All Vibration is a Summation of Mode Shapes

Brian Schwarz, Pat McHargue, Jason Tyler, Mark Richardson
Vibrant Technology, Inc.
Centennial, CO
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ABSTRACT
This paper expands on the ideas presented in two previous papers [1] & [2]. Here, we again show with examples how all vibration, whether it is represented in the form of time waveforms, frequency spectra, or ODS's, can be represented as a summation of mode shapes. The title of this paper is actually a universal law which is used for all modal analysis,

Fundamental Law of Modal Analysis (FLMA): All vibration is a ***summation*** of mode shapes.

The modal parameters of a structure can be obtained in two ways,

1. **Experimental Modal Analysis (EMA)**: EMA mode shapes are obtained by ***curve fitting*** a set of experimentally derived time waveforms or frequency spectra that characterize the structural dynamics

2. **Finite Element Analysis (FEA)**: FEA mode shapes are obtained as the ***eigensolution*** of a set of differential equations that characterize the structural dynamics

In this paper, it will be shown how the benefits of analytical FEA mode shapes can be combined with experimental data to yield more robust dynamic models [5], [7]. FEA mode shapes will be used to ***"decompose"*** and then ***"expand"*** experimental data to include DOFs that cannot or were not determined experimentally [4].

A ***unique advantage*** of this approach is that ***only mode shapes*** themselves are required. Modal frequency & damping are not used. Another ***unique advantage*** is that mode shapes from an FEA model ***with free-free boundary conditions*** and ***no damping*** can be used.

It usually requires a great deal of skill and effort to modify an FEA model and its boundary conditions so that its modal frequencies and mode shapes accurately match EMA modal frequencies and mode shapes. In addition, adding accurate damping to an FEA model is usually so difficult that damping is left out of the model altogether. The approach presented here circumvents both of these difficulties.

KEY WORDS
Auto power spectrum (APS)
Cross power spectrum (XPS)
Frequency Response Function (FRF)
Operating Deflection Shape (ODS)
Experimental Modal Analysis mode shape (EMA mode shape)
Finite Element Analysis mode shape (FEA mode shape)
Modal Assurance Criterion (MAC)
Shape Difference Indicator (SDI)\
Multi-Input Multi-Output (MIMO) Modeling & Simulation

INTRODUCTION
A mode of vibration is a mathematical representation of resonant vibration in a mechanical structure. When dynamic forces are applied and therefore energy is trapped within the boundaries of a structure, it will resonate. Any structure that is made out of one or more elastic materials will exhibit resonant vibration.

Resonant vibration can be thought of as a ***mechanical amplifier***. That is, when forces are applied to a structure, some of its modes will ***readily absorb the energy*** causing the structure to resonate with ***excessive levels of deformation***. This causes premature failure of bearings and gears, and material fatigue and failure.

Each mode is defined by three distinct parts; a ***natural frequency***, a ***damping or decay constant***, and a ***mode shape***. A mode shape represents the ***"standing wave deformation"*** of the structure at the natural frequency of a resonance. This standing wave behavior is caused when energy becomes trapped within the material boundaries of the structure, and must be absorbed by the damping mechanisms that are present.

Both EMA and FEA are based upon the FLMA. FLMA is ***assumed by all of the curve fitting methods*** that are used to extract EMA mode shapes from experimental data.

Modal Testing

All structural dynamic response data is initially acquired as multiple time waveforms. Using the FFT algorithm, each time waveform is transformed into its corresponding Fourier spectrum without loss of information. The Fourier spectra of multiple time waveforms can be further processed into a variety of frequency domain functions, including Auto & Cross spectra, FRFs, Transmissibility's, and ODS FRFs. All of these frequency functions preserve the structural dynamics originally captured in the time waveforms.

All modal testing is based on the FLMA, namely that all vibration data in the form of either time waveforms or frequency spectra is a ***summation of resonance curves***, each curve due to a mode of vibration.

Curve Fitting

Multiple time waveforms or frequency spectra defined for different DOFs (motions at different points in different directions) are needed to extract EMA mode shapes from experimental data.

An **Operating Deflection Shape (ODS)** is the data at the same sample from ***two or more time*** waveforms or frequency spectra that characterize the vibration of a machine or structure.

ODS data over a span of time or frequency samples is then ***"curve fit"*** to estimate the modal parameters of each resonance [6]. The curve fitting process involves matching an ***analytical parametric model*** to the experimental data, usually in a ***"least-squared-error"*** sense. In other words, the unknown modal parameters of the analytical model are estimated in a manner which minimizes the difference between the experimental data and the analytical model over a band of samples.

EXPANDING EXPERIMENTAL DATA

In this paper, it will be shown with two examples how analytical FEA mode shapes can be used to ***"expand"*** order-based ODS's and experimental time waveforms to include DOFs that were not determined experimentally [4]. Unlike the ***"curve fitting"*** used to extract EMA mode shapes, in this process the unknown ***participation of each mode shape*** in the experimental data is determined by a different ***"least-squared-error"*** process**.** The participation of each mode is then used to ***"expand"*** the data to include the original DOFs plus extra DOFs calculated from the DOFs of the mode shapes.

The following examples are used to illustrate shape decomposition & expansion,

1. Order-based ODS's of a rotating machine are decomposed & expanded from a few experimentally derived DOFs to 1000's of DOFs
2. The sinusoidal response time waveforms of a structure are decomposed & expanded to include twice as many DOFs as the original data

These examples illustrate the combined use of FEA mode shapes and experimental data to create more robust structural dynamic models, which in turn can be used in further structural dynamics applications.

In these examples only FEA mode shapes are used to decompose & expand the experimental data.

EXPANDING ORDER-BASED ODS's

In a rotating machine, the dominant forces are applied at multiples of the machine running speed, called orders. An ***order-based ODS*** is assembled from the ***peak values*** at one of the orders in a set of output-only frequency spectra. Auto spectra, Cross spectra, or ODS FRFs can be calculated from output-only data that is acquired while the machine is running. When displayed in animation on a 3D model of the machine, an order-based ODS is a convenient way of visualizing distributed vibration levels caused by unmeasured internal forces. These distributed vibration levels can be represented as a summation of resonances and can be used for monitoring the health of the machine.

