Using Modal Parameters for Structural Health Monitoring

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ABSTRACT

In two recent papers, we introduced the idea of numerically comparing currently acquired operating data with archived data to identify faults in rotating machinery [1]-[2]. We introduced a new metric for comparing two operating deflection shapes called the ***Shape Difference Indicator (SDI)***. In another previous paper [3], we used SDI to measure the difference in modal frequencies from before and after a stiffness change was made to a mechanical structure.

In this paper we provide more details of how experimental modal frequency & damping parameters can be used together with the SDI metric as a means of detecting and quantifying changes in the physical properties of a structure.

Also, we have implemented SDI together with a search method for ranking the differences between currently acquired modal parameters and archived modal parameters. We call this new method Fault Correlation Tools (FaCTs™). FaCTs™ can be used in multiple applications, including structural health monitoring, production qualification testing, and recertification of machinery in field maintenance applications.

KEY WORDS
Fourier spectrum (DFT)
Auto power spectrum (APS)
Cross power spectrum (CPS)
Frequency Response Function (FRF)
Experimental Mode Shape (EMA Mode)
Operational Mode Shape (OMA Mode)
Modal Assurance Criterion (MAC)
Shape Difference Indicator (SDI)

INTRODUCTION

It is well known that any change in a physical property of a mechanical structure (e.g. its mass, stiffness, or damping) has a direct effect on its resonant vibration. If the stiffness of a structure is increased, its resonant frequencies will increase. Likewise, if its stiffness is decreased, its resonant frequencies will decrease. Alternatively, if the mass of a structure increases, its resonant frequencies will decrease. If the mass decreases, its resonant frequencies will increase. If the damping forces applied to a structure are increased, the damping of its resonances will increase.

Each resonance is mathematically characterized by its three modal parameters (natural frequency, modal damping, and mode shape).

Modal parameters can be extracted from acquired vibration data using a wide variety of sensors and instrumentation, under a wide variety of operating conditions.

In this paper we show how experimental modal frequency & damping can be used together with the SDI metric as a means of detecting and quantifying changes in structural stiffness. SDI is similar to the Modal Assurance Criterion (MAC), which provides a numerical comparison between two mode shapes [4], [5]. Like MAC, SDI is a ***correlation coefficient*** with values that range between 0 & 1. A value of 1 indicates no difference between two shapes. A value less than 1 indicates that two shapes are different.

The SDI metric has been implemented together with a search algorithm to correlate currently acquired modal parameters with parameters stored in an archival data base. We call this new search method Fault Correlation Tools (FaCTs™). The result of each database search is a display of the **Top Ten FaCTs™ bars** (highest SDI values) between the current modal parameters and modal parameters stored in the database.

FaCTs™ can be used in several ways.

1. To indicate ***slowly occurring changes*** in the modal properties of a machine or structure from monitored data
2. To ***identify specific mechanical faults*** which have been associated with certain modal parameters archived in a database
3. In ***qualification testing*** were the experimental modal parameters of each test article are used to determine its ***pass-fail*** condition

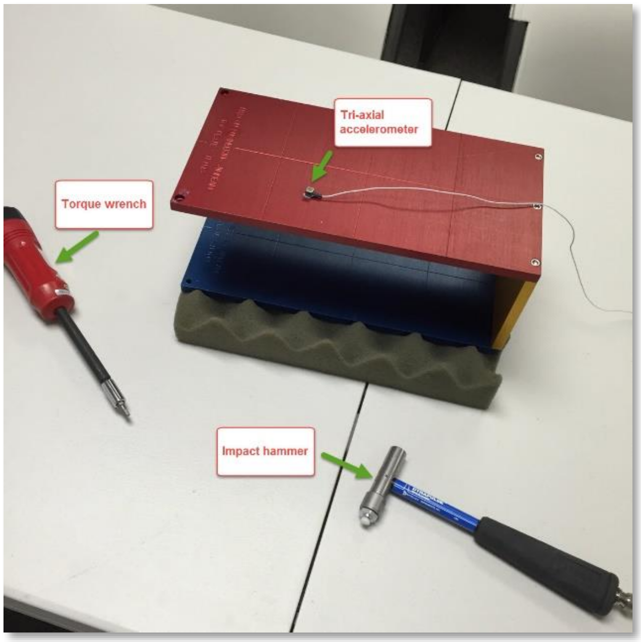

Figure 1. Jim Beam Test Structure.

