Modal Testing using the Slinky™ Method

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ABSTRACT

Conventional broad-band modal testing is done by acquiring a single-reference or multiple-reference set of FRFs and curve-fitting them to obtain modal parameters. Since a (fixed) reference sensor is required throughout the data acquisition process, testing a large structure requires that a (potentially) long wire be used to connect the reference sensor to the acquisition system.

In a previous paper [1], a new modal testing method was introduced which does not require the use of a fixed reference sensor. This method is based on the calculation of a series of Transmissibility's, called a TRN chain. This method has several important advantages,

1. Excitation forces need not be acquired
2. Only two response sensors are required for data acquisition
3. The two sensors can be physically close to one another throughout data acquisition

Since the excitation forces need not measured, data for calculating a TRN chain can be acquired from an operating machine, or during any test where excitation is provided by impacting or by using one or more shakers.

A Slinky test is a unique way of acquiring a TRN chain. In a Slinky test, one sensor is merely ***"hopped over"*** the other sensor with each new acquisition, as shown in Figure 1. As the two sensors are moved over the surface of the structure in this manner, a ***"chain"*** of Transmissibility's is calculated from the acquired data.

A TRN chain can be ***"seeded"*** with an Auto spectrum, Cross spectrum, Fourier spectrum, or FRF to yield a single-reference set of measurements, from which Operating Deflection Shapes (ODS's) or experimental modal parameters can be extracted. A Slinky test is much faster, easier, and less costly than a conventional modal test.

KEY WORDS

Transmissibility chain (TRN chain)
Fourier spectrum (DFT)
Auto power spectrum (APS)
Cross power spectrum (XPS)
Frequency Response Function (FRF)
Operating Deflection Shape (ODS)
Experimental Modal Analysis mode shape (EMA mode shape)
Operational Modal Analysis mode shape (OMA mode shape)
Modal Assurance Criterion (MAC)
Shape Difference Indicator (SDI)

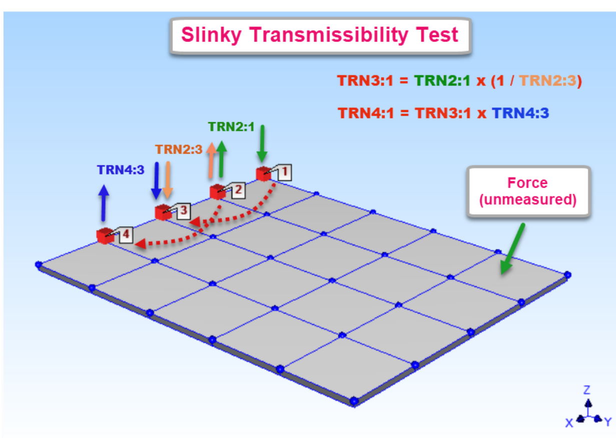

Figure 1. Slinky Test

INTRODUCTION

To obtain the experimental ODS's or mode shapes of a machine or structure, each degree-of-freedom (DOF) of a shape must contain the ***correct magnitude & phase relative to all other DOFs***. If all sensor data is ***simultaneously acquired***, each shape will contain the correct relative magnitudes & phases. However, simultaneous acquisition requires that all sensors be connected to a multi-channel acquisition system during the test so that all vibration time waveforms can be acquired from all data channels at once.

In traditional broad-banded testing, a set of cross-channel measurements is calculated from acquired time waveforms where a ***fixed reference sensor*** is used throughout the test. For large test articles, the wire from the reference sensor to the acquisition system ***could be very long***. In a roving impact test, the wire from the instrumented hammer to the acquisition system ***could also be long***.

Operational modal parameters (frequencies damping, & mode shapes) are typically obtained by curve fitting a ***single reference set*** of output-only Cross spectra or ODS FRFs. An ***experimental modal model*** (a set of scaled mode shapes reflecting the actual mass, stiffness and damping properties of the structure) is typically obtained by curve fitting a ***single reference set*** of ***calibrated FRFs***.

Cross-channel Measurements

Several different kinds of cross-channel functions can be calculated from two channels of acquired time waveforms,

1. An **output-only Cross spectrum** is a frequency-based cross-channel measurement defined as the Fourier spectrum of one response multiplied by the complex conjugate of the Fourier spectrum of the other response.
2. An **FRF** is a frequency-based cross-channel measurement defined as the Fourier spectrum of a structural response (in displacement, velocity, or acceleration units) divided by the Fourier spectrum of an excitation force that caused the response.
3. An **ODS FRF** is a frequency-based cross-channel measurement defined as the Auto spectrum of a vibration response together with the phase between the response and a reference response.
4. A **Transmissibility** is a frequency-based cross-channel measurement defined as the Fourier spectrum of a vibration response divided by the Fourier spectrum on another response

A **Transmissibility** is depicted in Figure 2.

Transmissibility TRN (2:1) is the Fourier spectrum of the response at DOF 2 (point & direction) divided by the Fourier spectrum of the responses at DOF 1 (point & direction)

$$TRN(2{:}1) = {FFT(2)}/{FFT(1)} = {Accel}/{Accel}$$

Figure 2. Transmissibility

TRANSMISSIBILITY PROPERTIES

A Transmissibility has ***two unique properties*** that make it useful for post-processing as part of a Slinky test to yield a ***single-reference set*** of cross-channel measurements.

1. The Transmissibility between DOF 1 & DOF 2 multiplied by the Transmissibility between DOF 2 & DOF 3 equals the Transmissibility between DOF 1 & DOF 3
2. The Inverse of the Transmissibility between DOF 1 & DOF2 is the Transmissibility between DOF 2 & DOF 1

These two Transmissibility properties are shown in Figure 3 and are used to post-process the data in a TRN chain of measurements. A TRN chain is described in the next section.