In this example, it is shown how ***modes participate differently*** at different operating speeds in an order-based ODS of a rotating machine. The ***modal participation*** is then used to ***expand*** the order-based ODS's, and they become a valid representation of the ODS for both the ***measured and un-measured*** DOFs of the machine.

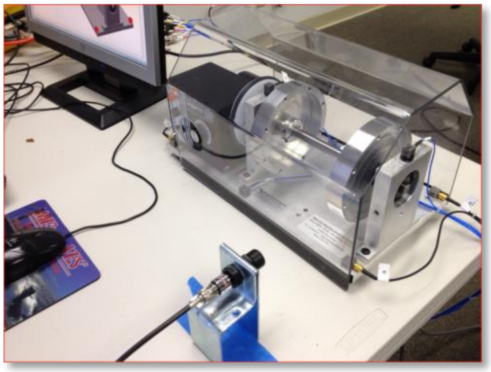

Figure 1. Variable Speed Rotating Machine

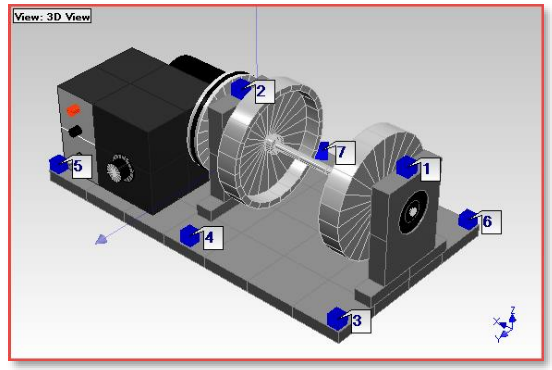

Figure 2. Model of the Rotating Machine Showing Accelerometers

Experimental ODS data was obtained from the rotating machine shown in Figure 1. In Figure 2 the model shows the eight tri-axial accelerometers used to acquire operating data, one on each bearing block and three on each side of the base plate. When post-processed, the accelerometer data provided ODS's with 24 DOFs, thus defining 3D motion on the machine at eight points.

The machine was supported on ***four rubber mounts*** (one under each corner), so its ***rigid body motion*** participated significantly in its ODS's. ***Six rigid-body*** and ***four flexible-body*** FEA mode shapes were obtained from an FEA model of the base plate & bearing blocks. These 10 mode shapes were used to expand the experimental ODS's.

One of the ***rigid-body*** mode shapes is shown in Figure 3, and a ***flexible-body*** mode shape is shown in Figure 4. These FEA mode shapes have 1938 DOFs in them, including the 24 DOFs of the eight accelerometers.

A comparison display of a 1938-DOF expanded ODS versus its original 24-DOF experimental ODS is shown in Figures 5 to 7. Each of these Figure contains an expanded versus an original ODS at a different machine speed. The expanded ODS is displayed on the left and the 24-DOF ODS is displayed on the right in each Figure. The participation bar charts on the right side of each Figure show the participation of each FEA mode shape in each ODS. The bars show that the ***rigid-body modes dominate*** each ODS, but they participate differently at each speed.

The SDI (shape difference indicator [3]) is also shown in the shape comparison display. SDI values range between 0 and 1. A ***value of "1"*** means the two shapes are ***identical***. The high SDI values in Figures 5 to 7 indicate that at each speed, the expanded ODS ***closely matches*** the 24-DOF ODS; (**0.87 at 985 RPM**), (**0.91 at 1440 RPM**), (**0.94 at 2280 RPM**).

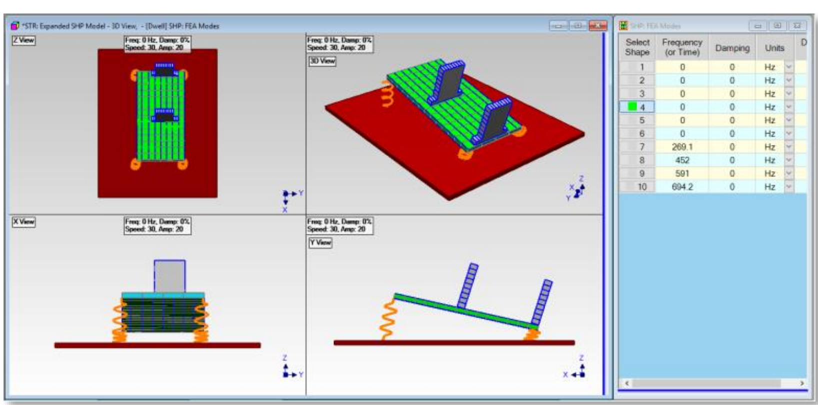

Figure 3. Rigid-Body Mode of Base Plate & Bearing Block.

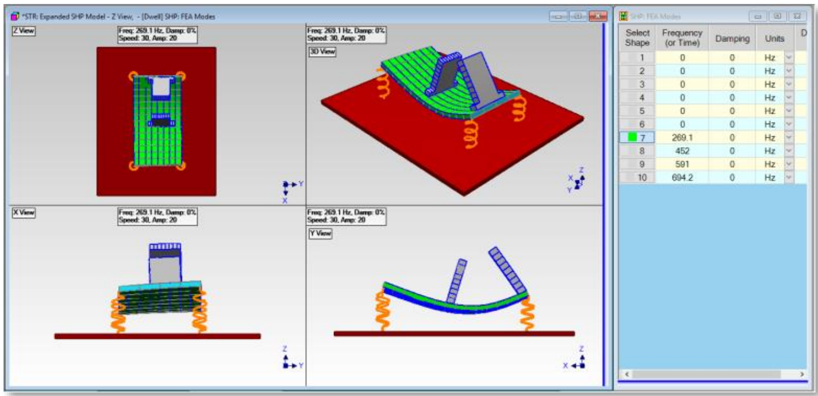

Figure 4. First Flexible Mode of Base Plate & Bearing Block.