In this paper, we will revisit the use of FaCTs™ to identify the torque used to tighten one of the cap screws of the Jim Beam structure shown in Figure 1 [3]. It will be shown that FaCTs™ can ***uniquely identify*** the amount of torque applied to the cap screw from among five different test results.

The Jim Beam was impacted with the instrumented hammer shown in Figure 1. Three FRFs were calculated between the acquired impact force and the tri-axial accelerometer response signals. The locations of the impact force and the accelerometer are not critical. However, if their locations are chosen closer to the anti-node (large mode shape amplitude) of a mode, any physical change will potentially have more influence on that mode.

After impacting the structure, the resulting FRFs were curve fit to extract the frequency & damping of six modes. The number of modes used is also not critical, but higher frequency modes will typically be more sensitive to physical changes.

When multiple modal frequencies are used, they are stored as a "shape". In this use, the term "shape" simply means that two or more modal frequencies are stored and treated mathematically like a vector. MAC correlates two mode shapes [4], [5]. SDI was developed for the same purpose; to correlate two shape vectors of data.

It will be shown that FaCTs™ can identify each of the five different test cases by comparing the modal frequencies acquired from each impact test with archived frequencies associated with a specific amount of torque applied to the cap screw.

Before discussing the test results, the Modal Assurance Criterion (MAC) and the Shape Difference Indicator (SDI) will be reviewed to point out their differences. It will then be shown that by increasing its sensitivity, SDI can be used as a reliable metric for uniquely identifying modal frequencies associated with specific joint stiffnesses.

REVIEW OF MODAL ASSURANCE CRITERION (MAC)

You might be wondering, ***if MAC is used for numerically correlating two shapes, why introduce another correlation method?*** SDI was developed to overcome two limitations of MAC, namely,

1. MAC only indicates the ***co-linearity*** of two shapes MAC = 1 if two shapes lie on the same straight line
2. MAC cannot measure the difference between two numbers MAC = 1 if two shapes only have one component

MAC is defined with the formula,

$$MAC = \frac{\left\| \{A\}^h \{B\} \right\|^2}{\{A\}^h\{A\}\{B\}^h\{B\}} \qquad (1)$$

{A}= shape A (complex m- vector)
{B}= shape B (complex m-vector)
m = number of matching DOFs between the shapes
h - denotes the transposed conjugate vector

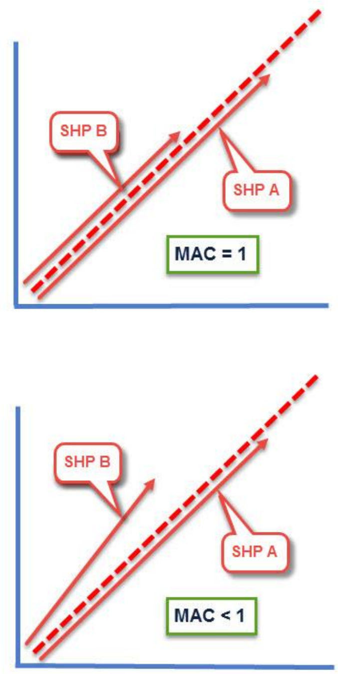

Figure 2. MAC Cases.

Figure 2 depicts two possible cases for MAC values. If two shapes lie on the same straight line, then **MAC = 1**. If two shapes do not lie on the same straight line, then **MAC < 1**.

MAC cannot answer the question, ***"If two shapes lie on the same straight line, are they different from one another?"*** To answer that question, a measure of the ***difference*** between two shapes was developed [3].

REVIEW OF SHAPE DIFFERENCE INDICATOR (SDI)

The Shape Difference Indicator is defined with the formula,

$$SDI = \left(1 - \frac{\left\| \{A\} - \{B\} \right\|^2}{\{A\}^h\{A\} + \{B\}^h\{B\}} \right)^2 \qquad (2)$$

or

$$SDI = \left(\frac{2\, real(\{A\}^h\{B\})}{\{A\}^h\{A\} + \{B\}^h\{B\}} \right)^2 \qquad (3)$$

real = real part of the shape product

SDI values also range between **0 & 1**. If two shapes have ***identical shape components***, **SDI = 1**. If two shapes have ***different shape components***, **SDI < 1**. Several examples illustrate typical SDI values.