The Product of two Transmissibilitys is another Transmissibility

$$TRN(3{:}1) = TRN(3{:}2)\ x\ TRN(2{:}1)$$

The Inverse of $TRN(2{:}1)$ is $TRN(1{:}2)$

$$TRN(1{:}2) = \left({}^{1}/_{TRN(2:1)}\right)$$

Figure 3. Transmissibility Properties

SLINKY TRN CHAIN

In a Slinky test, data is acquired in a chain fashion as depicted in Figure 1. The three measurements shown in Figure 1 are made with two sensors, (for example accelerometers), and a 2-channel acquisition system.

> Not only is the equipment required to do a Slinky test much less costly, but this method can be used to test ***any sized test article***, especially large ones.

The sensor data used in the **denominator** of a Transmissibility is called the **Input**, and the sensor data used in the **numerator** is called the **Output**. The test procedure depicted in Figure 1 is as follows;

- Attach sensors to ***points 1 & 2***
- Acquire vibration data from points 1 & 2. Designate ***point 1 as Input*** and ***point 2 as Output***
- Calculate **TRN(2:1)**
- Move the sensor from ***point 1*** to ***point 3***
- Acquire vibration data from points 2 & 3. Designate ***point 3 as Input*** and ***point 2 as Output***
- **C**alculate **TRN(2:3)**
- Move the sensor from ***point 2*** to ***point 4***
- Acquire vibration data from points 3 & 4. Designate ***point 3 as Input*** and ***point 4 as Output***
- Calculate **TRN(4:3)**

> In a Slinky test, ***only one sensor must be moved*** between acquisitions. ***Either sensor can be moved*** between acquisitions. ***Hopping one sensor*** over the other is ***not necessarily required***.

Regardless of how the TRN chain is acquired and calculated, it can always be post-processed into a single-reference TRN chain using the two unique properties of Transmissibility's shown in Figure 3. It is also shown in Figure 1 how the TRN chain is post-processed to yield a ***single-reference TRN chain***.

BENEFITS OF A TRN CHAIN

A TRN chain can be measured using a ***pair of uni-axial sensors*** or a ***pair of tri-axial sensors*** as described in Figure 4. Using ***two tri-axial sensors*** will capture the ***3D motion*** of a machine or structure at each test point, thus yielding ODS's and mode shapes that describe its ***3D motion***.

The benefits of TRN chain testing are summarized in Figure 5.

> The greatest benefit of Slinky testing is that the excitation forces do not have to be measured.

This benefit means that Slinky testing can be used to acquire data from operating machines or vehicles where the excitation forces cannot be measured. Also, since acquisition of the excitation force (or forces) is not required, any artificial excitation forces can be provided, either by impacting at a fixed DOF or by providing steady state excitation with one or more shakers.

> Any pair of sensors, two ***uni-axial sensors***, two ***tri-axial sensors***, or ***one uni-axial*** and ***one tri-axial***, sensor can be used for a Slinky test

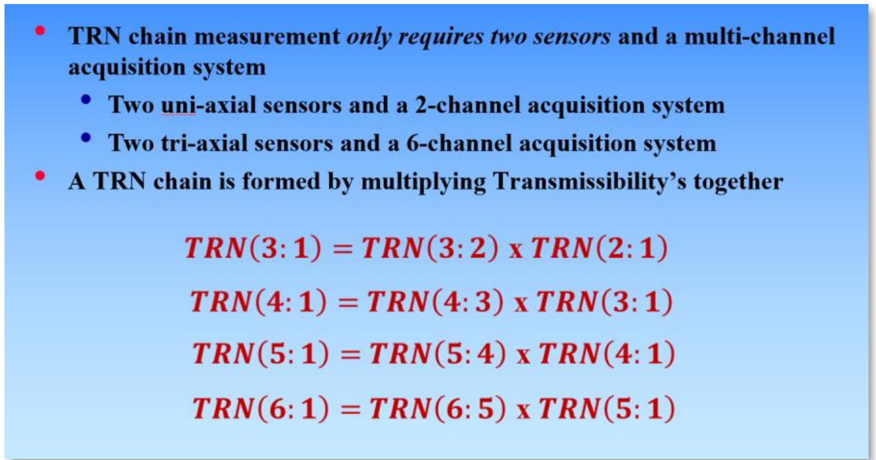

Figure 4. TRN Chain from Two Sensors

- Excitation forces need not be acquired
- Only two response sensors are required for data acquisition
- Variations of response levels are accounted for by the Transmissibility's
- The same TRN chain can be seeded with an Auto spectrum,
  Cross spectrum, Fourier spectrum, or FRF

Figure 5. TRN Chain Benefits

SEEDING A TRN CHAIN

Once a TRN Chain has been calculated for all points & directions (DOFs) on the test article, it can be **"seeded"** using either an Auto spectrum, Cross spectrum, Fourier spectrum, or an FRF to yield a single reference set of measurements from which ODS's and mode shapes can be extracted.