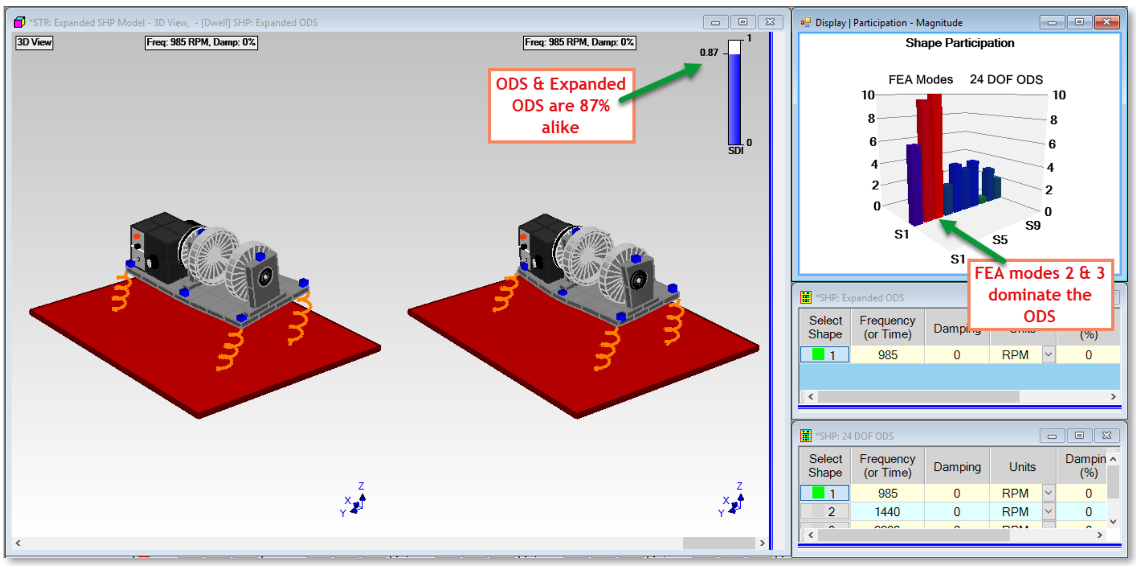

Figure 5. ODS Expansion at 985 RPM.

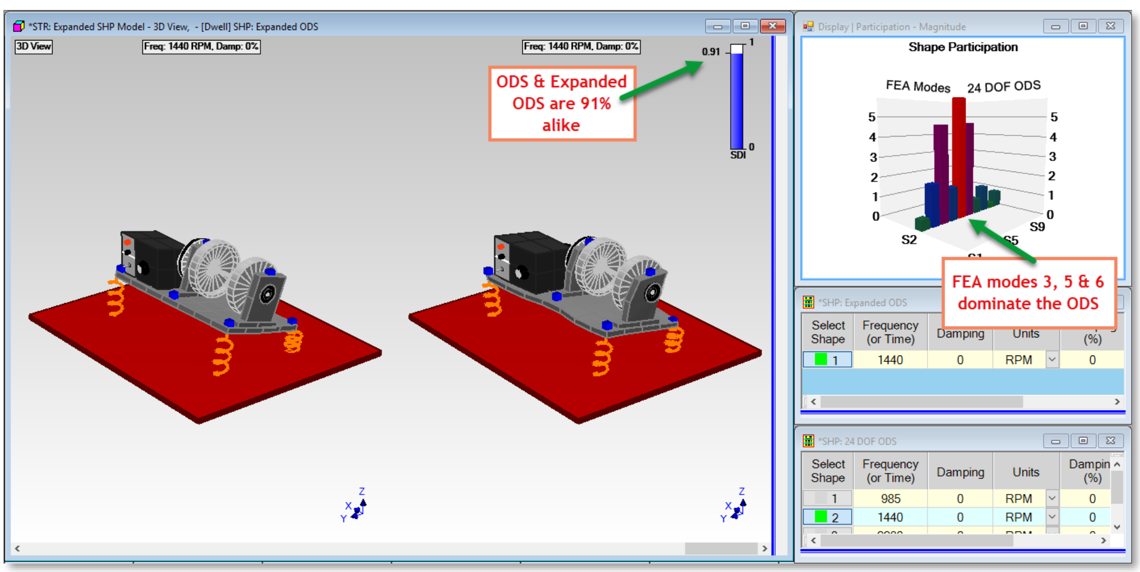

Figure 6.ODS Expansion at 1440 RPM.

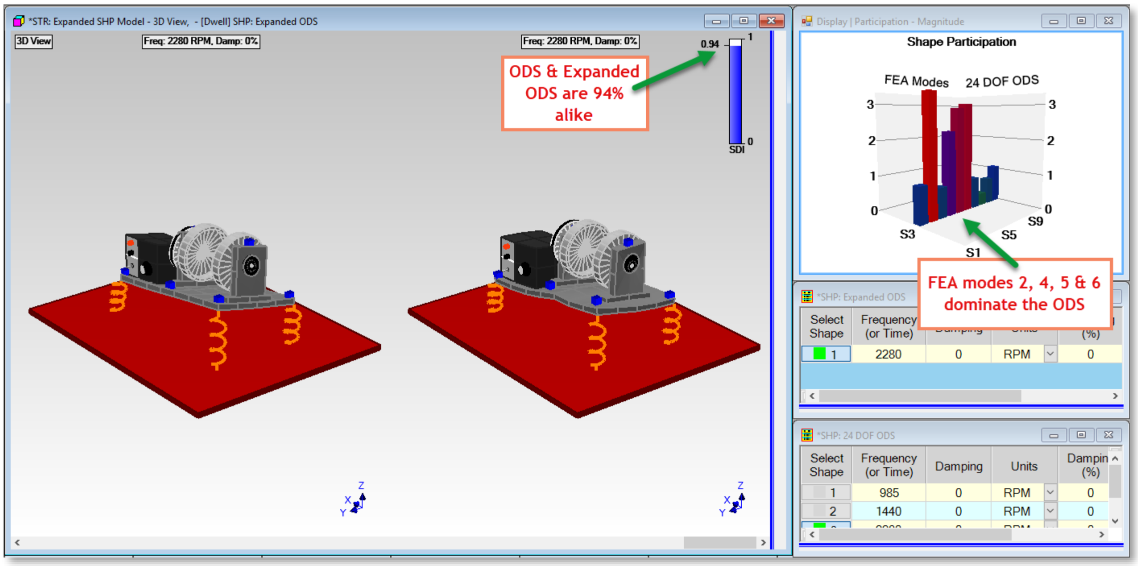

Figure 7. ODS Expansion at 2280 RPM.

Complex valued modal participations were used to curve fit the normal FEA mode shapes to the complex experimental ODS data.

