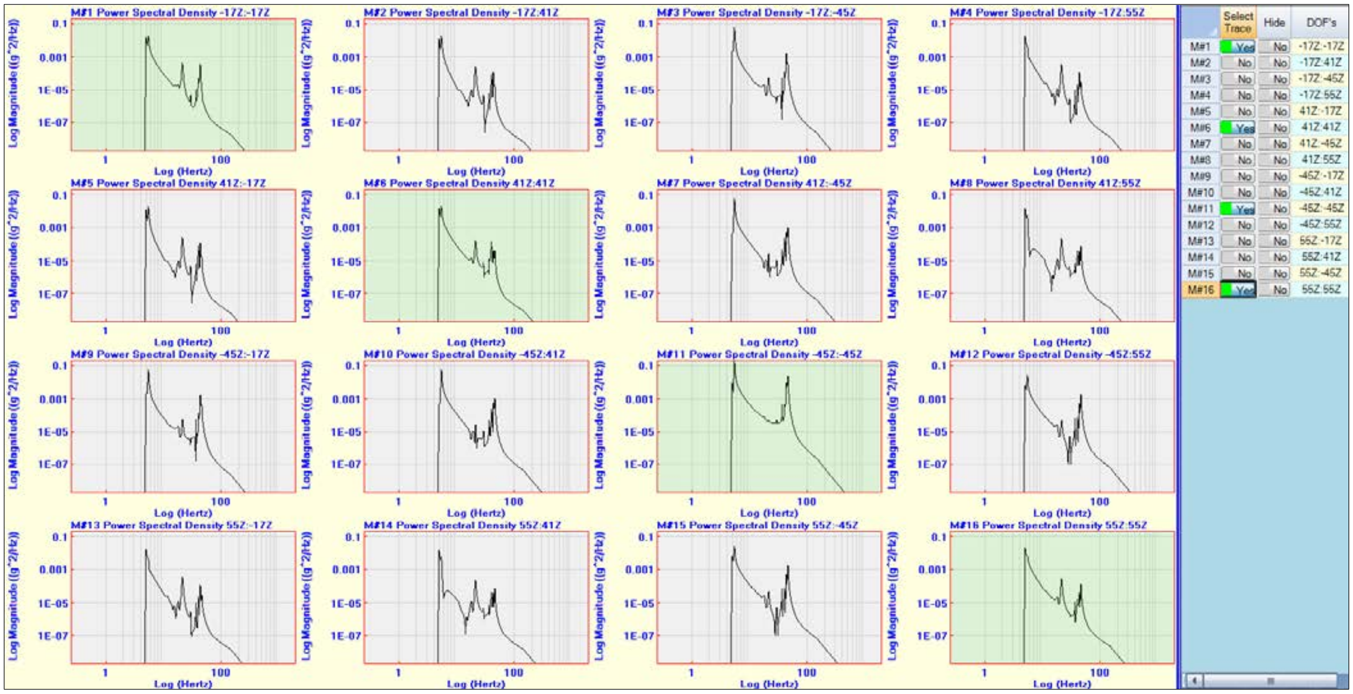

Figure 9 C-17 Control PSDs for 4 Z-direction Driving Points.

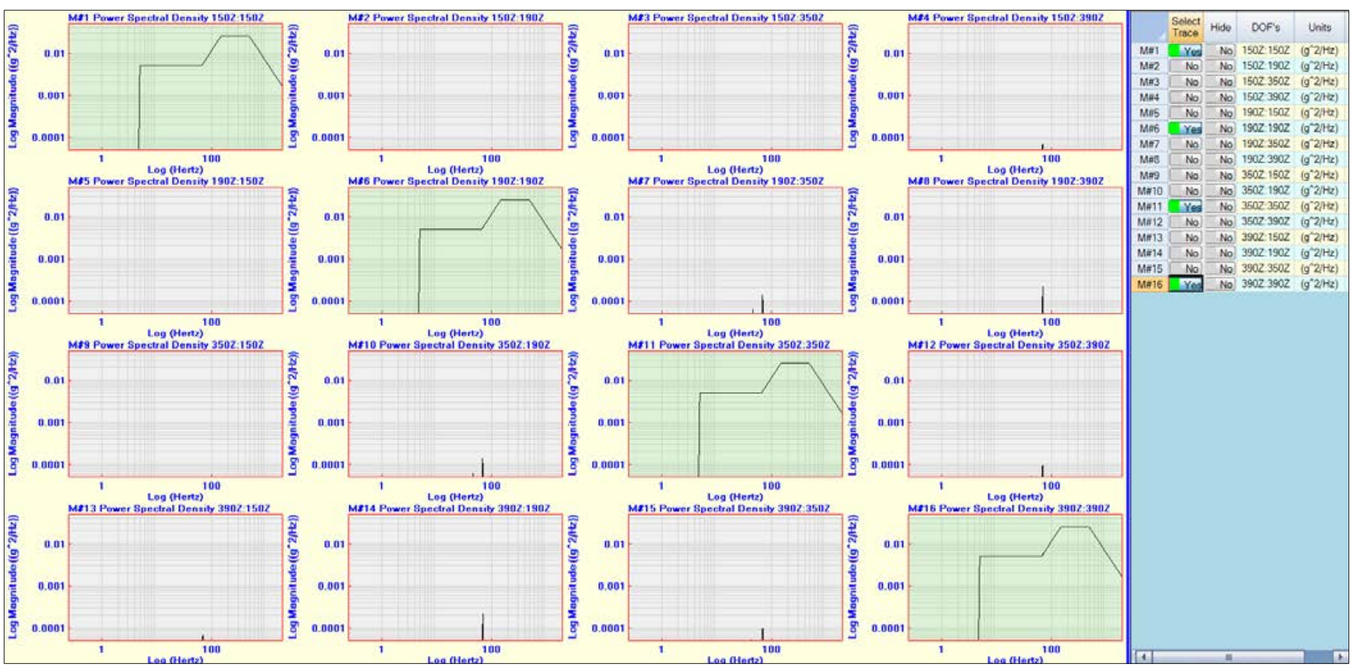

Figure 10 C-17 Simulated Base-Shake PSDs.