- Seeding with an Auto spectrum yields a ***single reference set*** of ODS FRFs
- Seeding with a Cross spectrum yields a ***single reference set*** of Cross spectra
- Seeding with a Fourier spectrum yields a ***single reference set*** of Fourier spectra
- Seeding with an FRF yields a ***single reference set*** of FRFs

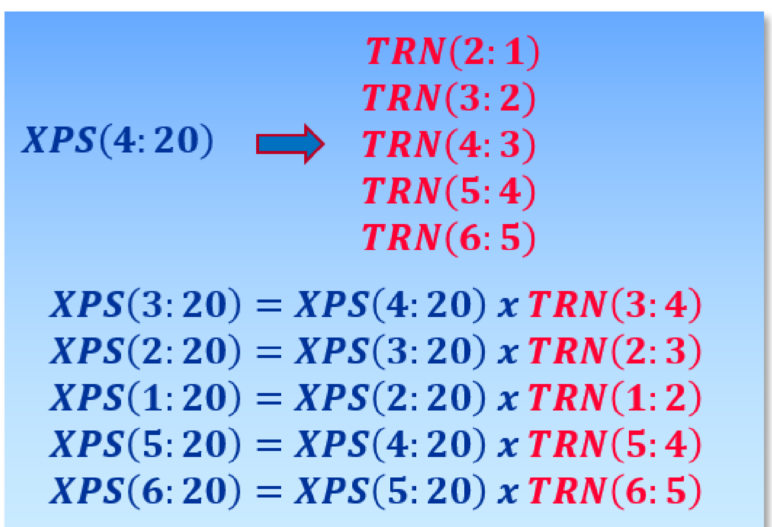

Figure 6. Seeding with a Cross Spectrum

$$\boldsymbol{FRF}(4{:}20) \Rightarrow \begin{array}{l} \boldsymbol{TRN}(2{:}1) \\ \boldsymbol{TRN}(3{:}2) \\ \boldsymbol{TRN}(4{:}3) \\ \boldsymbol{TRN}(5{:}4) \\ \boldsymbol{TRN}(6{:}5) \end{array}$$

$$\boldsymbol{FRF}(3{:}20) = \boldsymbol{FRF}(4{:}20)\ \boldsymbol{x}\ \boldsymbol{TRN}(3{:}4)$$
$$\boldsymbol{FRF}(2{:}20) = \boldsymbol{FRF}(3{:}20)\ \boldsymbol{x}\ \boldsymbol{TRN}(2{:}3)$$
$$\boldsymbol{FRF}(1{:}20) = \boldsymbol{FRF}(2{:}20)\ \boldsymbol{x}\ \boldsymbol{TRN}(1{:}2)$$
$$\boldsymbol{FRF}(5{:}20) = \boldsymbol{FRF}(4{:}20)\ \boldsymbol{x}\ \boldsymbol{TRN}(5{:}4)$$
$$\boldsymbol{FRF}(6{:}20) = \boldsymbol{FRF}(5{:}20)\ \boldsymbol{x}\ \boldsymbol{TRN}(6{:}5)$$

Figure 7. Seeding with an FRF

Seeding a TRN chain with a Cross spectrum is depicted in Figure 6. The same TRN chain is seeded with an FRF in Figure 7. Notice that the ***reference DOF*** of the Cross spectrum or FRF seed can be from ***anywhere on the machine or structure***.

The only requirement for seeding a TRN chain is that the ***Roving DOF of the seed be the same as one of the DOFs*** in the chain.

ROUND TRIP SIMULATIONS

To demonstrate the Slinky testing method, two ***round trip simulations*** are used,

- Slinky test of an aluminum plate using ***two uni-axial*** accelerometers
- Slinky test of the Jim Beam using ***two tri-axial*** accelerometers

In both simulations, the following steps were carried out,

1. Experimental FRFs were acquired by impact testing the structure
2. The FRFs were used to calculate responses of the structure due to random excitation
3. A TRN chain was calculated from the random responses
4. The TRN chain was seeded with an FRF
5. The single-reference FRFs derived from the TRN chain were compared with the original FRFs

Experimental FRFs

To provide its simulated response data, each structure was first impact tested to obtain a single-reference set of FRFs. A ***roving impact test*** was performed on the aluminum plate with an accelerometer attached at one corner (DOF 1Z). The plate was impacted at 30 points (DOFs 1Z to 30Z), and 30 FRFs were calculated between each impact DOF and the fixed reference DOF 1Z. The log magnitudes of several FRFs from the aluminum plate are shown in Figure 8. Notice in the property's spreadsheet that all the FRFs have the ***same reference DOF 1Z***.

A ***roving accelerometer test*** was performed on the Jim Beam. A model of the Jim Beam is shown in Figure 16. It was impacted at one corner of the top plate (DOF 15Z) throughout the test. A tri-axial accelerometer was moved from point 1 to point 33 between acquisitions of data. Ninety-nine FRFs were calculated, each FRF between the impact DOF and the 3D motion at each point on the beam. The log magnitudes of several FRFs from the aluminum plate are shown in Figure 9. Notice in the property's spreadsheet that all the FRFs have the ***same reference DOF -15Z***.