The ***high SDI values*** in Figures 5 to 7 verify that even complex valued ODS data can be ***accurately represented as a summation of normal mode shapes***.

EXPANDING SINUSOIDAL RESPONSES

In this second example, MIMO modeling & simulation was used to calculate simulated responses to two sinusoidal excitation forces applied to the Jim Beam structure shown in Figure 8. The forces were applied at points 5 & 15 in the Z-direction on the top plate of the beam, that is at DOFs 5Z & 15 Z. Two cases were simulated,

1. Two 500 Hz In-Phase sinusoidal excitation forces
2. Two 500 Hz Out-of-Phase sinusoidal excitation forces

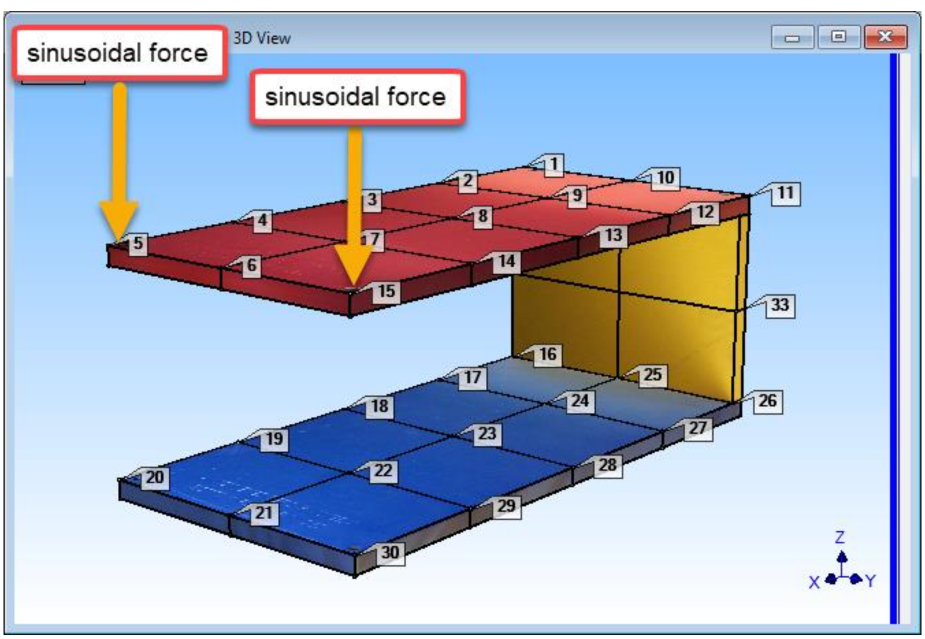

Figure 8. Sine Wave Excitation at Points 5 &15

The sinusoidal In-Phase and Out-of-Phase force time waveforms are shown in Figures 9A & 9B. Each time waveform contains ***5000 samples***. The cursor values in Figure 9A show that the two forces are In-Phase with one another. The cursor values in Figure 9B show that the two forces are Out-of-Phase with one another.

Four of the EMA mode shapes with frequencies ***surrounding 500 Hz*** are displayed in Figures 10A to 10D. These modes are expected to participate in the response of the Jim Beam at 500 Hz. The 493 Hz mode is nearest to 500 Hz, so it ***should dominate*** the sinusoidal response. We will see that in fact it does dominate the response, but depending on the phases of the excitation forces, ***other modes will also participate*** in the response.

The MAC values between 10 of the EMA & FEA mode shapes of the beam are shown in Figure 11. MAC ***values above 0.9*** indicate that two mode shapes are ***co-linear***. That is, two shapes ***lie on the same straight line*** (have the ***same shape***) but they may be scaled differently.