- If {A} = {B}, SDI = 1
- If {A} = {0} *or* {B} = {0}, SDI = 0
- If {A} = 2{B}, SDI = 0.64
- If {A} = 10{B}, SDI = 0.04

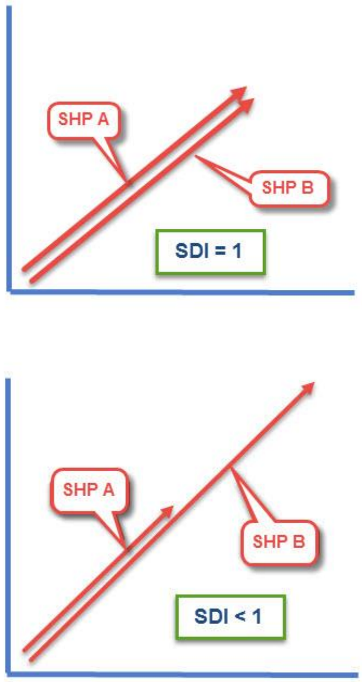

Figure 3 SDI Cases.

IDENTIFYING CAP SCREW TORQUE

SDI can be used to detect differences between two shapes, no matter what type of data they contain. To illustrate this, different amounts of torque where applied to one of the Allen screws that attach the top plate to the back plate of the Jim Beam, shown in Figure 4.

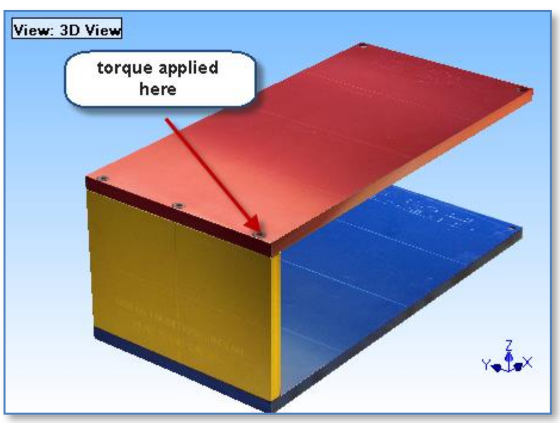

Figure 4. Torque Applied To Allen Screw

Six different torque values were applied to the Allen screw on the Jim Beam;

Case 1: 10 in-lbs
Case 2: 15 in-lbs
Case 3: 20 in-lbs
Case 4: 25 in-lbs
Case 5: 30 in-lbs
Case 6: 35 in-lbs

The Jim Beam was impact tested after each of the 6 torques was applied to the Allen screw. For each case, three FRFs were calculated (between the force and the three acceleration responses), and the FRFs were curve fit to extract the modal frequency & damping of six modes.

The frequency & damping of six modes of the Jim Beam were stored as shape components in two shape tables. The ***"modal frequency shapes"*** are listed in Figure 5, and the ***"modal damping shapes"*** are listed in Figure 6.

SHP: All Frequencies

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	Label
1	0	0	Hz	0	10 IN-LBs
2	0	0	Hz	0	15 IN-LBs
3	0	0	Hz	0	20 IN-LBs
4	0	0	Hz	0	25 IN-LBs
5	0	0	Hz	0	30 IN-LBs
6	0	0	Hz	0	35 IN-LBs

Modal frequencies stored as shapes

Select M#	Shape 1 Magnitude	Shape 2 Magnitude	Shape 3 Magnitude	Shape 4 Magnitude	Shape 5 Magnitude	Shape 6 Magnitude
M#1	155.4	155.5	156.1	156.3	156.5	156.3
M#2	189.2	190.4	191.2	191.7	192.1	192.4
M#3	341.2	341.6	342	342.2	342.4	342.4
M#4	429.2	430.1	430.8	431.2	431.6	431.8
M#5	481.8	482.6	483.3	483.8	484.2	484.4
M#6	562.7	564.6	565.7	566.5	567.1	567.7

Figure 5. Modal Frequency Shapes

SHP: All Damping

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	Label
1	0	0	Hz	0	10 IN-LBs
2	0	0	Hz	0	15 IN-LBs
3	0	0	Hz	0	20 IN-LBs
4	0	0	Hz	0	25 IN-LBs
5	0	0	Hz	0	30 IN-LBs
6	0	0	Hz	0	35 IN-LBs