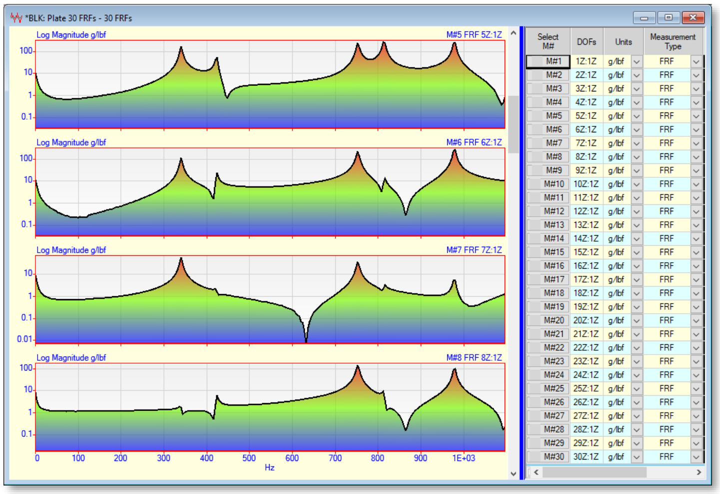

Figure 8. Experimental FRFs from the Aluminum Plate

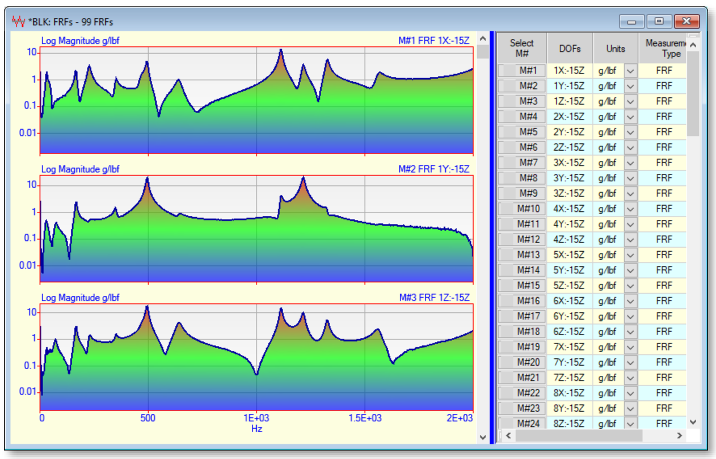

Figure 9. Experimental FRFs from the Jim Beam

MIMO MODELING & SIMULATION

Using its FRFs as its dynamic model, random excitation of each structure was simulated by calculating its responses to a random excitation force. This calculation was done using the FRF matrix-based MIMO Modeling & Simulation depicted in Figure 10. The following steps were used to calculate time domain waveforms due to a random excitation force,

1. A sequence of ***ten random time waveforms*** was created as ***a simulated force input*** to each structure
2. The experimental FRFs were multiplied by the Fourier spectrum of the Input force to obtain the Fourier spectrum of each response Output
3. The Fourier spectrum of each Output was Inverse transformed to obtain the time waveform of a random response

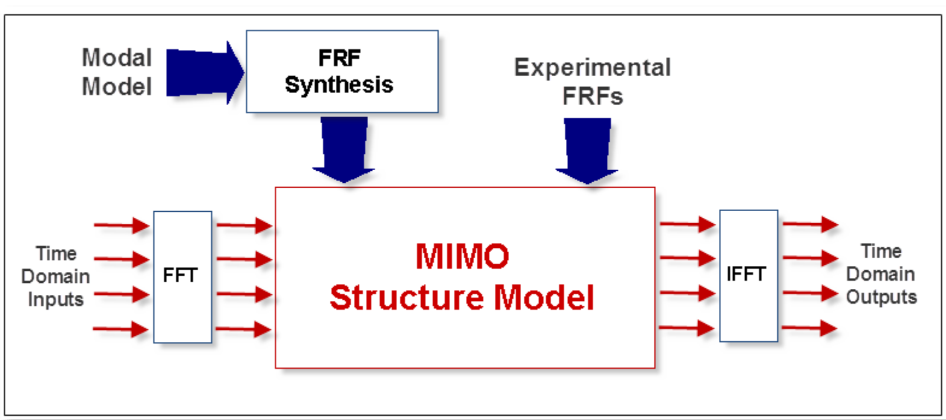

Figure 10. MIMO Modeling & Simulation

This process is referred to as **Multi-Input Multi-Output (MIMO) Modeling & Simulation.** The Fourier spectrum of the random Input and the Fourier spectra of several Outputs for the aluminum plate are shown in Figure 11. The spectra each have 10,000 samples in them. The original random force time waveform was created with 20,000 samples, enough samples to calculate Transmissibility's with 1000 samples each, using 10 spectrum averages to remove the random noise from the data. Each time window of 2000 samples also had a ***Hanning window*** applied to it before transforming it to a Fourier spectrum.

A Hanning window must be applied to the random time waveforms to ***minimize the leakage (smearing) effects*** of the ***non-periodic signals*** on their Fourier spectra.