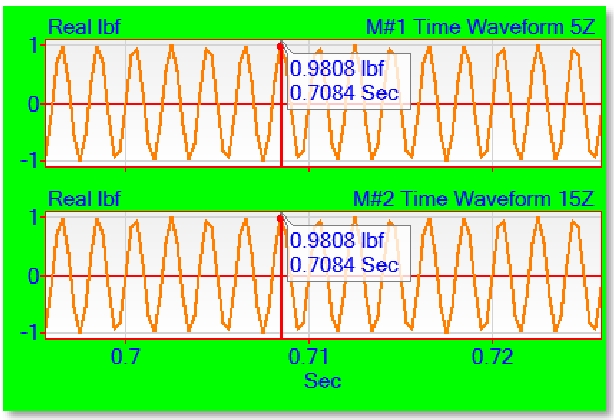

Figure 9A. In-Phase Excitation at DOFs 5Z & 15Z

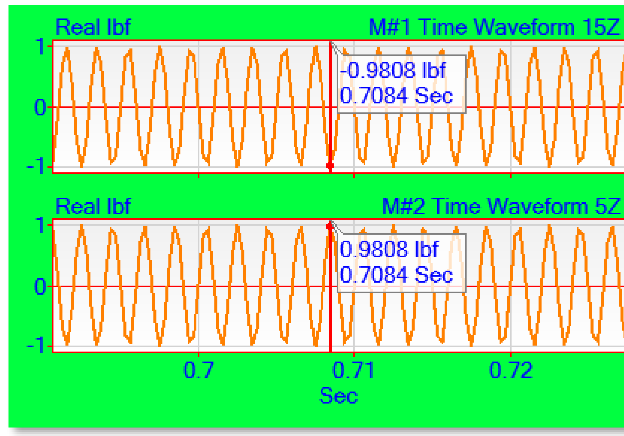

Figure 9B. Out-of-Phase Excitation at DOFs 5Z & 15Z

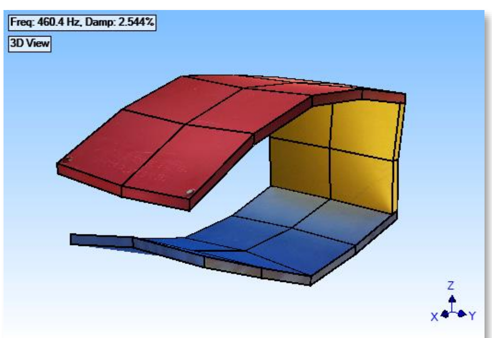

Figure 10A. EMA Mode Shape at 460 Hz

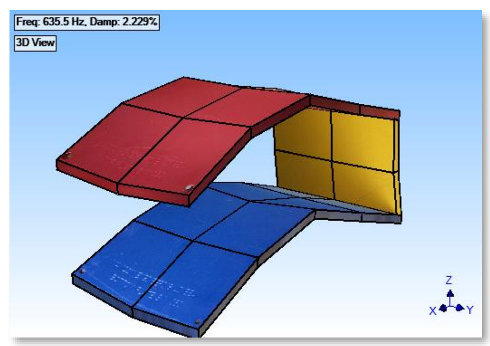

Figure 10C. EMA Mode Shape at 636 Hz

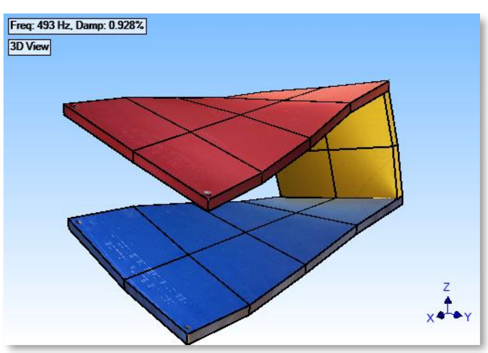

Figure 10B. EMA Mode Shape at 493 Hz

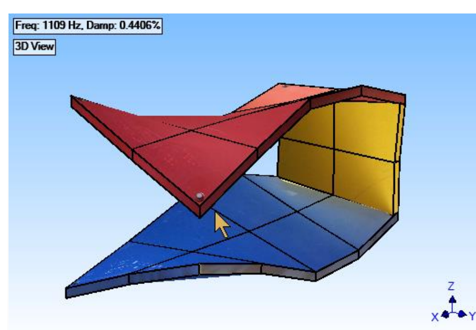

Figure 10D. EMA Mode Shape at 1109 Hz

The frequencies of the FEA & EMA modes are listed in Figures 12A & 12B. Notice that the frequency of each FEA mode shape is ***much less*** than the frequency of the corresponding EMA mode shape with which it has a high MAC value. The frequency differences will not matter however, because only the FEA mode shapes are used to decompose & expand the time waveforms.

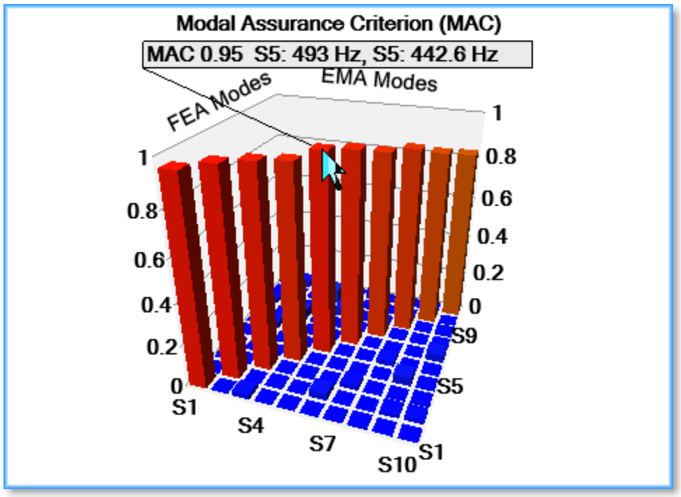

Figure 11. MAC values between EMA & FEA modes

*SHP: FEA Modes

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	Label
1	143.8	0	Hz	0	Mode 8 143.8134
2	203.7	0	Hz	0	Mode 9 203.7122
3	310.6	0	Hz	0	Mode 10 310.624
4	414.4	0	Hz	0	Mode 11 414.401
5	442.6	0	Hz	0	Mode 12 442.600
6	583.4	0	Hz	0	Mode 13 583.444
7	1002	0	Hz	0	Mode 14 1002.22
8	1091	0	Hz	0	Mode 15 1090.80
9	1168	0	Hz	0	Mode 16 1168.27
10	1388	0	Hz	0	Mode 17 1388.21

Figure 12A. FEA Modal Frequencies

SHP: EMA Modes

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	Label
1	165	3.052	Hz	1.85	Mode#01 165Hz
2	224.6	6.618	Hz	2.945	Mode#02 225Hz
3	347.9	5.113	Hz	1.47	Mode#03 348Hz
4	460.4	11.72	Hz	2.544	Mode#04 460Hz
5	493	4.575	Hz	0.928	Mode#05 493Hz
6	635.5	14.17	Hz	2.229	Mode#06 635Hz
7	1109	4.885	Hz	0.4406	Mode#07 1.11kHz
8	1211	7.052	Hz	0.5824	Mode#08 1.21kHz
9	1323	7.164	Hz	0.5415	Mode#09 1.32kHz
10	1557	16.59	Hz	1.066	Mode#10 1.56kHz

Figure 12B. EMA Modal Frequencies

Response to In-Phase Forces

In Figure 13, the simulated response of the Jim Beam structure to two In-Phase forces is displayed on the left. These responses were calculated using the EMA mode shapes, which were used to synthesize FRFs, which then were multiplied by the Fourier spectra of the forces to obtain Fourier spectra of the response DOFs of the Jim Beam. The Fourier spectra of the responses were then inverse Fourier transformed to obtain the response time waveforms displayed on the ***upper right*** of Figure 13. The MIMO modeling and simulation process is depicted in Figure 14.