Modal damping stored as shapes

Select M#	Shape 1 Magnitude	Shape 2 Magnitude	Shape 3 Magnitude	Shape 4 Magnitude	Shape 5 Magnitude	Shape 6 Magnitude
M#1	0.5556	0.7114	0.5825	0.577	0.5693	0.6964
M#2	0.6311	0.6152	0.5897	0.5779	0.6038	0.6215
M#3	1.104	1.258	1.155	1.051	1.145	1.212
M#4	0.8748	0.83	0.8684	0.8628	0.8617	0.8264
M#5	0.09772	0.735	0.7471	0.797	0.7207	0.7349
M#6	1.464	1.536	1.509	1.491	1.459	1.526

Figure 6. Modal Damping Shapes

The expected result regarding modal frequency is in evidence by examining each row of modal frequencies in Figure 5. As the screw torque was increased (from Shape 1 to Shape 6), the modal frequencies of all six modes also increased compared to the previous case.

A (perhaps unexpected) result occurred with the modal damping values, however. By examining each row of modal damping values in Figure 6, the damping of the structure was not significantly affected by changes in the torque applied to the Allen screw. The modal damping values in each row ***did not change significantly*** from one shape to the next one.

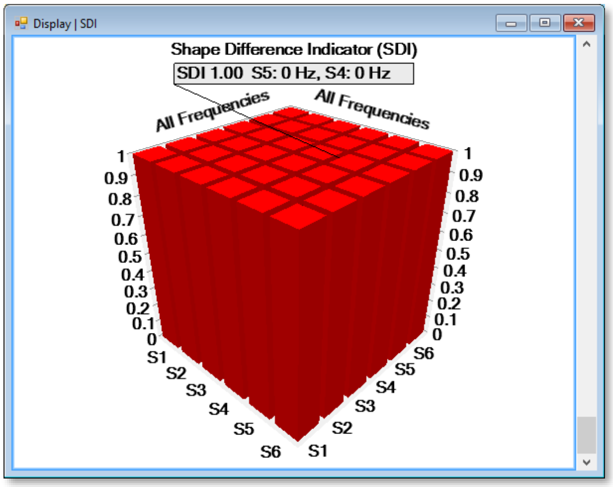

Figure 7. SDI Values of Modal Frequency Shapes

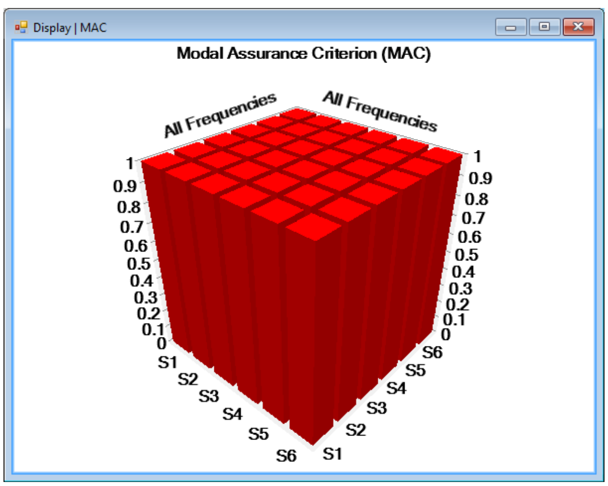

Figure 8. MAC Values of Modal Frequency Shapes

SDI & MAC WITH MODAL FREQUENCY SHAPES

The bar chart in Figure 7 is the SDI values for all pairs of modal frequency shapes in Figure 5. Each ***diagonal bar*** is the SDI value for a case with itself, which has the expected value of 1. Each ***off-diagonal bar*** is the SDI value for a pair of dissimilar stiffness cases.

OFF-DIAGONAL PROPERTY: If an ***off-diagonal*** bar is ***less than 0.9*** in a chart of bars between different pairs of shapes, the SDI (or MAC) metric can be used to ***uniquely distinguish*** between the two cases associated with the shapes.

The desired off-diagonal property ***is not exhibited*** in the SDI bar chart in Figure 7, nor in the MAC bar chart in Figure 8. Both bar charts show that ***neither SDI nor MAC could be used as a metric*** for distinguishing between the modal frequencies for the six stiffness cases.