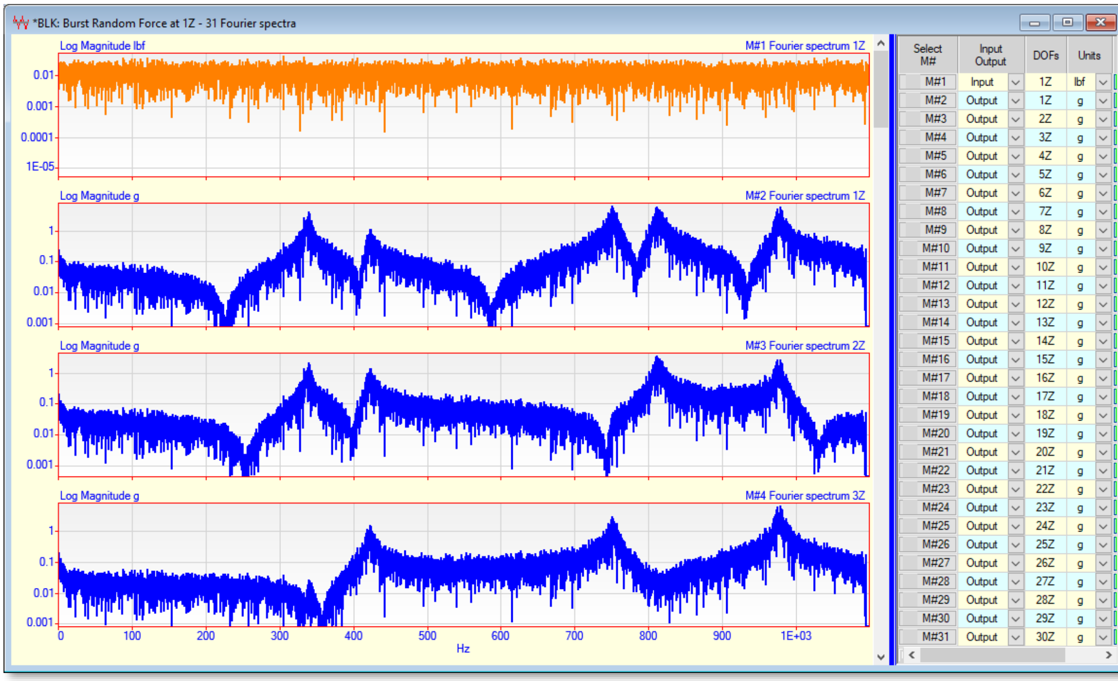

Figure 11. Fourier Spectra of Excitation Force & Three Plate Responses

SLINKY TEST OF THE ALUMINUM PLATE

A Slinky test using a ***pair of uni-axial accelerometers*** was simulated on the aluminum plate by ***"simultaneously acquiring"*** random responses from a pair of points at a time. Each pair of responses was used to calculate one Transmissibility. The random excitation force was applied at **DOF 1Z**, as shown in Figure 12.

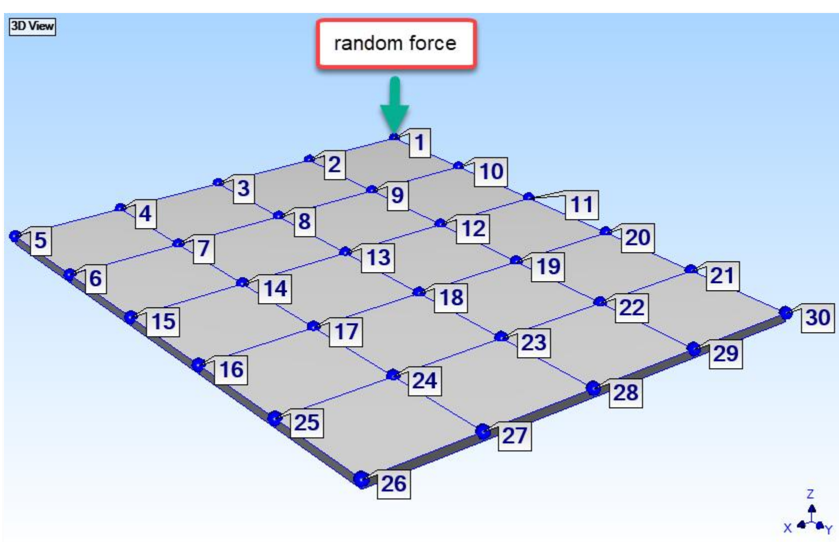

Figure 12. Test Points on the Plate

The plate has a (5 by 6) grid of 30 numbered test points. A TRN chain of 29 Transmissibility's was calculated from data ***"acquired"*** from each successive pair of grid points on the plate. Some of the Transmissibility's are displayed in Figure 13. Notice in the property's spreadsheet to the right that each Transmissibility has a ***different reference DOF***.

- A Transmissibility is a ***different complex waveform*** than an FRF
- ***Peaks*** in a Transmissibility are ***not resonance peaks***.
- Transmissibility's ***cannot be curve fit*** using an FRF-based curve fitting method to obtain mode shapes.

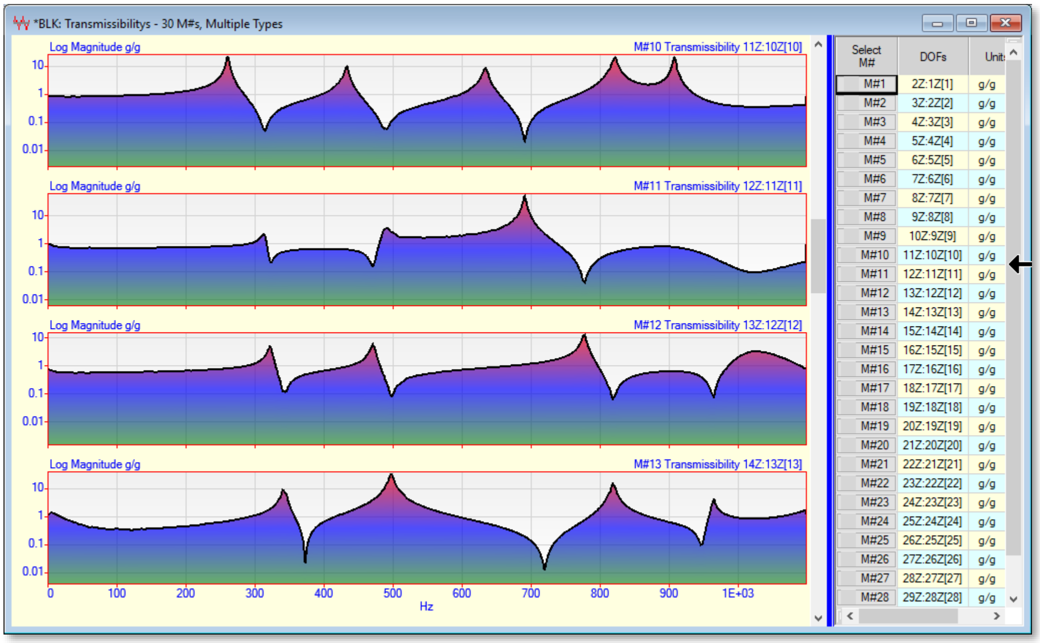

Figure 13. TRN Chain from the Plate

Seeding the Plate TRN Chain with an FRF

The TRN chain of 29 Transmissibility's was ***seeded*** with **FRF 15Z:1Z**. This FRF is between **Roving DOF 15Z** and **Reference DOF 1Z**. Seeding the TRN chain with **FRF 15Z:1Z** yielded a set of ***FRFs with Reference DOF 1Z***, some of which are shown in Figure 14.