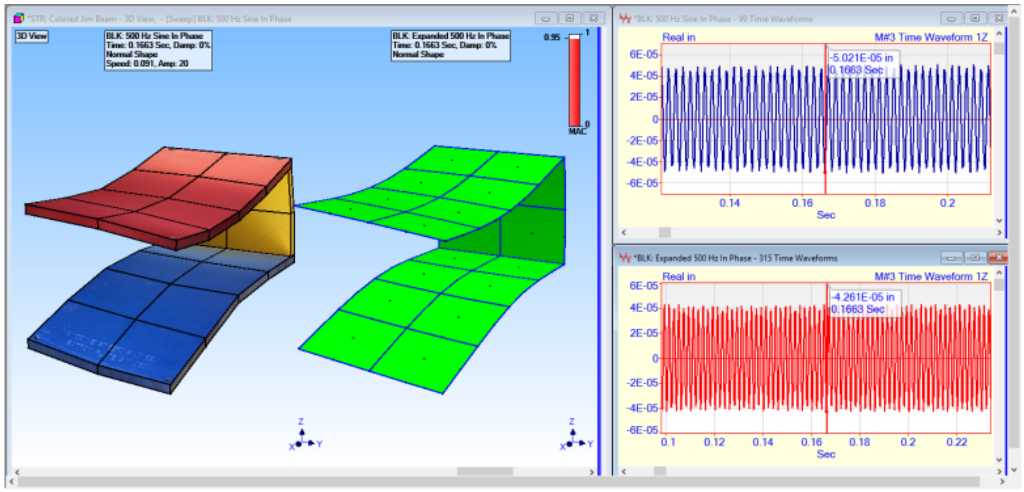

Figure 13. Simulated vs. Expanded Response to In-Phase Forces

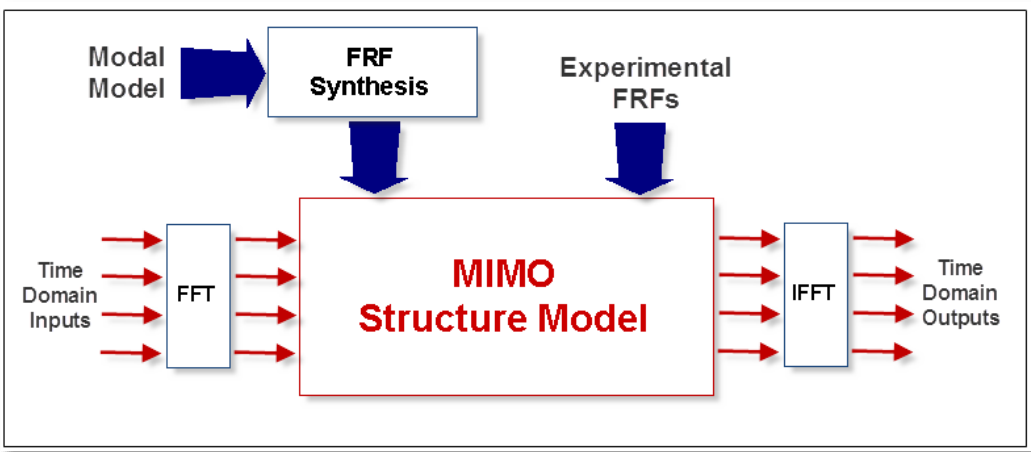

Figure 14. MIMO Modeling & Simulation

The FEA mode shapes were then curve fit to the simulated responses and used to expand them from 99 to 315 DOFs. The expanded response time waveforms are displayed on the ***lower right*** in Figure 13. The time-based ODS at the ***current Line cursor position*** in the simulated responses is displayed on the ***left-hand model***, and the expanded ODS from the ***same cursor position*** is displayed on the ***right-hand FEA model***.

The MAC bar between the two time-based ODS's (simulated & expanded) is also displayed in the ***upper right corner*** of the ODS display. This ***MAC value of 0.95*** indicates that the two ODS's are ***nearly co-linear***, meaning that they are ***essentially the same shape***.

In Figure 15, the two blocks of response time waveforms are correlated using MAC & SDI [3]. The ***upper graph is the MAC values*** between each sample of the simulated time waveforms versus the same sample of the expanded time waveforms. The ***lower graph is the SDI values*** between the same samples in each data block of time waveforms. Both MAC & SDI indicate that the data at the same sample was ***nearly identical*** in both data blocks**.** Not all samples are shown, but this was true ***for all 5000 samples*** of both time waveforms.

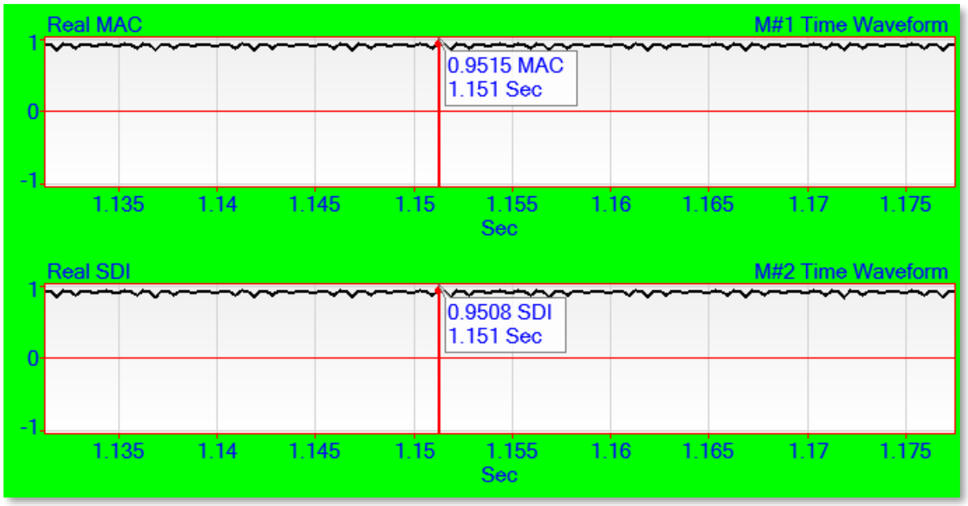

Figure 15. Correlation of Simulated & Expanded Time Waveforms

Modal Participation in Responses to In-Phase Forces

Figures 16A & 16B show the participation of the EMA & FEA mode shapes in two time-based ODS's taken from the simulated & expanded waveforms shown in Figure 13. The simulated responses were calculated using MIMO simulation and the EMA mode shapes, while the expanded responses were calculated by curve fitting the simulated responses with FEA mode shapes. These results show that both the EMA & FEA mode shapes participate in the simulated & expanded ODS's in a similar way. ***Mode #5 dominates*** each ODS, but ***modes #4, #6 & #7 also participate*** in each ODS.

The mode shapes of modes **#4 & #6 *are in-phase*** with the two In-Phase excitation forces at DOFs 5Z & 15Z, so they also participate in the ODS's. However, the mode shapes of modes **#5 & #7** are ***out-of-phase*** with the two In-Phase excitation forces at DOFs 5Z & 15Z. Yet both of these mode shapes also participate in the ODS's. How can this be true?