However, a ***unique characteristic*** of the SDI metric can be exploited to increase its sensitivity. With increased sensitivity, SDI can be used to distinguish the difference between all pairs of frequency shapes in Figure 5.

INCREASED SDI SENSITIVITY

Figure 9 is a plot of three SDI curves for different values of two shapes {u} & {v}. In general, {u} & {v} are vectors, but in this case each vector only has a real component. Three curves are plotted for the shape {v} = 1, 10, 100.

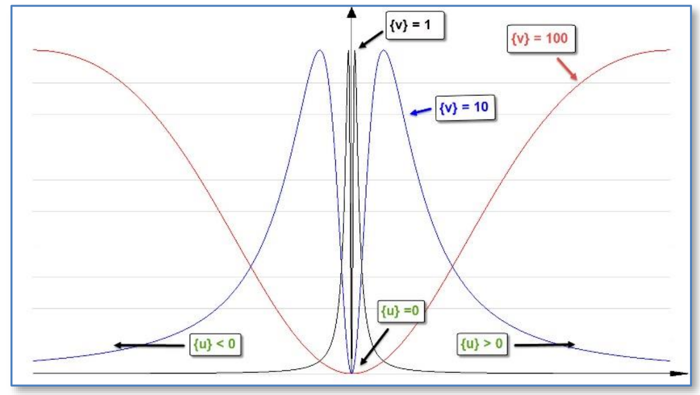

Figure 9. SDI Curves for {u} & {v}

These SDI curves have two unique properties,

1. When {u} = 0, SDI = 0
2. The SDI curve ***flattens out*** for large values of {v}, and ***sharpens up*** for small values of {v}

Another way of interpreting the curves in Figure 9 is that ***SDI is more sensitive to shape differences when the shape values are smaller***.

All of the frequency shape values in Figure 5 are above 100, so the SDI values in Figure 7 when calculated using a ***very flat SDI surface.*** For shapes with multiple components **(m>1)**, SDI is a surface instead of a line like that in Figure 9. Hence the SDI bars in Figure 7 are not a very sensitive measure of the shape differences.

However, the curves in Figure 9 show that the sensitivity of SDI can be increased if the vectors {u} & {v} are replaced with the following new vectors, which are both closer to the origin of the SDI surface.

$\{\underline{\mathbf{v}}\}$ = {small number} where (small number > 0)

$\{\underline{\mathbf{u}}\}$ = $\{\underline{\mathbf{v}}\}$+ ({u} – {v})

Replacing {u} & {v} with $\{\underline{\mathbf{u}}\}$ & $\{\underline{\mathbf{v}}\}$, where $\{\underline{\mathbf{v}}\}$ contains small numbers, moves the peak in the SDI surface closer to the origin where it ***makes faster transitions between 0 & 1***. This makes the SDI calculation more sensitive to the difference between {u} & {v}.

MODAL FREQUENCY SHAPES WITH INCREASED SENSITIVITY

Figures 10, 11, & 12 contain SDI bar charts of the modal frequency shapes for the six stiffness cases, but with increased sensitivities using ***small numbers = 0.5, 0.125, and 0.01***.

Clearly, increasing the sensitivity of SDI turned it into a useful metric for ***uniquely identifying all six torque cases***.

An off-diagonal SDI value ***less than 0.90*** makes it useful for distinguishing one stiffness case from another.

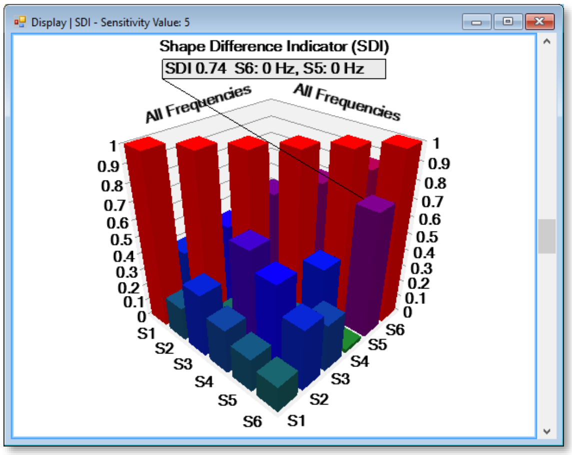

Figure 10. SDI for Frequency Shapes (Sensitivity = 0.5)