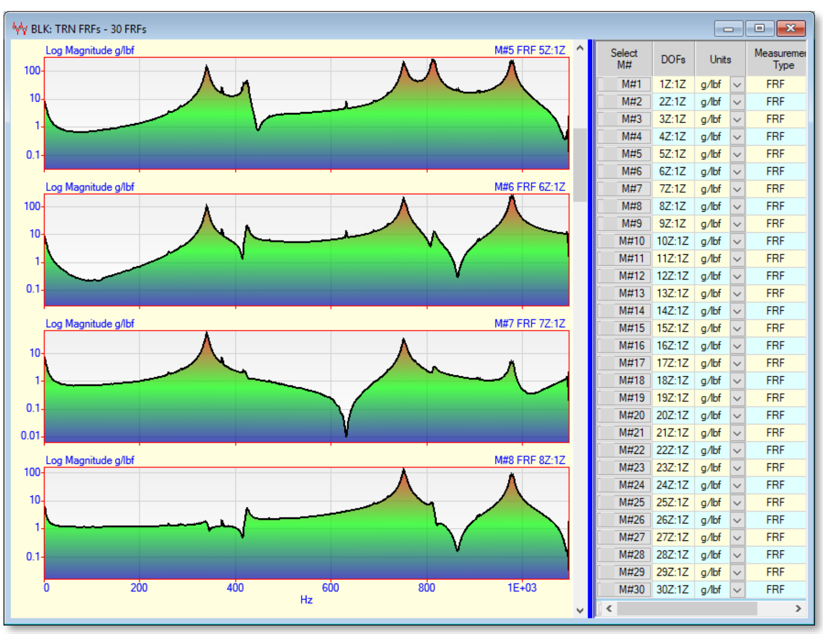

Figure 14. FRFs From Seeding the Plate TRN Chain

A TRN chain can be seeded with any FRF, provided that the ***Roving DOF of the seed matches*** one of the DOFs in the TRN chain.

FRF 15Z:1Z was chosen as the seed so that the resulting single-reference set of FRFs would have **reference DOF 1Z**, hence they could be numerically compared with the original experimental FRFs that were used to calculate the random responses of the plate.

Comparing Experimental & Slinky FRFs of the Plate

To confirm the accuracy of the simulated Slinky test on the aluminum plate, the FRFs derived from the Slinky TRN chain were numerically compared with the original experimental FRFs. The results are shown in Figure 15. The two sets of FRFs were numerically compared at each frequency sample using the SDI metric [2]. Like the MAC metric [3], SDI has values between 0 & 1. MAC measures the ***co-linearity*** of two shapes. SDI measures the ***difference*** between two shapes.

An ***SDI value of "1"*** at a sample means that the ODS (complex values of the FRFs) in one set of FRFs equals the values of the FRFs for the same sample in the other set.

An ***SDI value above 0.90*** at a sample indicates a ***strong correlation*** between the two sets of FRFs at that sample.

It is clear from Figure 15 that the round-trip simulation yielded FRFs derived from the TRN chain that ***closely correlated at all frequencies*** with the original experimental FRFs that were used to create the random responses of the plate.

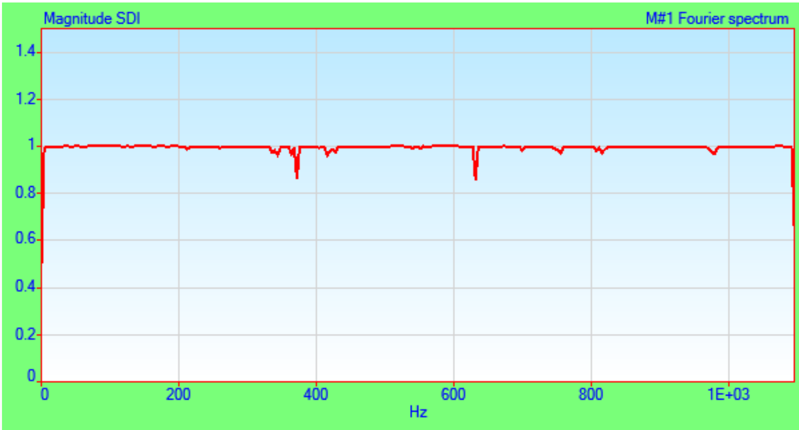

Figure 15. Correlation of Experimental & Slinky FRFs for the Plate

SLINKY TEST OF JIM BEAM

A Slinky test using a ***pair of tri-axial accelerometers*** was simulated on the Jim Beam by ***"simultaneously acquiring"*** its 3D random responses at successive pairs of points. In this example six channels of data were ***"simultaneously acquired"*** from each pair of points to simulate a test using ***tri-axial accelerometers***. Three responses from each point (in the X, Y, & Z directions) were used to calculate Transmissibility's. Each Transmissibility was calculated between a pair of response DOFs, providing a total of five Transmissibility's between each pair of points.