The only explanation for the participations of **modes #5 & #7** shown in Figures 16A & 16B is that even though these two mode shapes are ***out-of-phase*** with the two In-Phase excitation forces at DOFs 5Z & 15Z, these mode shapes ***sum together*** in a manner which contributes to the ODS's, with **mode #5** still dominating both ODS's.

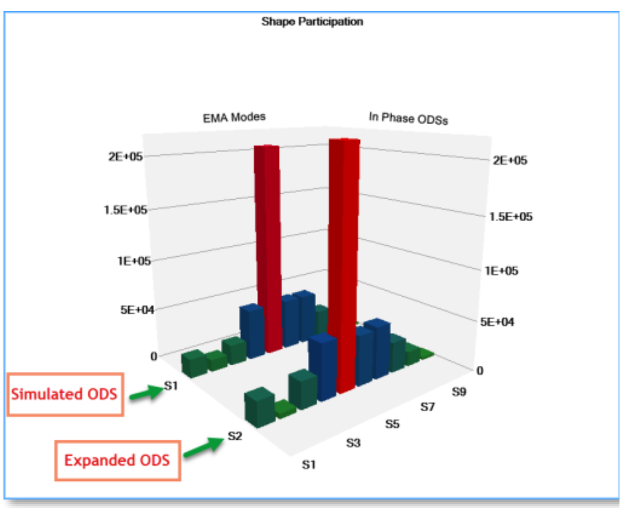

Figure 16A. EMA mode shape participation in the ODS's

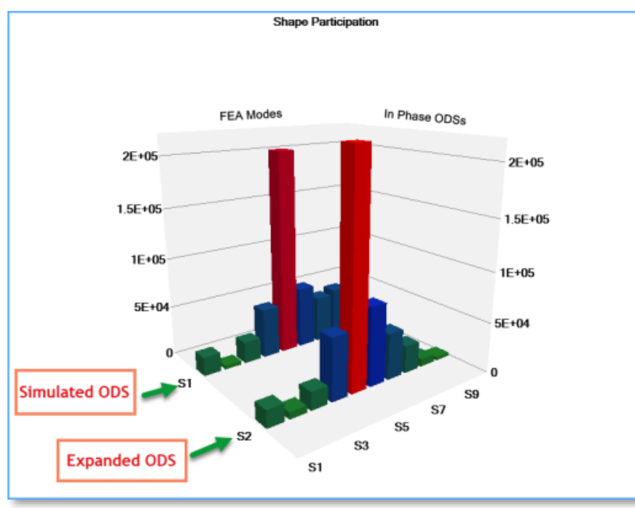

Figure 16B. FEA mode shape participation in the ODS's

Response to Out-of-Phase Forces

In Figure 17, the simulated response of the Jim Beam structure to two Out-of-Phase forces is displayed on the left. These responses were calculated using the MIMO simulation depicted in Figure 14 and the EMA mode shapes, which were used to synthesize FRFs, which then were multiplied by the Fourier spectra of the forces. The Fourier spectra of the responses were then inverse Fourier transformed to obtain the response time waveforms displayed on the ***upper*** right of Figure 17.

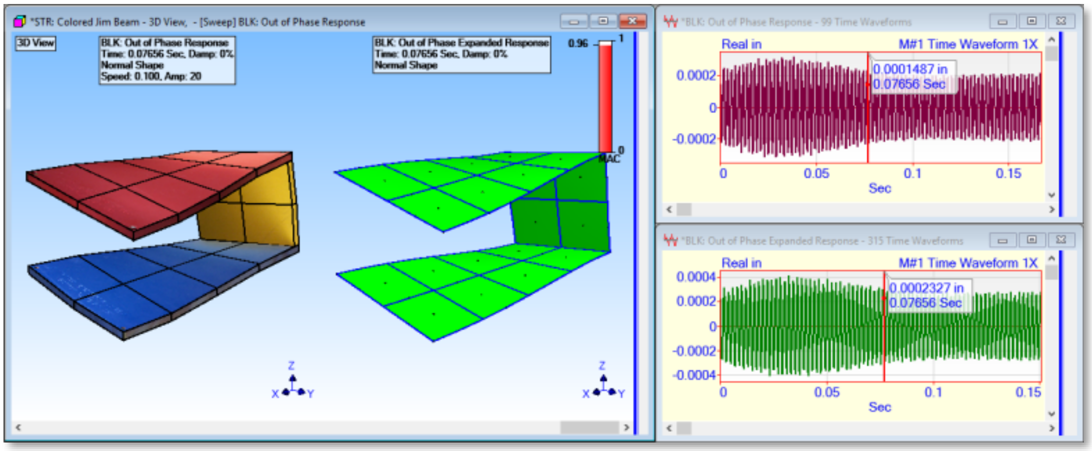

Figure 17. Simulated vs. Expanded Response to Out-of-Phase Forces

The FEA mode shapes were then used to expand each sample of the simulated response time waveforms. Those expanded responses are displayed on the ***lower right*** in Figure 17. The time-based ODS at the ***current Line cursor position*** in the simulated responses is displayed on the ***left-hand model***, and the ODS from the same cursor position in the expanded responses is displayed on the ***right-hand FEA model***.

The MAC bar between the two time-based ODS's (simulated & expanded) is also displayed in the ***upper right corner*** of the ODS display. This ***MAC value of 0.96*** indicates that the two ODS's are ***nearly co-linear***, meaning that they are ***essentially the same shape***.

In Figure 18, the two blocks of response time waveforms are correlated using MAC & SDI [3]. The ***upper graph is the MAC values*** between each sample of the simulated time waveforms versus the same sample of the expanded time waveforms. The ***lower graph is the SDI values*** between the same samples in each data block of time waveforms. Both MAC & SDI indicate that the data at the same sample was ***nearly identical*** in both data blocks**.** Not all samples are shown, but this was true ***for all 5000 samples*** of both time waveforms.