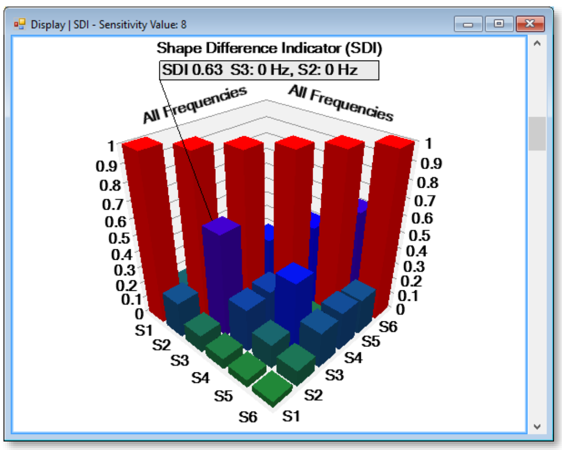

Figure 11. SDI for Frequency Shapes (Sensitivity = 0.125)

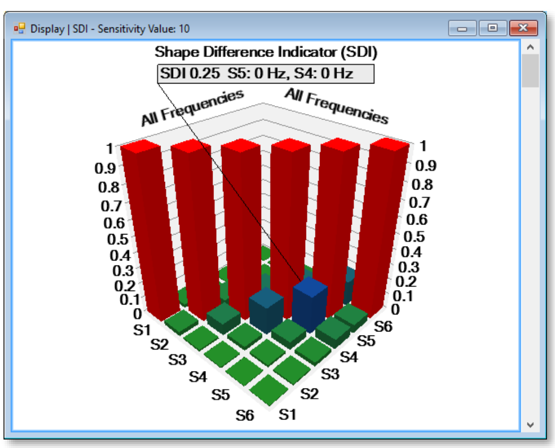

Figure 12. SDI for Frequency Shapes (Sensitivity = 0.01)

MODAL DAMPING SHAPES WITH INCREASED SENSITIVITY

Figures 13 & 14 contain SDI bar charts of the modal damping shapes for the five stiffness cases. No sensitivity was used in Figure 13, and a ***sensitivity of 0.01*** was used in Figure 14.

Even with a high sensitivity, some of the SDI bars in Figure 14 have values ***higher than 0.9***. This means that those pairs of stiffness cases are ***not distinguishable*** from one another.

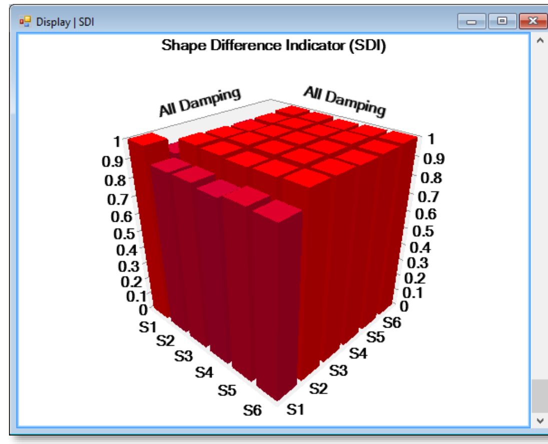

Figure 13. SDI for Damping Shapes (no Sensitivity)

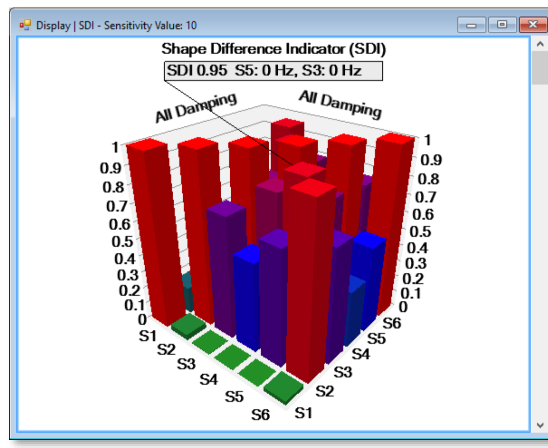

Figure 14. SDI for Damping Shapes (Sensitivity = 0.01)

FAULT CORRELATION TOOLS (FaCTs™)

Using SDI as a search criterion for correlating a currently acquired shape with shapes archived in a database has been trade marked as **FaCTs™**, an acronym for **Fault Correlation Tools**. If each of the archived shapes has been associated beforehand with a ***known mechanical fault or condition*** (cap screw torque in this case), SDI can be used to identify the specific fault. Using the FaCTs™ search method, the FaCTs™ bars with highest SDI values between the current shape and archived shapes are displayed in a bar chart. Each bar is labeled with the fault or condition that correlated highest with the current shape.