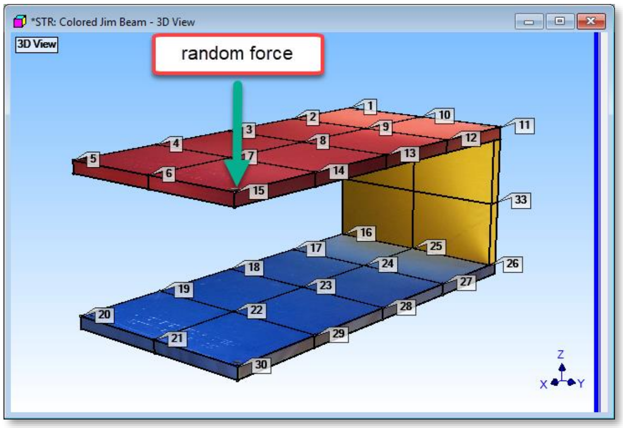

Figure 16. Slinky Test Points on the Jim Beam

The same random force that was used to excite the aluminum plate was also applied to the Jim Beam. It was applied at **DOF 15Z**, as shown Figure 16. Ninety-nine experimental FRFs where used to model the dynamics of the Jim Beam between Input at DOF 15z and all 33 points. The MIMO response calculation procedure depicted in Figure 10 was used to calculate 99 random responses of the Jim Beam.

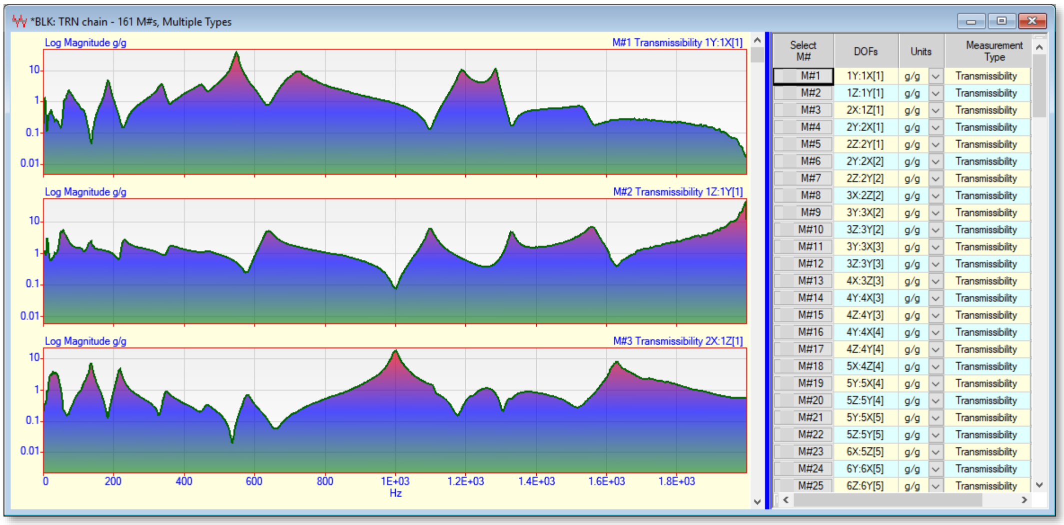

Figure 17. TRN Chain for Jim Beam

A TRN chain of ***160 Transmissibility's*** was calculated between each successive pair of points on the Jim Beam. Five Transmissibility's were computed for each of the 32 pairs of points. A portion of the TRN chain is shown in Figure 17. Notice in the property's spreadsheet to the right that each Transmissibility has a ***different reference DOF***.

Seeding the Jim Beam TRN Chain with an FRF

The TRN chain of 160 Transmissibility's was ***seeded*** with **FRF 4Z:15Z**. This FRF is between Roving DOF (4Z) and the Reference DOF (15Z). Seeding the TRN chain with **FRF 4Z:15Z** yielded a set of ***FRFs with reference DOF (15Z)***, some of which are shown in Figure 18.

A TRN chain can be seeded with any FRF, provided that the ***Roving DOF of the seed*** matches one of the DOFs in the chain.