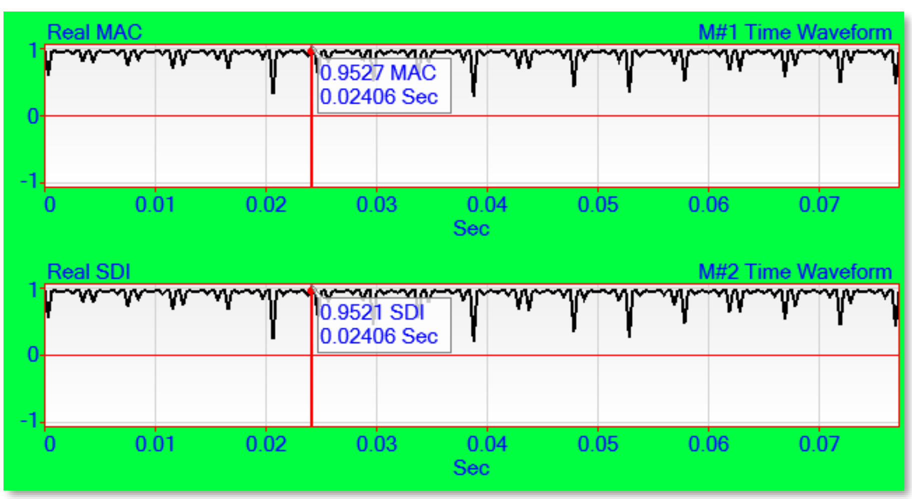

Figure 18. Correlation of Simulated & Expanded Time Waveforms

Modal Participation in Responses to Out-of-Phase Forces

Figures 19A & 19B show the participation of the EMA & FEA mode shapes in two time-based ODS's taken from the simulated & expanded response data blocks shown in Figure 17. The simulated responses were calculated using the EMA mode shapes, and the expanded responses were calculated by curve fitting the 10 FEA mode shapes to the simulated responses. These results clearly show that both the EMA & FEA mode shapes participate in the response ODS's in a similar way. In this case, because ***its mode shape is in-phase*** with the two Out-of-Phase excitation forces at DOFs 5Z & 15Z, **mode #5 *dominates the response,*** and there is ***very little participation*** from the other modes.

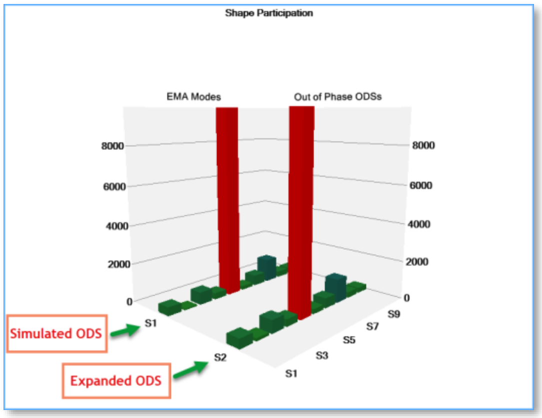

Figure 19A. EMA mode shape participation in the ODS's

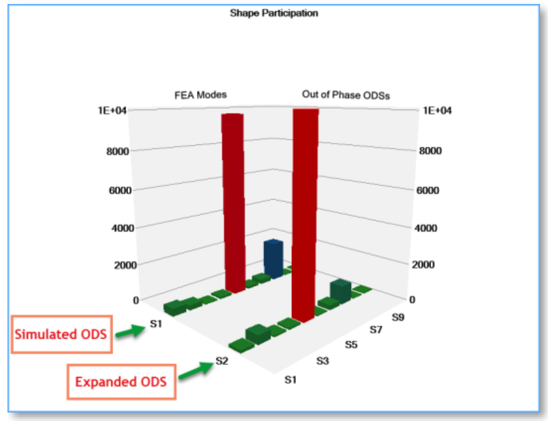

Figure 19B. FEA mode shape participation in the ODS's

CONCLUSIONS

In a previous paper [1] it was also shown how mode shapes can be used to decompose & expand a set of FRFs, Cross spectra, and ODS FRFs. In those cases, it was demonstrated that frequency domain vibration functions can be accurately decomposed into a ***summation of resonance curves*** by curve fitting them with a set of mode shapes, one frequency at a time. These participations of mode shapes at each frequency were then used to construct an ***expanded set of measurements*** using all of the DOFs of the mode shapes.

In this paper, the same procedure was used to decompose and then to expand several order-based ODS's, and to expand simulated sinusoidal response time waveforms. Both of these examples illustrated how the free-free FEA mode shapes of a machine or structure can be used to decompose & expand data according to the following law,

Fundamental Law of Modal Analysis (FLMA): All vibration is a ***summation*** of mode shapes.

This decomposition & expansion using FEA mode shapes offers some important advantages,

- Real-world time or frequency vibration data can be ***accurately curve fit*** using FEA mode shapes
- Normal mode shapes derived from an FEA model with ***free-free boundary conditions*** and ***no damping*** can be curve fit to real-world vibration data that includes ***real-world boundary conditions and damping***
- FEA normal mode shapes can be used to curve fit complex valued vibration data

It usually requires a great deal of skill and effort to modify an FEA model and its boundary conditions so that its modal frequencies and mode shapes accurately match EMA modal frequencies and mode shapes. Adding accurate damping to an FEA model is usually so difficult that damping is left out of the model altogether. The approach used here circumvents both of these difficulties.

Experimental vibration data ***always includes*** both ***real world boundary conditions*** and ***real-world damping***.

It was shown in the first example that complex ODS data can be accurately decomposed & expanded using the ***normal mode shapes*** of the bearing blocks & base plate of a rotating machine. These normal mode shapes were derived from a relatively simple FEA model with free-free boundary conditions and no damping.

Normal modes can be used to expand ***complex vibration data*** because the ***modal participation factors*** used to decompose & expand the complex data are also complex valued.

In the second example, normal mode shapes were used to decompose & expand real valued time waveforms. Curve fitting FEA mode shapes to this data yielded an unexpected participation of the mode shapes when two In-Phase sine wave forcing functions were applied to the top two corners of the Jim Beam structure.

The intuitive belief that the mode with natural frequency closest to a sinusoidal forcing frequency should dominate the vibration response turned out to be true, but it was found that other modes also participate in a response based on their mode shapes, not their frequencies.

Applying this approach to any vibration data that is acquired directly from a machine or structure can provide the necessary information for continuously monitoring its resonant properties. Furthermore, this decomposition & expansion capability is useful for creating the resonant properties of ***un-measured or un-measureable*** portions of a machine or structure.

This combined use of a ***simplified FEA model*** and a ***small amount of experimental data*** means that less time & expense are required to obtain meaningful data for use in machinery & structural health monitoring, and for more quickly diagnosing and mitigating noise & vibration problems.

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