In Figure 15, the highest FaCTs™ bar identified the current shape as being associated with the **Case 4: 25 in-lbs** of cap screw torque.

FaCTs™ can be used in a number of ways for post-processing data in an online machine health monitoring or structural health monitoring system. As part of an on-line monitoring system, FaCTs™ can be used to graphically indicate any change in a machine operating condition, or to identify a specific structural change such as the torque changes illustrated in this paper.

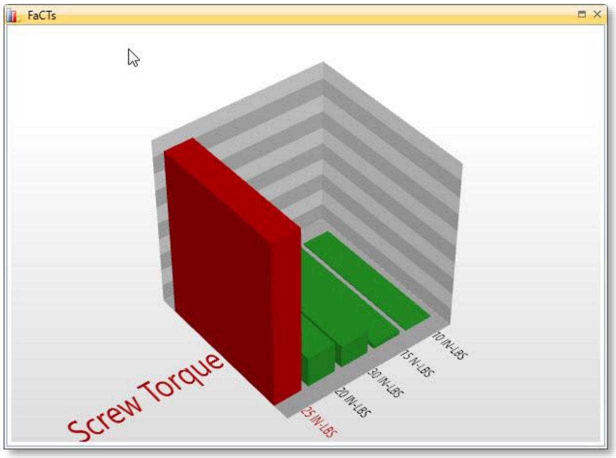

Figure 15. FaCTs™ Identifying 25 in-lbs of Torque.

FaCTs™ can also be used as a pass-fail indicator in a qualification testing system. Vibration parameters, or any other types of engineering parameters acquired from a test article can be numerically compared with archived parameters as a means of passing or failing the test article. FaCTs™ has been implemented in the Vibrant Technology ME'scope software.

CONCLUSION

The example used in this paper illustrated a well known fact, namely that ***resonant vibration is very sensitive*** to changes in the physical properties of a structure. When a physical change in a structure occurs, that change can be detected by measuring changes in the modal parameters of the structure. Furthermore, modal parameters can be used to identify a specific physical change by correlating them with archived parameters stored as shapes.

Stiffness changes were introduced into the Jim Beam by applying different amounts of torque to one of the cap screws used to attach the top plate to the back plate. The amounts of torque applied were different by a ***very small amount, only 5 in-lbs***.

The Jim Beam was impact tested after each of the five different torques was applied to the Allen screw. Then the resulting FRFs were curve fit to extract the modal parameters of six modes. These parameters were assembled into two tables of shape vectors, one for the modal frequencies and the other for modal damping.

Two different measures of shape correlation, MAC & SDI, were applied to the modal parameters in the two shape tables. Both measures indicate the likeness of a pair of shapes. Each is a ***correlation coefficient*** for a pair of shapes, with values between 0 & 1. A value of 1 means that the two shapes are the same, and a value less than 1 means that they are different.

MAC only measures the ***co-linearity*** of two shapes however, and did not indicate any difference between pairs of modal frequency shapes for the six stiffness cases.

SDI measures the ***true difference*** between two shapes, but it also did not initially indicate any difference between pairs of modal frequency shapes for the six stiffness cases.

However, a unique property of SDI was used to increase its sensitivity, namely that when the ***values of the shapes are closer to the origin,*** SDI transitions more rapidly between 1 and 0 when the two shapes are different.

Thus, by using ***two modified shapes***, one with ***values close to 0***, and the other containing those values plus the ***difference between the two original shapes***, the sensitivity of SDI was increased. With increased sensitivity, SDI could be used to clearly identify all six torque cases based on differences between their modal frequency shapes.

SDI was also applied to the modal damping shapes, but even with increased sensitivity, most but not all of the torque cases could be distinguished.

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