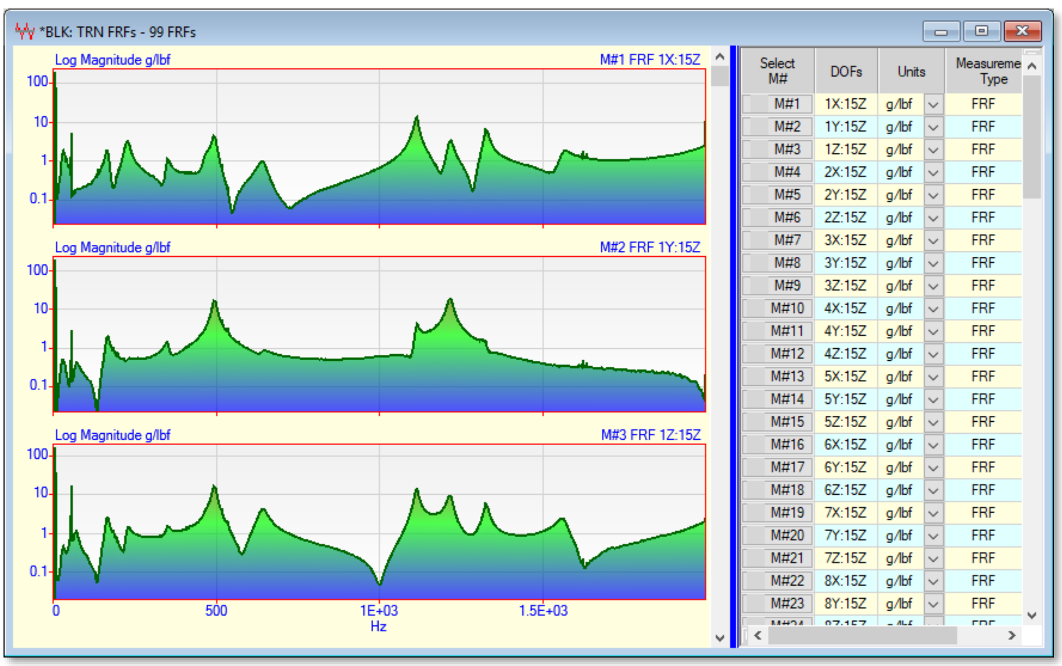

Figure 18. FRFs From Seeding the Jim Beam TRN Chain

Comparing Experimental & Slinky FRFs of the Jim Beam

To confirm the accuracy of the simulated Slinky test on the Jim Beam, the FRFs calculated by seeding the Slinky TRN chain were numerically compared with the original experimental FRFs. The results are shown in Figure 19. The two sets of FRFs are numerically compared at each frequency sample using the SDI metric [2]. SDI has values between 0 & 1.

An ***SDI value of "1"*** at a sample means that the ODS (complex values of 99 FRFs) in one set of FRFs equals the values of the 99 FRFs for the same sample in the other set. An ***SDI value above 0.90*** at a sample indicates a ***strong correlation*** between the two sets of FRFs at that sample. Except for a few samples at low frequencies, it is clear from Figure 19 that the round-trip simulation yielded FRFs derived from the TRN chain that ***closely correlated at all frequencies*** with the original experimental FRFs.

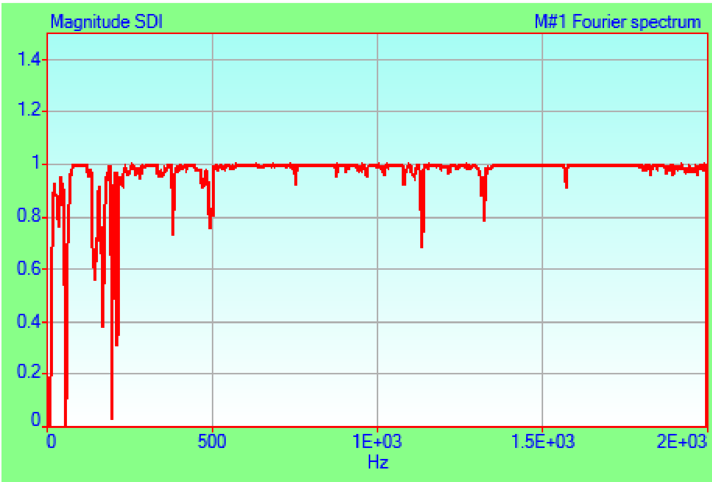

Figure 19. Correlation of Experimental & Slinky FRFs for the Jim Beam

CONCLUSION

In this paper, a new testing method was introduced which is based on the measurement of a chain of Transmissibility's called a TRN chain. Using this new method, ***any machine or structure, large or small***, can be tested with as little as ***two uni-axial response sensors***, a ***2-channel acquisition system***, and ***two short wires*** from the sensors to the acquisition system.

Moreover, this method can be used to test running machinery or large structures such as bridges and buildings because only the responses are measured. The ***excitation forces do not have to be measured***.

A ***single reference set*** of measurements is required to obtain experimental ODS's or mode shapes. If a conventional broadband testing method is used, ***one sensor must remain fixed*** throughout the data acquisition process. Consequently, the fixed reference sensor(s) must be connected by wire to the acquisition system, and this wire could be very long.

It was shown with two examples how a TRN chain is calculated from the simulated random responses of two different structures. The aluminum plate example illustrated testing with ***two uni-axial accelerometers***, and the Jim Beam example illustrated testing with ***two tri-axial accelerometers***. In both cases, single-reference sets of FRFs were obtained by seeding each TRN chain with an FRF seed. In both cases, the resulting FRFs ***closely correlated at all (or most) frequencies*** with the set of experimental FRFs that were used to create the random responses of the two structures.

Two unique properties of Transmissibility's turn the seeding of ***any*** TRN chain with an Auto spectrum, Cross spectrum, Fourier spectrum, or FRF into a straightforward calculation to yield a single-reference set of measurements.

Slinky testing makes the measurement of a TRN chain much easier than traditional single-reference testing. This simple process requires that ***only one sensor must be moved*** between acquisitions. It does not matter which one of the sensors is moved, if the Transmissibility is labeled with the correct Input (denominator) & Output (numerator) DOFs. Both sensors can also be moved provided that one sensor is moved to a DOF that was already acquired by the other sensor.

A drawback of this testing approach is that experimental noise, either in the seed of in some of the Transmissibility's, will propagate through the post-processing of the TRN chain. Noise propagated through the post-processing is the reason why there are low SDI values at some frequencies in Figure 19. However, using more spectrum averages (only 10 were used) in the TRN chain and seed calculations will reduce the noise in the results.

REFERENCES

[1] - P. McHargue, B. Schwarz, M Richardson, “ODS & Modal Testing Using a Transmissibility Chain”, IMAC XXXVI, February 2017

[2] - S. Richardson, J. Tyler, P. McHargue, B. Schwarz, M Richardson, “A New Measure of Shape Difference”, IMAC XXXII, February 3-6, 2014

[3] - R. J. Allemang "The Modal Assurance Criterion (MAC): Twenty Years of Use and Abuse", Proceedings of the International Modal Analysis Conference, 2002